CARIBBEAN EXAMINATIONS COUNCIL

REPORT ON CANDIDATES' WORK IN THE CARIBBEAN ADVANCED PROFICIENCY EXAMINATION

JUNE 2003

MATHEMATICS

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CARIBBEAN ADVANCED PROFICIENCY EXAMINATION
MAY/JUNE 2003

INTRODUCTION

This is the fifth year that Mathematics Unit I was examined on open syllabus and the fourth year for Unit II. Twelve hundred and thirty-five candidates registered for the Unit I examinations. Four hundred and four candidates registered for the Unit II.

Each Unit comprised three papers – Paper 01 and Paper 02 were assessed externally and Paper 03 was assessed internally by the teacher and moderated by CXC.

Paper 01 consisted of fifteen compulsory, short-response questions. There were five questions in each of Sections 1, 2 and 3 corresponding to Modules 1, 2 and 3 respectively. Each question carried marks in the range 4-8. Candidates could earn a maximum of 90 marks for this paper, representing 30 per cent of the assessment for the Unit.

Paper 02 consisted of six compulsory extended-response questions. There were two questions in each Section/Module. Each question carried 25 marks. Candidates could score a maximum of 150 marks, representing 50 per cent of the assessment for the unit. Marks were awarded for reasoning, method and accuracy.

Paper 03 was compulsory and was assessed internally by the teacher and moderated by CXC. For Unit I, candidates wrote at least three tests, one for each Section/Module. For Unit II, candidates wrote at least one test for each of Modules 1 and 2 and were required to submit a project based on any aspect of the syllabus. Candidates could score a maximum of 60 marks on this paper representing 20 per cent of the assessment for the Unit.

Paper 03B for Unit I was written by private candidates for the first time this year.

GENERAL COMMENTS

UNIT I

Indices, completing the square and limits continue to be topics which present difficulties for candidates. Basic algebraic manipulation and numerical calculation have not been carefully applied, and some aspects of elementary coordinate geometry seem not to be very well understood. Trigonometry, on the whole, seems to be a difficult topic for many. It was also apparent that many of the skills which should have been inculcated at the CSEC level were not carried forward to this Unit and this led to poor execution of logical reasoning in setting out the details in the solutions of some of the questions posed.

Despite these perceived weaknesses, there were several outstanding performances, particularly in Paper 02. Topics such as curve-sketching, the cosine rule and basic calculus received excellent treatment.

DETAILED COMMENTS

UNIT I
PAPER 01

SECTION A (Module 1.1: Basic Algebra and Functions)

Question 1

Specific Objective(s): (a) 1, 2; (c) 2, 5, 6; (e); (f) 2 (i)

(a) There were several ways to solve this problem. These included equating coefficients, use of the Factor Theorem and/or expansion. Candidates used all methods indicated. The question was well-done.

(b) This question tested basic knowledge of indices. The solution of quadratic equations was also involved. The question was fairly well done although several elementary errors were made in obtaining the correct quadratic equation in x.

Answers: (a) h = 4, k = 3; (b) 3, -1

Question 2

Specific Objective(s): (b) 1, 2; (c) 2; (f) 1, 2 (i)

This question tested the candidates' ability to recognise a quadratic equation in which the variable quantity is not a single parameter and to convert such an equation to the usual quadratic in a single variable.

The question was very poorly done. The use of the modulus in such a context was an unusual demand for several candidates. Most of the successful attempts used a substitution such as $y = 2x-3$ or $y = |2x - 3|$. Some solutions were lost by replacing $|2x - 3|$ with $2x - 3$.

Answers: -1, 1, 2, 4

Question 3

Specific Objective(s): (c) 1; (f) 1, 2 (ii)

The main difficulty encountered in this question involved completion of the square. Graphical methods were more successful.

Answers: (a) a = -1, h = 1, k = 4; (b) max. value = 4

Question 4

Specific Objective(s): (g) (i), 1, 2, 4, 5

(a) This part was well done although some candidates used θ in degrees without conversion to radians which gave an incorrect answer.

(b) Some candidates used the arc length formula, $r\ \theta$, to calculate the length of the chord AD. Otherwise, there were many accurate answers.

Answers: (a) 141m^2 (b) AD = 18.1m

Question 5

Specific Objective(s): (d) 1, 3; (f) 2 (i), (ii)

This question was well done. Few candidates had difficulty solving the equation $4x^2 + 6x = 8x + 6$.

