ABSTRACT

Schur Multipliers and the Fourier Interpolation

Problem

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The Schur-multiplier problem and the Fourier interpolation problem were found to be bounds for each other and in some specific cases a solution to one problem provides a solution to the other problem.

For any $m \times n$ matrix T, the Schur multiplier norm, denoted $\|T\|_S$, is $\max_{X \neq 0} \|T \circ X\| / \|X\|$ where $\circ$ denotes the Schur product. In particular, for any $n \times n$ Hankel matrix T_μ with $\mu = (\mu_{-n+1}, \mu_{-n+2}, \ldots, \mu_{n-1})$, the Schur-multiplier problem is to find the Schur multiplier norm of T_μ. Let

$$\mathcal{B}_\mu = \{f \in L_1(\mathbf{R}) : \hat{f}(x_j) = \mu_j \text{ for } -n+1 \leq j \leq n-1\}$$

where $\hat{f}$ is the Fourier integral transform of f, $\{x_{-n+1}, x_{-n+2}, \ldots, x_{n-1}\}$ is a set of real numbers in arithmetic progression, and $L_1(\mathbf{R})$ is the set of complex-valued Lesbesgue-integrable functions on $\mathbf{R}$. Then the Fourier interpolation problem is to find $\inf_{f \in \mathcal{B}_\mu} \|f\|_1$. Thus, one connection between these two problems is that

$$\|T_\mu\|_S \leq \inf_{f \in \mathcal{B}_\mu} \|f\|_1.$$

In this thesis, necessary and sufficient conditions needed for equality in the 2-dimensional case are stated and proved along with necessary conditions for the n-dimensional case and also an alternative proof to that presented by R. McEachin in [RMn].

In the 2-dimensional case, the analysis couples a hermitian unitary matrix X with a unit vector $\mathbf{y}$ that is in the zone of feasibility of X to produce a distinguished Hankel T_μ consequently equating the two problems.

In the 3-dimensional case, the analysis uses a unit vector $\mathbf{y}$ as pivot to construct a feasible pair $(X, \mathbf{y})$ and hence a distinguished T_μ.

Keywords: Nadine McCloud; Schur multiplier; Fourier interpolation problem; hermitian unitary matrix; norming pair; feasible pair; distinguished matrix; zone of feasibility.