

ABSTRACT

Stochastic Approximation Procedure with some Application in Credibility Theory

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The Stochastic Approximation Procedure plays an important role in its relationship to the Theory of Credibility. If the prior distribution is a member of the Exponential Dispersion Family, then the traditional credibility formula

$$C_n = Z_n \bar{x}_n + (1 - Z_n) \mu,$$

(where C_n denotes the fair premium, μ is the overall prior mean, \bar{x}_n is the mean associated with n years of experience data x_1, x_2, \dots, x_n , and Z_n is the credibility factor), holds true. However, if the prior distribution is a member of the Location Dispersion Family, then the traditional credibility formula fails. In this case, the stochastic approximation procedure is

used for the evaluation of the fair premium. As a result, this dissertation will examine the modified stochastic approximation procedure

$$x_{i+1} = x_i + \alpha_i g(x_i + \xi_{i+1}), \text{ for } x_0 = \mu, \quad (1)$$

where $\{\alpha_i\}$ is the sequence of positive numbers, $\{\xi_i\}$ is a sequence of martingale-differences, the function g is twice differentiable, $xg(x) \leq 0$ for $x \neq 0$, and μ is an initial value. In doing so, the boundedness and exponential stability of procedure (1) will be discussed; hence the theorem on the convergence of non-negative semi-martingale will have been utilized.

Keywords: Stochastic Approximation Procedure, Credibility Theory, Exponential Dispersion Family, Location Dispersion Family.