Answers: $^3/_2$, -1.

SECTION B (Module 1.2: Geometry and Trigonometry)

Question 6

Specific Objective(s): (a) 5 (ii), 6, 7

This question tested some of the basic properties of perpendicularity of lines, and tangents to circles, in coordinate geometry.

(a) This part was generally well done. Several candidates left the equation of the line in the form $y = -\frac{1}{3}x + 1$ rather than in the simplified form $3y + x = 3$ or $3y = 3 - x$.

(b)(i) This part was very well done. Most candidates left the equation of the circle in the form $(x - 1)^2 + (y + 2)^2 = 2^2$ and did not proceed to obtain the expanded form $x^2 + y^2 - 2x + 4y + 1 = 0$

(ii) Most candidates had difficulty with this part. Many substituted $x = 3$ and $y = -2$ into the equation of the circle without any evident deduction.

Very few of them used the substitution $x = 3$ to find a quadratic equation in y and then obtain coincident roots.

Answers: (a) Equation of line: $3y + x = 3$ or equivalent form.

(b) (i) Equation of circle: $(x - 1)^2 + (y + 2)^2 = 2^2$ or $x^2 + y^2 - 2x + 4y + 1 = 0$

Question 7

Specific Objective(s): (b)

This question examined linear and quadratic inequalities.

(a) The majority of candidates seemed uncomfortable with this type of inequality. Few of them used the sign change table to find the solution.

(b) This part was generally well done. In many cases, the values, - 2 and 1, were not included in the solution set.

Answers: (a) $x < -1$ or $x > 0$; (b) $-2 \leq x \leq 1$

Question 8

Specific Objective(s): (d) 1, 2, 6, 7, 8

Candidates demonstrated a good knowledge of the basic topics in the trigonometry required to solve this question; however, there were several instances of carelessness in the working which robbed candidates of full marks on the question. Many candidates expressed angles in degrees rather than in radians.

Answers: $R=\sqrt{2}$, $\theta = \frac{\pi}{2}$ or π; $\alpha = \frac{\pi}{4}$

Question 9

Specific Objective (s): (e) 2, 3, 4, 5

This question dealt with the elementary representation of complex numbers and with the argument of a complex number.

Both parts of this question were very well done. Candidates displayed good skill in rationalising a complex number whose numerator and denominator are also complex numbers.

Answers: (a) $1+ i$

Question 10

Specific Objective (s): (f) 2,3,4,5,7,10

The topics in vector analysis tested by this question included unit vector and perpendicularity.

(a) (i) Candidates demonstrated a good understanding of the concept of a unit vector.

(ii) Several candidates had difficulty relating the vector $\overrightarrow{OC}$ produced to $\overrightarrow{OB}$ so that the use of the unit vector in the direction $\overrightarrow{OB}$ was not recognised as a link to finding the point C.

(b) A very simple question which was well done.

Answers: (a) (i) unit vector = $\frac{1}{\sqrt{2}}$ $(\underset{\sim}{i} + \underset{\sim}{j})$, (ii) $\overrightarrow{OC} = \sqrt{5/2}\ (\underset{\sim}{i} + \underset{\sim}{j})$

SECTION C (Module 1.3: Calculus 1)

Question 11

Specific Objective(s): (a) 4, 5, 6, 7, 8

About thirty per cent of the candidates achieved a full score of 5 marks on this question. Unfortunately, many candidates had difficulty in executing the following:

In Part (a) -

(i) successfully factorizing simple quadratic expressions, and

(ii) recognizing that simplification of the quotient and evaluation of the limit were required in order to achieve the desired answer.

In Part (b) - successfully solving a modulus equation to achieve more than one solution.

A few candidates used L'Hopital's Rule in Part (a) and though they were rewarded if done correctly, it must be duly noted that this method is NOT outlined in the syllabus.

Generally, candidates recognized that the denominator in Part (b) had to be equated to zero in order to evaluate the critical values of x for establishing continuity. However, very often in stating their answers, they did not use $\neq$ but rather stated equality.

