C A R I B B E A N   E X A M I N A T I O N S   C O U N C I L

REPORT ON CANDIDATES' WORK IN THE CARIBBEAN ADVANCED PROFICIENCY EXAMINATION MAY/JUNE 2009

APPLIED MATHEMATICS

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CARIBBEAN ADVANCED PROFICIENCY EXAMINATION

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INTRODUCTION

The revised Applied Mathematics syllabus was examined this year for the second time. The revised Applied Mathematics syllabus is a two-Unit course comprising three papers. Paper 01, multiple choice items, and Paper 02, essay questions, were examined externally, while Paper 03 was examined internally by the class teachers and moderated by CXC. Contributions from Papers 01, 02 and 03 to each Unit were 30 per cent, 50 per cent and 20 per cent respectively.

Unit 1: Statistical Analysis, consists of Collecting and Describing Data, Managing Uncertainty and Analysing and Interpreting Data.

Unit 2: Mathematical Applications, consists of Discrete Mathematics, Probability and Distributions and Particle Mechanics.

GENERAL COMMENTS

For Unit 1, three hundred and seventy candidates wrote the 2009 paper and four wrote the Alternative to SBA paper, namely Paper 03/2. For Unit 2, 165 candidates wrote the 2009 paper and two wrote the Alternative to SBA paper, namely Paper 03/2.

Approximately 85 per cent of the candidates registered for the Unit 1 Statistical Analysis, and 93 per cent of the candidates registered for the Unit 2, Mathematical Applications, obtained acceptable grades, Grades I – V. The standard of work seen from most of the candidates in this examination was generally good.

In Unit 1, candidates appeared to be well prepared in Describing and Collecting Data and in Managing Uncertainty while Analysing and Interpreting Data posed problems for many candidates.

In Unit 2, again this year, candidates appeared to be well prepared in Discrete Mathematics and Probability and Distributions and generally answered the questions well, with most candidates attempting all of the questions in these sections. There were a number of candidates who appeared to be well prepared in all three modules. Once again, however, Particle Mechanics was not generally well answered. Overall, there was a large number of areas of strength displayed by many candidates. As in previous years, candidates need to pay more attention to their algebraic manipulation.

UNIT 1

Statistical Analysis

Paper 01

The performance on the 45 multiple choice items on Paper 01 produced a mean of 46 out of 90 with scores ranging between zero and 88.

DETAILED COMMENTS

Paper 02

Module 1

Question1

This question tested candidates' ability to:

(a)	Distinguish between qualitative and quantitative data, discrete and continuous data.
(b) (i)	Explain how a random sample can be selected using the method of random numbers.
(ii)	Select a random sample using a table of random numbers.
(c) (i)	Distinguish between a population and a sample.
(ii) (iii)	State the population of interest in a given survey and identify sampling techniques used in a given situation.
(iv)	Perform calculations involving stratified random sampling.
(v)	Calculate sector angles of a pie chart from given categories.

This question was attempted by approximately 99 per cent of the candidates, of whom 70 per cent gave satisfactory responses.

Part (a) of this question was very well done, with only a few candidates failing to identify age as a continuous random variable.

Part (b) seemed to have posed a great deal of difficulty for most candidates as they were unable to explain clearly how to obtain a sample of 10 accounts using the method of random numbers. In fact, many candidates tried to explain how to obtain a sample of 10 accounts using the lottery method of sampling.

In Part (c) (i), some candidates gave a geographical definition rather than a statistical definition for population. However, the majority were able to relate the sample to the population.

In Part (c) (ii), candidates correctly stated the population of interest for this survey as the students at the school, while in Part (c) (iii) candidates correctly identified the sampling technique as stratified random sampling.

In addition, in Part (c) (iv) candidates correctly calculated the number of students in the sample from the fourth year group as 9 using stratified random sampling.

Those who attempted Part (c) (v) correctly calculated the angle for each response category using. $\frac{given\ value}{50} \times 360^{\circ}$

Question 2

This question tested candidates' ability to:

(a)	Assess the appropriateness of the kind of chart to best illustrate statistical data.
(b) (i) (ii)	Construct a frequency distribution table and bar charts.
(c) (i)	Assess a given situation in which sampling techniques are more appropriate and why.
(ii)	Perform calculations involving stratified random sampling.
(d) (i) (ii)	Calculate the mean, median, mode and interquartile range for ungrouped data.
(iii)	Construct a box and whisker diagram.
(iv)	Interpret the shape of the distribution in terms of skewness.

