CARIBBEAN EXAMINATIONS COUNCIL

REPORT ON CANDIDATES' WORK IN THE CARIBBEAN SECONDARY EDUCATION CERTIFICATE JANUARY 2008

MATHEMATICS

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GENERAL PROFICIENCY EXAMINATIONS

JANUARY 2008

GENERAL COMMENTS

The General Proficiency Mathematics examination is offered in January and May/June each year. The Basic Proficiency examination is offered in May/June only.

There was a candidate entry of approximately 14 800 in January 2008. This year, fifty-seven per cent of the candidates achieved Grades I – III. The mean percentage for the examination was 87.6.

DETAILED COMMENTS

Paper 01 – Multiple Choice

Paper 01 consists of 60 multiple-choice items. This year, thirty-one candidates earned the maximum available mark and seventy-seven per cent of the candidates scored at least half the total marks for this paper.

Paper 02 – Essay

Paper 02 consisted of two sections. Section I comprised eight compulsory questions totalling 90 marks. Section II comprised six optional questions: two each in Relations, Functions and Graphs; Trigonometry and Geometry; Vectors and Matrices. Candidates were required to choose any two questions. Each question in this section was worth 15 marks.

This year, no candidate earned the maximum available mark on Paper 02. However, three candidates scored 119 marks out of a possible 120. Approximately thirty-two per cent of the candidates earned at least half the maximum mark on this paper.

Compulsory Section

<u>Question 1</u>

This question tested candidates’ ability to:

- perform basic operations with fractions
- perform basic operations with decimals
- calculate the hire purchase price
- express one quantity as a percentage of another

The question was attempted by 99 per cent of the candidates, 28 per cent of whom scored the maximum available mark. The mean mark was 8.21 out of 11.

Generally, performance on this item was good. The majority of candidates demonstrated proficiency in performing operations on fractions and decimals. In part (a)(i), some candidates converted the fractions to decimals and then simplified. However, this procedure could not yield an exact answer in this situation.

In part (a)(ii), some of the weaker candidates simplified $2-\frac{0.24}{0.15}$ by first subtracting 0.24 from 2 and then proceeded to divide the result by 0.15.

In part (b), many candidates scored full marks. Errors were seen in expressing the difference between the hire purchase price and the cash price as a percentage of the cash price. For example, some candidates did not use the cash price in the denominator while others set up the percentage with 100 in the denominator.

Solutions:

(a) **(i)** $\frac{11}{14}$ **(ii) 0.4**

(b) **(i) $354.00** **(ii) $34.05** **(iii) 10.64%**

Recommendations

Teachers need to point out to students that computations involving fractions always yield exact results and converting fractions to decimals will sometimes yield an approximate result. They should also emphasize that in expressing one quantity as a percentage of another, the quantities to be compared must first be written as a fraction, with the whole as the numerator, before multiplying by 100%.

Question 2

This question tested the candidates' ability to:

- solve a linear inequality in one unknown
- identify sets of numbers, in particular whole numbers
- factorise expressions of the form ax+bx, a^2- b^2, ax+bx+ay+by
- use algebraic expressions to represent information
- use linear equations to solve worded problems

The question was attempted by 99 per cent of the candidates, less than 1 per cent of whom scored the maximum available mark. The mean score was 4.84 out of 12.

Solving the inequality in part (a), proved to be the most challenging part of this question. Candidates omitted to change the direction of the inequality when dividing by a negative quantity. They also had difficulty in identifying the smallest whole number that satisfied the inequality.

Factorisation by common factors or by the difference of two squares appeared to be easier than factorisation by grouping. The major difficulty experienced by candidates in using the grouping technique was extracting a negative factor.

There were several good attempts in part (c), where candidates were able to write the expression and equation for the total amount of money collected for the sale of sponge cakes. A few candidates did not multiply the cost by the quantity, while others collected their terms incorrectly while solving the equation.