Answers:

(a) -3

(b) $f(x)$ is continuous for $x \in \mathbf{R}, x \notin \{3, -3\}$; also expressed as $x \in \mathbf{R}, x \neq \pm 3$

Question 12

Specific Objective (s): (b) 2

Over 80 per cent of the candidates attempted this question. However, the majority scored a maximum of two of the five marks as candidates were not cautious in making the substitution for the y-value in the correct function, $y = 2x^3$. Instead, they substituted y in the derivative $y' = 6x^2$. Some candidates, however, had difficulty in obtaining the derivative.

Answer: The gradient of the tangent is 24.

Question 13

Specific Objective(s): (b) 13-19

Approximately 90 per cent of candidates responded to this question and of these responses about 30 per cent received full marks. Difficulties arose due to

(i) incorrect differentiation of a constant,

(ii) poor factorizing skills, and

(iii) uncertainty in determining the nature of the turning points.

Many candidates also did irrelevant calculations of the y – values for the stationary points as only the x – values were required.

Answers:

(a) The x – values of the stationary points are $x = 0$ and $x = 1$.

(b) When $x = 0$, the stationary point is a maximum.

(c) When $x = 1$, the stationary point is a minimum.

Question 14

Specific Objective(s): 5, 7; (c) 3, 6

This question had an extremely high percentage response, however, several candidates seemed to have misinterpreted the question and did not proceed to find the derivative of the given function, f (x), first. It also seemed that some candidates were unaware of the quotient rule for differentiating. The relationship between differentiation and integration as well as the difference in the notations or both aspects of calculus seemed unclear to many candidates.

Answer: $f(x) = \frac{7 - x^2}{(x^2 + 7)^2}$; $\frac{1}{2}$

Question 15

Specific Objectives(s): (c) 6, 9

Very few candidates responded to this question, possibly due to time constraints. From the attempts made, the following was noted:

Part (a)

(i) Many candidates used an incorrect method of solving the equation $x^2 = 3x$ as they divided throughout by x and thus lost the solution $x = 0$.

(ii) Candidates generally did not express their answer in coordinate form and simply stated that $y = 9$.

Part (b)

(i) Some candidates had difficulty in establishing how to determine the area of the required region.

(ii) A few candidates attempted to find the volume of the region.

(iii) Some candidates were unable to integrate the two simple polynomial expressions.

Answers:

(a) The coordinates of P are (3,9).

(b) The area of the required region is 4½ squared units.

PAPER 02

SECTION A (Module 1.1: Basic Algebra and Functions)

Question 1

Specific Objective(s): (c) 2, 4, 5; (g) (i) 1, 4, 5, 6

The question tested properties of algebraic operations and plane trigonometry. In general, the responses were good.

(a) Most candidates knew the Factor Theorem, but some resorted to long division to calculate the remainder. Both methods were accepted.

(b) This part was well done, although some candidates tried to use Pythagoras' theorem to calculate BC or treated ABC as an isosceles triangle. A few candidates could not round off to three significant figures correctly.

(c) The use of surds posed a problem. Some candidates evaluated the surd answers and proceeded to calculate estimated values of area and length.

A common error noted was $\sin\frac{\pi}{4} = \frac{1}{4}\sin\pi$

Answers: (a) p = -3, q = 2; remainder = 4 (b) (i) 4.36 cm (ii) 0.397

Question 2

Specific Objective(s): (d) 1, 2 (i), 3, 5

This question tested properties of functions. In general, candidates found this question difficult.

(a) This part was well done. Some candidates superimposed the three diagrams although the question was set out in three separate parts, (i), (ii) and (iii). A few candidates gave negative values in their sketch of $|f(x)|$ and some used the translation (-1, 0) in Part (i).

(b) Many candidates seemed unaware of how to handle the concepts of one-to-one and onto as they apply to functions. These are topics that need to be emphasised in the classroom.

Answers: (a) Stationary values are (i) (3, -2); (ii) (2, 1); (iii) (2, 2)

(b)(i) Stationary value is (2, 4).

(ii) [$x : 0 \leq x \leq 2$] or [$x: 2 \leq x \leq 4$] or any proper subset of each.

(v) $0 \leq y \leq 4$.

SECTION B (Module 1.2: Geometry and Trigonometry)

Question 3

Specific Objective(s): (a) 2, 3, 4, 5 (i), (ii), 6; (c) 1, 2, 3

The question covered the topic of co-ordinate geometry. Candidates found the question difficult.

