ABSTRACT

Improving Approximations To Real-Valued Functions
Using Statistical Ideas

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Approximation theory is a branch of real analysis which has attracted the attention of mathematicians for the last 150 years. Early work in this field was initiated by Weierstrass in the 19th. century with later contributors including Sz*á*sz, Baskakov and Lupas. Complicated functions can be approximated by much simpler polynomial functions. This provides an estimate of a solution using a method known as polynomial approximation. S.N. Bernstein proved the Weierstrass theorem in 1912 using Bernstein polynomials, while in 1950, Sz*á*sz generalized these polynomials on an infinite interval. In 1957, Baskakov proposed an infinite polynomial approximation of functions while in 1967, Lupas used the Weierstrass theorem to approximate convex functions.

The aim of this thesis is to construct a 'two-phase' iterative algorithm based on the original two-phase iterative algorithm proposed by Sahai (2010), which was identified as an improvement to the original bias-reduction technique in Sahai (2004) from which it was derived. The idea here is to improve the approximation of the Baskakov and Lupas operators. This is done by first modifying the operator itself and applying the algorithm which was previously applied to the Sz*á*sz operator. This algorithm uses the statistical concepts of Bias and Mean Square Error (MSE) to approximate an unknown function. The procedure uses known values of the function at equally spaced points or specified knots in the set of continuous functions in the interval [0,1], which is representative of the general interval [a,b]. This is referred to as the Bias-MSE reduction approach.

Consequently, this thesis modifies this algorithm by applying the Relative MSE (RMSE) instead of the MSE. The computational achievements of both algorithms are explored in empirical studies. The function is assumed to be known. In this case, *exp(x)*, 2^x, *ln(2+x)*, *sin(1+x)* and $sin(2+\frac{x*\pi}{2})$ are used to demonstrate the relative errors and relative gains of the algorithms. Comparison of these results for the Bias-MSE Reduction approach and the Bias-RMSE Reduction approach for each of the Bakakov and Lupas Operators respectively indicates which approach is more efficient. Both approaches have generally similar gains in relative efficiency as the number of iterations increases, with slight variations of not more than 2% for different values of n between them. Although the Bias-RMSE method is an improvement to the previous Bias-reduction method used by Sahai (2004), it is slightly less efficient than the Bias-MSE approach in almost all cases of the assumed function.

Keywords: Approximation theory; Two-phase iterative algorithm; Bias-MSE Reduction; Bias-RMSE Reduction.