Part (a) was exceptionally well done. Only a few candidates who failed to follow the instructions lost marks.

Part (b) (i) was exceptionally well done, with approximately 99 per cent of the candidates being able to construct the frequency distribution table from the given data.

In Part (b) (ii), a few candidates illustrated the frequency distribution from Part (b) (i) as a histogram rather than a bar chart.

In Part (c), some candidates had difficulty explaining why a stratified sampling method might be a better technique for choosing the sample rather than using a simple random sampling method. Nevertheless, these candidates were able to select 25 employees by calculating the number of employees in each of the four stated categories using stratified random sampling.

Part (d) (i) was well done by the majority of candidates, who were able to state the mode and median of the distribution.

In Part (d) (ii), most candidates were able to correctly calculate the mean number of hours that students slept as 7.04, but a few candidates used an incorrect position for the lower and upper quartile and hence obtained an incorrect value for the inter-quartile range.

In Part (d) (iii), most candidates were able to draw an accurate box and whisker diagram, with the exception of about 3 per cent of the candidates who drew incorrect whiskers and used incorrect quartiles.

In Part (d) (iv), many candidates correctly identified the shape of the distribution as positively skewed, while the others incorrectly identified the shape of the distribution as either symmetrical or negatively skewed.

Module 2

Question 3

This question tested candidates' ability to:

(a) Calculate $P(A \cup B)$, $P(A \cap B)$ and P(A|B).

(b) Construct and use a probability distribution table for discrete random variables to obtain probabilities.

(c) Construct a tree diagram.

This question was generally well done.

In Part (a), most candidates correctly defined *P(A|B)* = $\frac{P(A \cap B}{P(B)})$ and $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ and were able to obtain the correct response of 0.5 for each of the above. A few candidates defined the P(A|B) and $P(A \cup B)$ incorrectly and so were unable to get the correct result.

Part (b) was generally well done, with candidates being able to calculate $P(G \cap H)$ as 0.28 and $P(G \cup H)$ as 0.82. There were those candidates who did not use that fact that *G* and *H* are independent events and so experienced great difficulty when trying to calculate $P(G \cap H)$ and $P(G \cup H)$.

In Part (c) (i), most candidates were able to illustrate the given information on a well-labelled tree diagram. However, a few candidates placed incorrect labels and probabilities on the branches of the tree diagram.

Part (c) (ii) was generally well done, with candidates being able to use the tree diagram to calculate probabilities, but a few candidates failed to use the information provided by the tree diagram.

Part (c) (iii) was generally well done by the candidates who performed well in the previous parts. Of the 95 per cent of the candidates who attempted this question, 70 per cent gave satisfactory responses.

Part (d) was generally well done by most candidates. The majority of candidates were able to calculate the value of k as 0.2. For (d) (ii), some candidates applied a different approach to find the expectation but still arrived at the correct answer 6. For part (d) (iii), a few candidates were unable to find the cumulative probabilities, 0.4 and 0.3, to give 0.7 as the required solution.

Question 4

This question tested candidates' understanding of the

(a) Binomial distribution –

(i) - (iv) the conditions for which discrete data can be modelled by the binomial distribution,

(b) (i) - (ii) the notation for the binomial distribution and its probabilities, the expected value and the variance of a binomial distribution and to use the binomial distribution to calculate probabilities.

(c) The Normal distribution -

Determine probabilities from tabulated values of the standard normal distribution Z ~ N(0, 1);

Standardising a value of the normal distribution

(d) Solve problems involving probabilities of the normal distribution using *z*-scores.

Generally, this question was fairly well done by most candidates, and did not appear to be too difficult.

In Part (a), a few candidates did not recognize the binomial model.
Part (b) (i) was well done by the majority of candidates. Many of them were able to use E(X) = np, though some candidates further expressed the answer as a whole number.