(a) Many candidates calculated the coordinates of A and B but were unable to calculate the coordinates of C. Some used vectors. A common error was finding the coordinates of the points of intersection of the lines $2x + 3y = 6$ and $5x + y = 7$.

(b) The properties of perpendicular and parallel lines posed a problem to some candidates.

(c) Although some candidates obtained incorrect equations for CD and AD, their method of finding the coordinates of D was correct.

(d) The distance formula was used correctly in most cases.

Some used the determinant formula to find the area of the triangle, namely;

$\frac{1}{2} [x_1, (y_2 - y_3) + x_2 (y_3 - y_1) + x_3 (y_1 - y_2)]$

See model solution in Appendix.

Question 4

Specific Objective (s): (d) 1, 2, 3, 4, 7

The question covered the topic of trigonometry. It was attempted by the majority of candidates. Many of the responses were poor.

(a) Some candidates stated their solutions in degrees rather than in radians.
A few candidates wrote $\cos 2\theta = 1 - 2\sin^2 \theta$ and attempted to use the relationship $R \cos (\theta + \alpha)$.

(b) Some candidates calculated the angle A and proceeded to find $\tan \frac{A}{2}$. This method was accepted, but many obtained one value of A as a consequence.

(c) Some candidates factorised $\cos^4 A - \sin^4 A$ and incorrectly obtained
$\cos^4 A - \sin^4 A = (\cos^2 A - \sin^2 A)^2$. The proof of the identity was sometimes given by calculating simultaneously both sides of the equation. Many did not attempt this part.

(d) This part was well done. Some candidates calculated angle A and angle B and then proceeded to calculate cos (A-B) and cos (A+B).

Some common errors noted were:

(1) $\cos (A - B) = \cos A - \cos B$

(2) $\cos (A - B) = \cos \frac{5}{13} \cos \frac{3}{5} + \sin \frac{12}{18} \sin \frac{4}{5}$

that is, use of ratios as angles.

Answers: (a) $\frac{2\pi}{3}, \frac{4\pi}{3}$ (b) $\pm \frac{1}{2}$ (d) $\frac{63}{65}, \frac{56}{65}$

SECTION C (Modules 1.3: Calculus 1)

Question 5

Specific Objective(s): (a) 9; (b) 3, 8 – 10, 13, 14, 16 – 19.

The question tested properties of Differentiation and the Intermediate Value Theorem.

(a) Candidates generally failed to observe that f (x) was a continuous function. Some found this question difficult.

(b) This part was well done.

(c) Candidates knew the Quotient and Product rules, but had considerable difficulty simplifying the 2nd Derivative so as to obtain the given answer.

A common error noted was: $\frac{dy}{dx} = \frac{1(x^2+2)-1(2x+2)}{(x^2+2)^2}$.

Answers: (b) (i) $\left(\frac{1}{3},\right) \frac{13}{27} \left(, \right) 3, -9$

(ii) $f''(x) = 6x - 10$,

$\left(\frac{1}{3}, \frac{13}{27}\right)$ is the local maximum point

$\left(3, -9\right)$ is the local minimum point

Question 6

Specific Objective(s): (b) 5, 10, 21; (c) 7, 9

This question tested aspects of the trapezium rule, differentiation integration and volume.

(a) This part was poorly done. The algebra involved in substituting for x in terms of n proved difficult for many candidates. Some candidates went on to evaluate the area under the curve which was not required.

(b) This part was very well done although some candidates failed to use the product rule properly. A few candidates did not see the connection between $f'(x)$ and the integral involved.

(c) (i) A few candidates did not provide a proper sketch of the graph of the curve; often only half of the diagram was drawn. Some others did not label the sketch properly leaving out the minimum at the point (0, 1).

(ii) Too many candidates calculated the volume about the x-axis although it was stated in bold type "about the y-axis." These candidates obtained the wrong answer although their calculations were consistent.

Answers: (b) (ii) ¾ (c) (ii) ½ π units ^{3}

PAPER 03

(INTERNAL ASSESSMENT)

Module Tests

Generally the tests were relevant and appropriate. Some teachers used past examination or similar questions for their tests. In most of these cases, the test scores for each module ranged from 14 to 20 with a maximum score of 20. In general, the tests were satisfactorily presented and assessed. There were occasions, however, where teachers submitted marking schemes that were difficult to follow. Some of the schemes included fractional marks.