Solutions:

(a) **(i)** **$x > -2$** **(ii)** **$x = 0$**

(b) **(i)** **$x(x - y)$** **(ii)** **$(a + 1)(a - 1)$** **(iii)** **$(p - q)(2 - p)$**

(c) **(i)** **$2(k + 5)$** **(ii)** **$2(k + 5) + 10k + 4(2k)$**
(iii) **$k = \$6.50$**

Recommendations

Teachers need to emphasize the difference in techniques used to solve equations and inequations. The practice of verifying solutions when solving word problems must also be emphasized.

Question 3

This question tested the candidates' ability to:

- construct and use Venn diagrams
- solve problems involving the use of Venn diagrams
- determine the elements in the union, intersection and complement of two sets
- use the properties of parallel lines to calculate unknown angles
- use the properties of isosceles triangles to calculate unknown angles

The question was attempted by 99 per cent of the candidates, 5.34 per cent of whom scored the maximum available mark. The mean score was 5.66 out of 12.

Responses to part (a) were generally good. Candidates displayed strengths in constructing the Venn diagram and listing elements of the subsets. A few candidates omitted some elements of the union in their list while others mistakenly listed the members of the intersection.

Part (b) was poorly done. Candidates made assumptions in calculating angles and were unable to provide any reasons for their calculations. The properties of angles on parallel lines were hardly mentioned and there was difficulty in naming the angles.

Solutions:

(a) **(ii)** **a)** **SUT ={l,m,k,p,q}**
b) **S2 ={q,n,r}**

(b) **(i)** **90^0** **(ii)** **48^0** **(iii)** **84^0**

Recommendations

Teachers need to teach Set Theory using practical examples to determine the intersection, union and complement of sets.

The properties of angles, parallel lines and polygons should be reinforced through regular practice and varied examples.

Question 4

This question tested the candidates' ability to:

- interpret the 24 hour clock notation and use it to calculate elapsed time
- calculate average speed
- calculate the area of a square, circle and a sector

The question was attempted by 98 per cent of the candidates, 4 per cent of whom scored the maximum available mark. The mean score was 3.66 out of 10.

In part (a), the majority of candidates were able set up the subtraction to calculate the length of the journey but many failed to obtain the correct result. Substituting their time into the formula for average speed did not pose problems for candidates. However, they failed to express 6 hours 50 minutes as hours and hence, could not obtain the correct speed.

In part (b), candidates were successful in calculating the area of the circle and square but did not recognize that the area of the sector was required to compute the area of the shaded region. They incorrectly subtracted the area of the square from the area of the circle to obtain the area of the shaded region.

Solutions:

(a) (i)	**6 hours 50 min**	**(ii)**	**60 km/h**		
(b) (i)	**38.5 cm^2**	**(ii)**	**12.25 cm^2**	**(iii)**	**2.625 cm^2**

Recommendations

Teachers need to emphasize that in time, the base unit is 60 and not 100. The importance of consistency in units in calculating speed should also be addressed.

Question 5

This question tested the candidates' ability to:

- construct a frequency table from a bar graph
- interpret information from a bar graph
- determine the mean from a bar graph
- calculate the probability of a simple event

This question was attempted by approximately 94 per cent of the candidates, 4.65 per cent of whom scored the maximum available mark. The mean mark was 4.89 out of 12.

Candidates displayed strengths in stating the mode, computing the mean, and calculating the probability. In setting up the frequency table, some candidates had incorrect labeling, for example, the use of the term "frequency" was often incorrect. Interpreting the data in parts (b) and (e) posed most problems for candidates. Although many candidates correctly used the formula, $\frac{\sum fx}{\sum f}$ in computing the mean, they could not state how many boys were in the club, nor could they state how many books were read. In the latter case they merely added the scores from 0 to 4 and obtained an answer of 10.

Solutions:

(a)

Number of Books	**0**	**1**	**2**	**3**	**4**
Number of Boys	**2**	**6**	**17**	**8**	**3**

(b) **36 boys**
(c) **Mode : 2**
(d) **76 books**
(e) **Mean: 2.1**
(f) $\frac{11}{36}$

Recommendations

Teachers must pay closer attention to developing skills in interpreting data. Students must be given opportunities to experience statistics by generating their own data on relevant and interesting situations. Statistical concepts such as the mean and the mode should be introduced and understood before formulae are introduced.