For Part b (ii), though many candidates knew the binomial formula for finding probability, many of them confused the probability values of *p* and 1 – *p*. About 60 per cent of the candidates who attempted this part did it correctly. However, many candidates did not correctly interpret the phrase "***at most 2*" .**

In Part (c), there were indications that candidates could read the value from the table, but they experienced problems writing the steps taken to get to the solution. In Part (d), candidates generally had problems writing the standardization correctly as $\frac{x-\mu}{\sigma}$. Many candidates wrote μ- x instead of $x - \mu$, and many candidates also used the variance, $\sigma^2 = 16$, instead of the standard deviation, $\sigma = 4$ in the standardizing formula. Most candidates were able to convert P(Z > 1.25) to 1 – P(Z < 1.25), and also to use φ correctly, φ(- 1.5) = 1 – φ(1.5). However, many candidates did not convert the probability to a percentage as required in the question.

Module 3

<u>Question 5</u>

This question tested the candidates' understanding of:

(a) confidence intervals - description and how to construct

(b) unbiased estimates of parameters

(c) using the normal distribution as an approximation to the binomial.

Generally, this question was very poorly done. Candidates had problems stating definitions or reasons, and explaining concepts.

In Part (a), candidates were asked to give an explanation of the term 95 per cent confidence interval in the context of the population mean. Though most candidates knew that this had something to do with an interval, they could not adequately state anything about the interval. Furthermore, many candidates did not say that the interval contained μ.

In Part (b) (i), less than 50 per cent of the candidates attempted this question, and many of them did not use the correct formula.

Most of the candidates who attempted Part (b) (ii) used the standard deviation of the sample, rather that the unbiased estimate that they were asked to calculate in Part (b)(i).

For Part (b) (iii), most of the candidates were able to write ``*increase the sample size*``, but many of them could not give a second method. Some candidates also confused *confidence level* with *significance level.* A few candidates were able to state that ``*increasing the sample size will be better*``, but not many of them could give a reason for this.

Part (b) (iv) was done well by those candidates who attempted this question.

Very few candidates attempted this Part (c) of the question. Many were unable to calculate the expected interval as 0.1 × 60 = 6.

In Part (d) (i), most of the candidates correctly stated that the Normal distribution would be used, but many of them could not give the parameters.

In Part (d) (ii), many candidates in calculating the probability did not use the correct standard error.

Many of them incorrectly standardized using $z = \frac{\mu - \bar{x}}{\sigma}$ instead of $z = \frac{\bar{x} - \mu}{\sigma}$.

Question 6

This question tested the candidates' understanding of:

(a) hypothesis testing using the chi squared distribution
(b) the application of the chi squared table

In Part (a) (i), many candidates interchanged the null and the alternative hypotheses.

In Part (a) (ii), candidates were knowledgeable of the procedures required to use the chi squared test to complete a contingency table by calculating expected values. In Part (a) (iii), many candidates were able to write $(r - 1)(c - 1)$ as the formula for finding the degrees of freedom, but some added rather than multiplied the quantities, or they substituted incorrect values for r and c. Many candidates wrote the critical value, rather than the critical region. Many candidates looked up the table value incorrectly, using the lower 5 per cent value rather than the upper 5 per cent value.

In Part (a) (iv), most candidates attempted to write a conclusion for the test. Using their null hypothesis, they were able to make a valid decision as to reject, or fail to reject the null hypothesis; however, many of them could not state a valid conclusion for the test.

In Part (b), candidates were required to generate the equation of a regression line. However, not many candidates attempted it, but for those who did, it was very well done yielding the correct answer y = 2 + x.

UNIT 2

Mathematical Applications

Paper 01

The performance on the forty-five multiple choice items on Paper 01 produced a mean of 56 out of 90 with scores ranging between 0 and 88.

DETAILED COMMENTS

Paper 02

Module 1

Question 1

This question tested candidates' ability to:

(a) Use the activity network in drawing a network model to model a real-world problem.

(b) Calculate the earliest start time, latest start time and float time.

(c) Identify the critical path in an activity network

This question was reasonably well done by most candidates, as 45 per cent of them scored between 20 and 25 marks (at least 80 per cent).