Teachers are reminded that

(i) final module marks should not be fractional

(ii) mark schemes and solutions should be submitted with the sample for moderation.

PAPER 03B

SECTION A (Module 1.1: Basic Algebra and Functions)

<u>Question 1</u>

(a) Specific Objective(s): (a) 1, 2, 5; (b) 1 – 3; (f), 1, 2(i)

This part of the question covered the topics of inequalities and modulus leading to equations in the variable x. The methods for solution seemed familiar to the candidates but poor algebraic manipulation produced incomplete solutions.

(b) Specific Objective(s): (c) 1, 2, 5; (f) 1

The factor theorem was being tested in this part which was well done. A few minor errors with signs occurred.

(c) Specific Objective(s): (e); (f) 2 (i)

Indices and quadratic equations were combined in this part. There were some good solutions among the submissions.

(d) Specific Objective(s): (g) 2, 4

Some good attempts were made to solve this problem with limited success at a complete solution. There was some difficulty in calculating the area of the sector.

Answers: (a) $-\frac{1}{4} \le x \le \frac{7}{6}$

(b) p = 3, q = 4

(c) $x = 3$ or $x = 2$

(d) 5.20 m

SECTION B (Module 1.2: Geometry and Trigonometry)

Question 2

(a) Specific Objective(s): (a) 7

Candidates were required to find the equation of a circle, given the centre and a point on its circumference. It was well done.

(b) Specific Objective(s): (e) 1

There were several good solutions to this part which covered the roots of quadratic equations and the relationships between such roots.

(c) Specific Objective(s): (d) 5, 7

This part required solutions of a trigonometric equation. Good starts were made in finding the values of θ but some solutions were lost in the final analysis.

(d) Specific Objective(s): (f) 7, 8, 10

Perpendicularity of vectors was tested in this question. The solution required solving a quadratic equation in the variable t.

There were some good attempts at this question.

Answers: (a) $(x - 3)^2 + (y - 4)^2 = 13^2$ or $x^2 + y^2 - 6x - 8y - 144 = 0$

(b) (i) $\propto + \beta = -2$, $\propto \beta = ½$

(ii) $\propto^2 + \beta^2 = 3$

(iii) $x^2 - 12x + 4 = 0$

(c) $\theta = 0, \frac{\pi}{2}, \frac{2\pi}{3}, \pi$

(d) $t = \pm 1$

SECTION C (Module 1.3: Calculus 1)

Question 3

(a) Specific Objective(s): (a) 1, 4, 5, 6, 7, 8

(i) This question covers the simple application of the limit of a rational function with a common factor in both numerator and denominator.

Candidates showed familiarity with the concept but poor factorisation of the numerator led to mistakes in proceeding to the correct solution.

(ii) The topic covered is continuity of a rational function over the real numbers.

This was a familiar topic with the candidates. A few errors were made in finding the ultimate solutions.

(b) Specific Objective(s): (b) 9, 10; (c) 2, 3, 4, 5

This part covered both the differentiation of a rational function and the integration of a related function. It was well done.

(c) Specific Objective(s): (c) 5, 9

The question required the use of integration in obtaining the volume of a solid. It was well done.

Answers: (a) (i) $\lim_{x \to 2} \frac{(x^3 - 4x)}{x - 2} = 8$

(ii) $x \in \mathbf{R},\ x \neq 1,\ x \neq 6$

(b) $\int \frac{16}{(3x+4)^2}\,dx = \frac{4x}{3x+4} + k$

(c) $\text{Vol} = \frac{32\pi}{5}$ units^3

GENERAL COMMENTS

UNIT 2

The performance of candidates on Unit 2 was very encouraging on the whole. There were some very creditable performances in both papers showing improvements in some of the basic building blocks of the discipline. Despite these gains, it is clear that some fundamental approaches need to be improved in order to achieve a better overall understanding and coverage of the syllabus. Candidates for Unit 2 must carry forward relevant knowledge from Unit 1; failure to do this resulted in the inability of candidates to find solutions in more than a few instances. Poor formatting of the logic in answers to questions often led to faulty deductions and wrong conclusions.