Question 6

This question tested candidates' ability to:

- identify and describe transformations given an object and its image
- use instruments to draw and measure angles and line segments
- use instruments to construct a parallelogram given two adjacent sides and one angle

The question was attempted by 86 per cent of the candidates, less than 1% per cent of whom scored the maximum available mark. The mean mark was 3.61 out of 12. Responses to this question were mainly poor. Candidates seemed unable to recognize transformations in this particular context. In cases where the transformation was named correctly, candidates could not state the characteristics which define the transformation.

In constructing the parallelogram, although candidates were able to make accurate measurements, many were unfamiliar with the techniques for constructing parallel lines. Some candidates constructed the 60^0 angle correctly and proceeded to draw the parallelogram without the use of instruments.

Solutions:

(a) (i) A rotation of 180^0 about the point L
An enlargement, scale factor -1, about L
(ii) A translation of 4 units horizontally to the right and 1 unit vertically downwards

(b) (ii) WY = 10.9 cm

Recommendations

In the teaching of transformation geometry, teachers need to re-introduce informal approaches without the use of graph paper, before they use formal approaches on the Cartesian plane. The critical attributes of each transformation and the particular vocabulary associated with describing these transformations must be emphasized. Techniques in the proper use of geometrical instruments to construct parallel lines must be explored using different approaches.

Question 7

This question tested the candidates' ability to:

- use substitution to complete a table of values for a given function
- draw the graph of a quadratic function given a specific domain
- draw the graph of a linear function of the type y = k
- state the solution of a pair of equations – one linear and one quadratic
- derive a quadratic equation given its roots

The question was attempted by 91 per cent of the candidates, 1.55 per cent of whom scored the maximum available mark. The mean mark was 6.26 out of 11.

In general, the performance on this question was satisfactory. Candidates displayed strengths in calculating the table values, using the given scale, plotting points and drawing the graphs. In some cases, incorrect graphs were obtained because of errors in the values calculated or plotting the points. In some cases, candidates drew a vertical line for the line y = 2.

In stating the x-coordinates of the points of intersection of the line and the curve, some candidates gave the x-intercepts instead. Hence, a limited number of candidates was able to attempt the last part of the question.

Solutions:

(a) when x = 0, y = 0 ; when x = 3, y = -3 ; when x = 5, y = 5

(b) (ii) x = -0.4 and 4.4 (iii) $x^2 - 4x – 2 = 0$

Recommendations

Teachers should ensure that the students can determine the basic shape of the graph on inspecting the equation. This would assist them in identifying errors made in completing their table of values. Teachers must emphasize that rulers cannot be used to draw curves.

Question 8

This question tested the candidates' ability to:

- recognise number patterns
- calculate unknown terms in a number sequence
- state the formula for the n^{th} term in a sequence
- use a pattern to generate the sum of a series

The question was attempted by 95 per cent of the candidates, 1.41 per cent of whom scored the maximum available mark. The mean mark was 6.08 out of 10. Responses to this question were generally good. The majority of candidates recognised the number pattern and were able to calculate specific terms or the sum of the series. Deriving formulae in terms of **n** posed the most difficulty for candidates and some substituted a specific number instead of using a generalized result.

Solutions:

(a) **(i)** **21, ½ (6)(6 + 1)** **(ii)** **½ (n)(n + 1)**

(b) **(i)** **36^2** **(ii)** **$[½ (n)(n + 1)]^2$**

(c) **$[½ (12)(12 + 1)]^2 = 6024$**

Recommendations

Teachers should continue to create opportunities for students to recognise patterns and solve for unknown terms. However, more attention must be placed on generalization of the $\mathbf{n}^{th}$ term of a sequence.

Question 9

This question tested the candidates' ability to:

- represent inverse variations symbolically
- perform calculations involving the inverse variation
- apply Pythagoras' Theorem to obtain an algebraic relationship between the sides of a right-angled triangle
- expand a binomial expression and simplify algebraic terms
- solve a quadratic equation and interpret the solution in a given context
- solve problems involving quadratic equations

The question was attempted by 28 per cent of the candidates, 5.4 per cent of whom scored the maximum available marks. The mean mark was 4.78 out of 15.