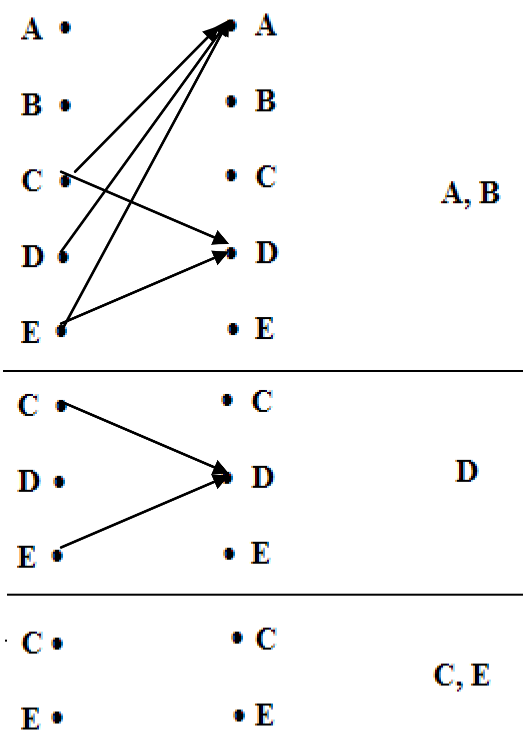
In Part (a), only a few candidates started out with an activity network algorithm. A number of candidates inserted additional edges and left out the start and finish nodes from the activity network.

In Part (b), many candidates did not calculate the earliest and latest start times correctly and so, their float times were incorrect also.

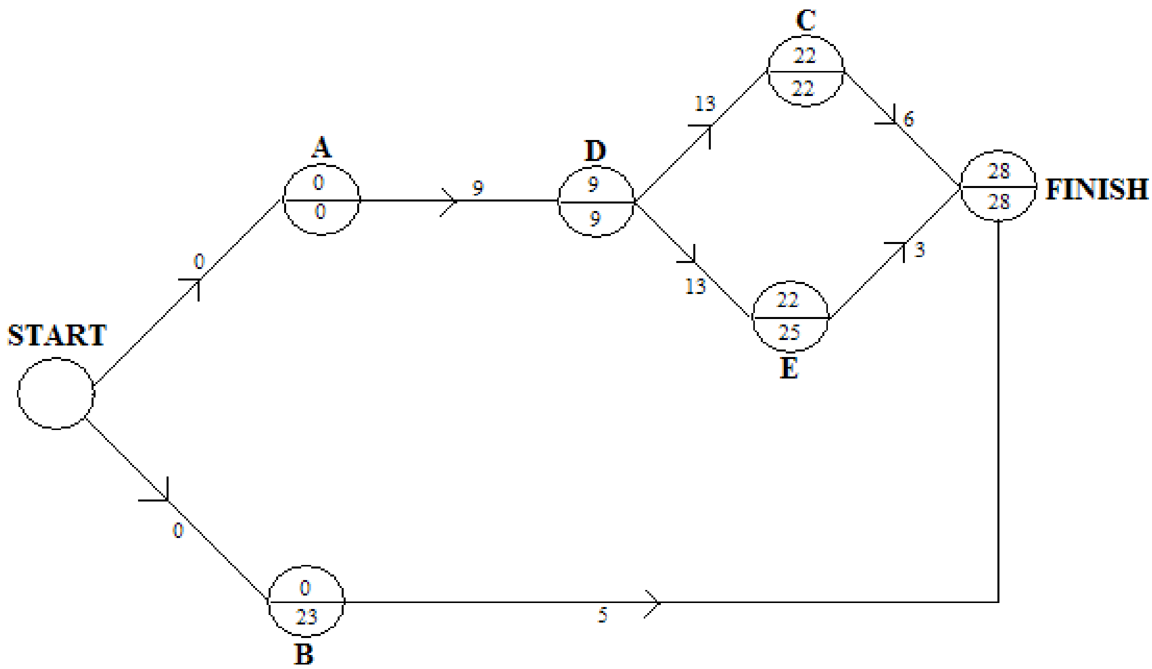
In Part (c), most candidates were able identify a critical path, although some answers were based on incorrect data from Part (b) and were therefore incorrect. Most of the candidates calculated the minimum completion time correctly.

Answers

(a) Activity Network Algorithm



Activity Network



(b)

Activity	Earliest Start Time	Latest Start Time	Float
A	0	0	0
B	0	23	23
C	22	22	0
D	9	9	0
E	22	25	3

(c) (i) Critical path: Start, A, D, C, Finish

(ii) Minimum completion time = 28 days

Question 2

This question tested candidates' ability to:

(a) Formulate simple propositions.