In general, questions on such topics as integration and differentiation in Calculus, partial fractions, proportionality, Newton-Raphson in approximating roots of equations, probability theory and mathematical induction were reasonably well done, but areas such as limits, indices, series, solutions of equations and trigonometry need to be consolidated. Some candidates had difficulty distinguishing between an A.P. and a G.P., and displayed considerable weaknesses in algebraic manipulation and simplification of equations; the strengthening of these skills would enhance performance of candidates even further, and should be encouraged at the school level.

By and large, the results were encouraging.

DETAILED COMMENTS

UNIT II

PAPER 01

SECTION A (Module 2:1 Calculus II)

Question 1

Specific Objective(s): (a) 2, 3, 4, 7, 8, 9, 10

This question tested the exponential function and the theory of quadratic equations.

Most candidates were able to find the coordinates (c, d) and (m, n) but experienced difficulty in finding (a, b). Many candidates made errors in solving the equation $e^{2x} = 2e^{x}$.

Answers: (c, d) = (0, 2);   (a, b) = (ln 2, 4);   (m, n) = (0, 1)

Question 2

Specific Objective(s): (a) 4, 10

This question related e^{x} and e^{-x} in the context of quadratic equations.

Many candidates used the substitution y = e^{x} to obtain the quadratic equation $y^2 - 4y - 1 = 0$ but some of them failed to follow through to the correct solution. The better candidates were able to solve the quadratic by completing the square.

Question 3

Specific Objective(s): (b) 1

This question required using the fact that $\lim_{x\to 0} \frac{\sin x}{x} = 1$ to find other limits involving sin 2x and sin 3x as $x \to 0$.

Many candidates obtained the correct result by faulty reasoning; for example, $\lim_{x\to 0} \frac{\sin 2x}{x} = \lim_{x\to 0} 2\frac{\sin x}{x} = 2\ (1) = 2,$ and $\lim_{x\to 0} \frac{\sin 2x}{\sin 3x} = \lim_{x\to 0} \frac{2}{3}\frac{\sin x}{x} = \frac{2}{3}$.

There were some good responses but too few in numbers.
Answers: (a) 2 (b) 2/3

Question 4

Specific Objective(s): (b) 4, 5, 7, 8

This question covered the topics of implicit differentiation and the chain rule.

(a) Some candidates did not appear to be familiar with the difference between $\sin^2 x$ and $\sin x^2$, and as a consequence several errors were made in the differentiation.

(b) The majority of candidates were able to perform the differentiation correctly but some did not obtain the correct expression for $\frac{dy}{dx}$ in terms of x and y.

Answers: (a) $2x \sin x^2 + 2x^3 \cos x^2$

(b) $-\,{}^{10}\!/_{3}$

Question 5

Specific Objective(s): (c) 1, 3

This question tested the resolution of a proper rational function into partial fractions and the integration of the resultant expressions.

This question was well done. Candidates were able to integrate properly and to recognise that

$$\frac{1}{2}\ln x - \frac{1}{2}\ln(x+2) = \frac{1}{2}\ln\left(\frac{x}{x+2}\right),\ x > 0\,.$$

In a few cases, candidates omitted the constant of integration k.

SECTION B (Module 2.2: Sequences, Series and Approximations)

Question 6

Specific Objective(s): (a) 1, 3

This question dealt with sequences and limits, and required the use of the limit theorems in Unit 1, Module 1.3 to obtain the solution.

The question was attempted by approximately half the candidates but few obtain any substantial marks for their efforts. The main principle,

$\lim\limits_{n\to\infty} x_n = l \Rightarrow \lim\limits_{n\to\infty} x_{n+1} = l$, was missed by several candidates.

See model solution in Appendix.

Question 7

Specific Objective(s): (b) 3, 4, 6

This question related to geometric progressions.

Many candidates achieved full marks for this question. Some of those who did not obtain complete answers used the incorrect formula for the sum to infinity of a geometric series despite the availability of a formula sheet or performed the incorrect algebraic manipulation in obtaining *2h* as the subject in the equation $\frac{h}{1-k} = 2h$. Some candidates did not simplify $\frac{h}{2h}$ to obtain $\frac{1}{2}$.

Answer: k = $\frac{1}{2}$

Question 8

Specific Objective(s): (c) 3, 4, 5

This question was devoted to arithmetic progressions.

Many candidates attempted this question and several obtained full marks, nevertheless, weaknesses were observed in the following areas:

(i) Simplification of algebraic equations

(ii) Application of the correct formula for the sum to n terms of an A.P.

