On Stability of Difference Equations of Non-hyperbolic type

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Abstract

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In this thesis we perform a stability analysis of solutions of the difference equation

$$x_{n+1} = x_n(1 - \mu x_n^{2k}), \quad n \in \mathbb{N}, \quad x_0 \in \mathbb{R}, \tag{0.0.1}$$

according to the location of the initial values. In our analysis the real axes can be represented as a union of 3 sets $\mathcal{A}$, $\mathcal{B}$ and $\mathcal{C}$. When the initial value belongs to $\mathcal{A}$ or $\mathcal{B}$, solutions of (0.0.1) are asymptotically stable; when the initial value belongs to $\mathcal{C}$, solutions of (0.0.1) are unstable. Solutions do not change sign when $x_0 \in \mathcal{A}$; when $x_0 \in \mathcal{B}$, solutions change sign finitely often; finally, solutions change sign infinitely often when $x_0 \in \mathcal{C}$. A similar analysis is done for generalizations of (0.0.1).

We prove an a.s. asymptotic stability result for a linear stochastic difference equation with non-homogenous coefficients. The proof is based on semi-martingale convergence theory.

For the stochastically perturbed version of (0.0.1) we apply Monte-Carlo methods to examine the probability of stability of the solutions.

Key words: Difference equation, asymptotic stability, stochastic difference equation, almost sure stability, convergence theorem, Monte-Carlo methods.