SUMMARY

This thesis deals with theoretical and computational aspects of optimal control problems. In particular, six major results are given:

The necessary conditions for the general control problem with state variable inequality constraints are derived in a generalized manner using the Bolza formulation in the Calculus of Variations. This approach serves to unify the various sets of necessary conditions previously presented and rationalizes the method of obtaining these conditions. Valentine's Slack Variable Technique forms the basis of the approach, allowing the inequality constraints to be restated as equality constraints after which variational analysis is used.

Pontryagin's Maximum Principle is restated to include a wider class of control constrained problems in a manner different from previous investigators. The principle is then extended to include state-constrained problems with the simplicity of the resulting necessary conditions being highlighted. It is shown that the Maximum Principle is not a restatement of the Weierstrass Condition of the Calculus of Variations as is currently held, but rather is a much more efficient optimization tool.

A derivation of the basic first order gradient method which differs from previous derivations is presented. This derivation establishes the ease with which the method deals with control constraints. The algorithm is extended to include state variable inequality constraints, using the necessary conditions of the extended Maximum Principle. The resulting first order algorithm is shown to be quite efficient and with respect to the treatment of state variable inequality constraints, is shown to be more effective than existing methods.

Based on the gradient method, a new algorithm is derived for solving "bang-bang" type control problems. Using a system which has two pairs of complex conjugate eigenvalues (leading to seven switchings in the optimal control), the technique is compared with several other methods, both general and specialized. The results indicate a faster convergence rate for the new method as compared to the others.

A family of second order algorithms derived from the gradient method, is developed. These algorithms are based on a new method of obtaining state and co-state perturbations due to control changes. The usual linear Two Point Boundary Value Problem is completely avoided and so is the attendant complicated programming. Control and state constraints are easily handled and convergence is quite rapid. The results of a comparative study of some of the algorithms, which indicate their efficiency, are given.

Based on the conditions of the extended Maximum Principle, Sufficiency Conditions are presented which are applicable to optimal control problems with any of control, control/state and state constraints for both fixed and variable final time. They appear to be simpler than previously published conditions and retain a simple form for the various problem conditions.

The dissertation is concluded with an extensive discussion.