(b) (i – iii) Formulate compound propositions that involve conjunction, disjunctions and negations.

State the converse, inverse and contrapositive of implications of propositions.

(c) Use truth tables to:

i. Determine whether a proposition is a tautology or a contradiction.

ii. Establish the truth values of converse, inverse and contrapositive of propositions.

iii. Determine if propositions are equivalent.

(d) Identify the vertices and sequence of edges that make up a path.

(e) Determine the degree of a vertex.

(f) and (g) Identify the critical path in an activity network.

This question was well done.

In Part (a), a number of candidates confused the conditional with the bi-conditional.

In Part (b), most candidates confused the converse with the inverse and the contrapositive.

Part (c) was generally well done as most candidates were able to recognize a contradiction.

In Part (d), very few candidates were able to list all the correct paths.

Part (e) was generally well done as most candidates were able to state the degree of the vertex D correctly. A small number of students confused the degree of the vertex with the measurement of an angle (for example, 60^0).

Part (f) was well done. Some candidates, however, drew switching circuits instead of logic gates.

Part (g) was well done. A few candidates, however, confused the "AND" and the "OR" gates. Some incorrectly showed more than two inputs. The majority of the candidates were able to conclude correctly whether the circuits were equivalent.

Answers

(a) Tom works hard if and only if he is successful.

(b) (i) The inverse of $p \Rightarrow q$ is $\sim p \Rightarrow \sim q$

(ii) The converse of $p \Rightarrow q$ is $q \Rightarrow p$

(iii) The contrapositive of $p \Rightarrow q$ is $\sim q \Rightarrow \sim p$

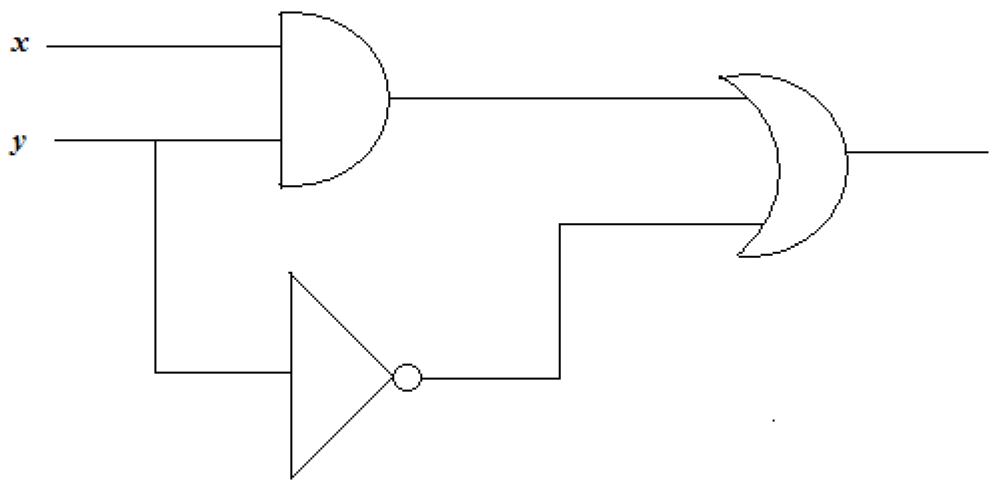
(a)

a	b	~b	$a \Rightarrow b$	$a \wedge (a \Rightarrow b)$	$[a \wedge (a \Rightarrow b)] \wedge \sim b$
T	T	F	T	T	F
T	F	T	F	F	F
F	T	F	T	F	F
F	F	T	T	F	F

The proposition is a contradiction.

(b) Paths: AE, ADE, ADCE, ABDE, ABCE, ABCDE, ADBCE, ABDCE

(c) The degree of vertex D is 4



(d) (i) $\sim(a \vee b)$ or $\overline{a+b}$

(ii) $\sim a \wedge \sim b$ or $\bar{a}.\bar{b}$

a	b	$a \vee b$	$\sim(a \vee b)$
1	1	1	0
1	0	1	0
0	1	1	0
0	0	0	1

~a	~b	$\sim a \wedge \sim b$
0	0	0
0	1	0
1	0	0
1	1	1

The circuits in (i) and (ii) are equivalent.

Module 2

Question 3

This question tested candidates' ability to:

(a) (i) Find the number of different arrangements using each letter once.