Responses to this question were mainly unsatisfactory. In part (a), candidates were unable to represent the inverse variation symbolically and few had successful attempts in finding the constant of variation.

In part (b), a limited number of candidates was able to apply Pythagoras' Theorem to state the correct relationship among the sides of the triangle. Expanding the binomials and simplifying their result proved to be challenging for candidates. Many candidates gave $(a-7)^2$ as $(a^2 + 49)$. Those who went on to solve the quadratic equation preferred to use the formula rather than factorise. Interpreting the result was difficult for many candidates and negative lengths of sides were often given.

Solutions:

(a) **(i)** $V = \frac{K}{P}$ **(ii)** **K =6 400** **(iii)** **V = 13.3**

(b) **(i)** $(a - 7)^2 + a^2 = (a + 1)^2$ **(ii)** **a = 12 or 4**

(iii) **Reject a =4, since (4-7) is negative, hence a = 12**
Sides are 5, 12 and 13

Recommendations

Teachers should use integrated approaches to teach the topic of variation, emphasizing graphical approaches to determining the constant of variation. In treating the expansion of binomials, teachers should use visual approaches using the sides of rectangles to represent linear expressions of the form (x+k) so that students can appreciate that the expansion has three distinct terms.

Question 10

This question tested the candidates' ability to:

- write inequalities from worded statements
- draw graphs of linear inequalities in one or two variables
- determine the solution of a set of inequalities
- use linear programming techniques to determine the maximum value of an expression

The question was attempted by 42 per cent of the candidates, 4.7 per cent of whom scored the maximum available mark. The mean mark was 5.58 out of 15.

The majority of candidates were able to write the inequalities, although the weaker candidates omitted the variables in the second inequality and incorrectly wrote
$6 + $24 ≤ 360. Those candidates who gave a full response to the question displayed strengths in using the scale, drawing the line x+y = 30, stating the coordinates of their region and calculating the maximum profit.

A significant number of candidates lost marks when they drew the line 6x + 24y = 360 incorrectly or shaded the incorrect region.

Solutions:

(a) (i) $x + y \leq 30$, $6x + 24y \leq 360$

(b) (iii) (0, 0) (0, 15) (20, 10) (30, 0)

(c) (i) 0 ; \$45.00 ; \$50.00 ; \$30.00

(ii) Maximum Profit: \$50.00

Recommendations

Teachers should emphasize that the solution of an inequation is a region and that the correct region can be obtained by testing points on both sides of the line. Further, once a region has been identified, students should test points in the region to verify that all inequalities are satisfied. The use of a consistent system for identifying the feasible region must also be addressed.

Question 11

This question tested the candidates' ability to:

- use theorems in circle geometry to calculate the measure of angles
- draw a diagram to represent information involving bearings and distances
- solve problems involving bearings
- use the sine and cosine rules in the solution of problems involving triangles

The question was attempted by 23 per cent of the candidates 3.47 per cent of whom scored the maximum available mark. The mean mark was 5.45 out of 15.

In part (a), most candidates were able to calculate the unknown angles although many of the candidates did not give reasons for their answers. The drawing of the diagram in part (b) posed particularly difficult for candidates, especially showing the bearing. In calculating the distances, most candidates correctly chose the cosine rule and substituted the lengths of the sides of the triangle correctly. However, many were unable to calculate the angle ABC, others made errors in simplifying their result. The last part of the question which required candidates to calculate the bearing was omitted by most candidates.

Solutions:

(a) (i) 100^0 [The angle at the centre is twice that at the circumference]

(ii) 40^0 [Triangle WOY is isosceles, $\frac{180^0 - 100^0}{2}$]

(b) (ii) 134.1 m (iii) 123^0

Recommendations

Teachers should allow students to represent situations involving bearings using models, before drawing them on paper. Approaches to teaching bearings need to be practical and done in an outdoor setting. Basic geometrical skills in calculating angles should be reviewed prior to teaching this topic.