(ii) Find the probability that an arrangement starts with a consonant and then the consonants and vowels are arranged alternately.

(b) (i) (ii) Calculate and use probabilities associated with conditional, independent or mutually exclusive events.

(c) (i) Use the result $P(a < X \leq b) = \int_a^b f(x)\,dx = F(b) - F(a)$.

(ii) (iii) Calculate the expected value and the variance.

Overall, this question was reasonably well done.

Part (a) (i) was well done by candidates, while Part (a) (ii) presented difficulty for only a few candidates who gave the incorrect result of $4! + 3!$.

In Part (b), the majority of the candidates were able to obtain the correct solution without displaying a tree diagram. Most of the errors in Part (b) (ii) were the result of errors carried forward from Part (b) (i).

In Part (c), the majority of the candidates showed that they understood that they were expected to use integration since the random variable was continuous and not discrete. However, a few candidates incorrectly treated the random variable as discrete. In Part (c)(iii), most candidates expanded $(2-q)^3 = 6$ to get a cubic equation that they were unable to solve (one candidate actually used the

Newton-Raphson method to find an approximation to the root of the equation), rather than finding the cube root of the equation $(2-q)^3 = 6$ and so obtain the value for *q*.

Answers

(a) (i) number of arrangements = 7! = 5040

(ii) number of arrangements = $4! \times 3!$ = 144

Probability = $\frac{144}{5040} = \frac{1}{35}$

(b) (i) P(L) = $P(F \cap L) + P(G \cap L) + P(T \cap L) = 0.665$

(ii) P(T|L) = $\frac{P(T \cap L)}{P(L)} = \frac{0.045}{0.0665} = 0.677$

(c) (i) P(X > 1) = $\frac{1}{8}$

(ii) E(X) = ½, Var(X) = $\frac{3}{20}$

(iii) q = 0.18

<u>Question 4</u>

This question tested candidates' ability to:

(a) Model practical situations in which the discrete, uniform, binomial, geometric or Poisson distributions are suitable.

Apply the formulae:

(i) $P(X = x) = {}^nC_r\, p^x q^{n-x}$

(ii) $P(X = x) = \frac{\lambda e^{-\lambda}}{x!}$

to calculate probabilities of discrete binomial and Poisson distributions respectively.

(b) Use the formulae for E(X) and Var(X).

(c) Use the Poisson distribution as an approximation to the binomial distribution, where appropriate.

This question was generally well done.

In Part (a), most students recognized that the distribution was binomial and were able to get the majority of the marks for Parts (a) (i) and (ii). However, a few candidates were able to model the situation using $Y \sim Bin(5, 0.541)$ and went further to find P (Y = 3).

In Part (b), a few candidates recognized that the Poisson distribution was to be applied. Some, however, used $X \sim Po(0.4)$ or $X \sim Po(0.04)$ incorrectly instead of $X \sim Po(4)$.

In Part (c), very few candidates recognized that the Poisson distribution should have been used to approximate the binomial in this case.

Answers

(b) (i)

a) $P(X = 8) = 0.237$

b) $P(8 \le X \le 10 = 0.541$

(ii) Mean = E(X) = np =7.8

Standard deviation = $\sqrt{npq} = 1.65$

(iii) $Y \sim Bin(5, 0.541)$

$P(Y = 3) = {}^5C_3 (0.541)^3 (0.459)^2 = 0.334$

(c) $X \sim Po(4)$

$P(X \ge 1) = 1 - P(X = 0) = 1 - 0.135 = 0.865$

(d) $X \sim Bin(100, 0.04)$

Use Poisson approximation since n = 100 > 30 and np = 4 < 5

$P(X \le 3) = 0.433$

Module 3

Question 5

This question tested candidates' ability to:

(i) perform calculations involving a body moving with constant velocity up a plane

(ii) apply the equation F = ma to situations where the body is moving in a horizontal plane

(iii) apply the equation P = Fv

(iv) calculate the maximum speed under the application of variable forces.

In Part (a), there were a few correct responses from those candidates who did not include a force diagram. The majority lost marks because they had:

(i) wrong signs for the weight component and the resistive forces

(ii) missing tractive force

(iii) use of mass (750 kg) as the weight instead of 7500 N (mass × g).

The correct solution was 35000 N.