Question 12

This question tested the candidates' ability to:

- recognize and use the trigonometrical ratios to solve right-angled triangles
- solve problems involving angles of elevation and depression
- represent lines of latitude and longitude, and points given their positions on a sketch of the Earth
- calculate the distance between two points on the Earth measured along a circle of longitude
- calculate the circumference of a circle of latitude

The question was attempted by 8 per cent of the candidates, 1.64 per cent of whom scored the maximum available mark. The mean mark was 4.03 out of 15.

In part (a), candidates had difficulty in identifying the angles of elevation and depression. However, once these angles were inserted they were able to correctly select the trigonometric ratio needed to solve the problem. In part (b), most candidates were able to draw the lines of latitude and longitude, although the line of longitude did not always pass through the North and South Poles. Locating the points P and Q was well done. In calculating the distance PQ along the circle of latitude, some candidates did not use the radius of a great circle and many could not calculate the angular difference.

Solutions:

(a) (i) 28.6 m (ii) 12.9 m

(b) (iii) a) 8 890 km b) 25 714 km

Recommendations

In teaching the application of trigonometry to solving problems in the real world, teachers should construct apparatus to measure angles of elevation and use models to represent the Earth. Students should be encouraged to use these materials to construct diagrams and solve problems. In addition, scale drawings should be done to reinforce these concepts.

Question 13

This question tested the candidates' ability to:

- locate points on a diagram given the relevant information
- add vectors
- use vectors to represent and solve problems in geometry

The question was attempted by 24 per cent of the candidates 1.38 per cent of whom scored the maximum available mark, The mean mark was 4.83 out of 15.

In part (a), candidates were able to locate the points on their diagram and write vectors for AC and PQ. Many candidates demonstrated weaknesses in simplifying their vectors and could not derive the proof.

In determining the position vectors for RT and SR in part (b), many candidates did not reverse their vectors in writing the route, for example RT was written as OR + OT instead of RO + OT.

Candidates were unable to choose an appropriate strategy to determine the position vector of F. Hence, this part was either omitted or poorly done.

Solutions:

(a) **(ii) a)** $\mathbf{2x + 3y}$ **b)** $\mathbf{x} + \frac{3}{2}\mathbf{y}$

(b) **(i)** **a)** $\begin{pmatrix} 2 \\ -6 \end{pmatrix}$ **b)** $\begin{pmatrix} -4 \\ 2 \end{pmatrix}$

(ii) **b)** **F(4, 1)**

Recommendations

The resolution of vectors should be taught using concrete examples such as airplane routes. Teachers should challenge students to find alternate routes to the same journey. Students should also be given sufficient opportunities to communicate their routes using vector notation, emphasizing that vectors can only be added end to end.

Question 14

This question tested the candidates' ability to:

- multiply matrices
- identify a singular matrix
- calculate the determinant of a 2×2 matrix
- find the inverse of a non-singular 2×2 matrix
- use matrix methods to solve a system of linear equations

The question was attempted by 50 per cent of the candidates, 4.35 per cent of whom scored the maximum available mark. The mean mark was 5.42 out of 15.

In part (a), the multiplication of a 1×2 matrix by a 2×2 matrix proved to be very challenging for candidates. The majority of candidates obtained a column matrix instead of a row matrix.

In showing that R is non-singular, many candidates merely found the determinant and did not make a statement to the effect that since the determinant was not zero, the matrix was non-singular. In multiplying R by R^{-1}, many candidates worked with the fraction inside the matrix thus setting themselves up for computational errors in multiplying the elements. Some candidates were unfamiliar with **I** as the identity matrix.

There were better attempts at part (b) where candidates had to use matrices to solve a pair of simultaneous equations. However, many candidates did not use their calculations of the inverse from the earlier question and treated the last part of the question as an independent question, losing valuable time redoing the calculations.

Solutions:

(a) **x = 4, y = 3**

(b) **(i)** **Determinant of R is 7.**
Since the determinant of R is not equal to zero, R is non-singular.

(iv) **x = 1, y = 2**

Recommendations

Teachers should emphasize that there is a condition for the multiplication of matrices. Students should be given opportunities to create their own matrices and determine if it is possible to multiply any two of these matrices. In solving simultaneous equations, students should appreciate that it is more efficient to perform the multiplication by $\frac{1}{\det}$ in the final stage of the solution.