In Part 5 (b) (i), very few candidates realized that the force found in Part (a) was to be reduced by 650 N on the horizontal ground, ignoring any further reduction (i.e. weight component). The correct solution was acceleration 1 ms^{-2}.

In Part 5 (b) (ii), some candidates realized that at the maximum speed there was no acceleration which led to the correct solution V = 32.3 ms^{-1}.

Question 6

This question tested candidates' ability to:

(a) Determine the velocity and displacement of a particle with variable acceleration.

(b) Apply their knowledge of vectors to show that there was no acceleration along Ox, to find the time when the velocity was perpendicular to the acceleration and to find the distance of the particle from O when t = 3.

This question was poorly done. Forty-eight per cent of the candidates scored between 0 and 4 marks.

In Part (a), most of the candidates realized that they were required to integrate to find expressions for the velocity and the displacement. However, a few tried to use the equations of motion to solve the problem, not realizing that this could not be done since the acceleration was not constant.

The answer to part (b), followed from their solutions to (a). There were a few who got the correct answers, but were unable to explain what the results meant.

In Part (c)(i), only a few candidates were able to differentiate twice, thereby obtaining the coefficient of " "$\boldsymbol{i}$" and stating that this indicated that there was no acceleration along Ox.

The candidates who attempted Part (c) (ii) displayed a knowledge of $a.b = 0$ for perpendicularity of vectors a and b, and solved the problem. However, no candidate observed that from (a)(i), the acceleration is along Oy, therefore the velocity is along Ox, hence : $4t^3 - 32 = 0 \Rightarrow t = 2$.

Part (c) (iii) was well done, as candidates were able to substitute t = 3 into $r = 8ti + (t^4 - 32t)j$, hence finding the required distance.

UNIT 1

Statistical Analysis

Internal Assessment

After careful evaluation of the Internal Assessment samples, it was found that:

1. Some project titles were too long or were not relevant to the course. Project titles must be concise and relevant.
2. Some projects had no title, which was a pity, since this is perhaps the easiest mark to score in the project.
3. Too often, questionnaires and long lists of data were included in the presentation of data. This is unnecessary and should be appropriately placed in the appendices.

4. Diagrams such as pie charts were frequently not properly labelled in the presentation of data. This impacted on the marks scored by candidates in this section.
5. In most cases, the purpose was stated but no variables were identified. Candidates, in general, seemed to be confused as to the meaning of the word “variable”.
6. In many cases, the method of data collection was too simplistic. The method was stated in many instances, but was not described in detail.
7. A small percentage of the teachers (approximately 5 per cent) misunderstood the process of recording the marks on the AMAT 1-3 forms (i.e. the form containing the data on the five samples).
8. The references were omitted in about 10 per cent of the Internal Assessment samples submitted. Far too many candidates did not use an up-to-date and consistent convention in the reference.

UNIT 2

Mathematical Applications

Internal Assessment

1. Some of the topics chosen for study by the candidates were not relevant to Applied Mathematics.
2. A very small minority of teachers misunderstood the process of recording the marks on the AMAT 23 forms.
3. Some diagrams were not labelled in the projects.
4. A small minority of candidates did not follow the stipulated format.
5. Generally, the analyses were comprehensively done, but there were a few occasions when the analysis was not relevant.
6. In many instances, the task was not clearly stated. This section should not have been placed in the introduction and should be more concise. Unnecessary details should not be included here.
7. Method of data collection was in some cases too simplistic or non-existent. This section needs to be clearly described.
8. The Hungarian algorithm was misused in many cases or improperly applied.
9. Many candidates who incorporated logic gates into their projects did so inappropriately.
10. Many candidates lost two marks allocated to “insights into the nature and resolution of problems encountered in the tasks”.

Recommendations

It was evident from the work produced in Unit 1 that candidates were well prepared in Describing and Collecting Data and in Managing Uncertainty while in Unit 2 they were well prepared in Discrete Mathematics and Probability and Distributions. Candidates are still experiencing problems with the Mechanics Module. Candidates have to spend more time solving problems from the Mechanics Module as well as developing the ability to explain concepts. In addition, they need to pay special attention to their algebraic manipulation. It would be helpful if the coverage of the syllabus could be completed in time to allow candidates to spend more time solving problems from all of the Modules.