(iii) Solutions to quadratic equations leading to difficulty in solving n(n + 2) = 168

Answers: (a) d = 2 (b) n = 12

Question 9

Specific Objective(s): (c) 1, 2

This question related to the binomial theorem and its use in expansions of the form $(a+x)^n$ for positive integers n. Indices played a role in the solution.

There were several attempts at this question. Many candidates were able to state the appropriate form of the general term in the expansion of $\left(2x^3 - \frac{1}{x}\right)^8$ but could not follow through to the correct solution. Many candidates did not raise the coefficient 2 in the term $2x^3$ to the required power in the final calculation.

Answer: 112

Question 10

Specific Objective(s): (e) 1, 3

The question tested techniques for finding approximate roots of an equation. Part (a) was more popular than Part (b).

(a) Several candidates did this part but careless evaluation of $f(1)$ and $f(-1)$, and failure to draw a conclusion about the existence of a root led to a loss of marks.

(b) Many candidates who used the Newton-Raphson method of approximation obtained full marks for this part. Other methods were also used. A few candidates lost credit due to carelessness in calculating $f(0.4)$ and $f'(0.4)$.

SECTION C (Module 1.3: Probability and Mathematical Modelling)

Question 11 (a)

Specific Objective(s): (a) 1, 2, 3, 8, 9

(a) This question was generally well done. Most candidates recognised and applied the formula $P(A \cup B) = (PA) + P(B) - P(A \cap B)$ to obtain the solution.

(b) This part was well done. Some candidates had difficulty in distinguishing between the addition law and the multiplication law of probability.

Answers: (a) 0.4

(b) 0.8

Question 12

Specific Objective(s): (a) 1 – 6, 8 – 10

(a) This question was generally well done, however, some candidates simplistically reasoned that the probability that 'No boy hit the wicket' was the same as 1 – P (all 3 boys hit the wicket). This resulted in that category of candidates obtaining incorrect answers.

(b) Some candidates found difficulty in setting out the respective probabilities of one boy hitting the wicket and the corresponding probabilities of the other two boys missing the wicket. The addition law was not applied correctly to obtain the required answers.

Answers: (a) 0.417

(b) 0.431

Question 13

Specific Objective(s): (a) 1, 2, 3, 10, 12

(a) Very few candidates found this question difficult. It was readily recognised that the formula $P(A \cap B) = (PA) + P(B) - P(A \cup B)$ was required.

(b) The majority of candidates did not recognise the conditional probability in this part which suggested that more emphasis and practice should be applied to questions of this type.

Answers: (a) 0.05

(b) 0.25

Question 14

Specific Objective(s): (a) 3, 5, 6, 7, 11

(a) Few difficulties were encountered with this part of the question. Candidates were able to identify the appropriate formulae to find the respective probabilities.

(b) Some candidates did not display a high degree of understanding that $P(A \cap B') = P(A) - P(A \cap B)$. However, those candidates who used a Venn diagram were able to obtain the correct results.

Answers: (a) 0.75 (b) 0.125

Question 15

Specific Objective(s): (a) 3, 4, 5, 6, 13; (b) 2, 3

(a) Some candidates did not recognise the distribution as binomial, however, those candidates who defined the correct distribution were able to arrive at the correct answers.

(b) Many candidates stated the correct formula for calculating the probabilities of $P(X \leq 2) = P(X = 0) + P(X = 1) + P(X = 2)$, however, errors crept into the calculations and this led candidates to approximate their answers too early to obtain correct results to three significant figures.

Answers: (a) 0.591 (b) 0.991

PAPER 02

SECTION A (Module 2.1: Calculus II)

Question 1

This question dealt with the differentiation of the exponential function e^{-x}, the laws of logarithms, differentiation by the chain rule and integration parts.

(a) Specific Objective(s): (b) 3; (c) 5

(i) This question was well done. A small number of candidates did not use the quotient rule to differentiate e^{-x}.

(ii) This part was very well done. There were a few instances where there was confusion with the signs of the terms. The constant of integration was often omitted.

(b) Specific Objective(s): (b) 4, 8, 9

This question was very popular with the better candidates, however, the changing of the signs often created difficulties for the weaker candidates.

(c) Specific Objective(s): (a) 9, 10

(i) Both parts of this question were reasonably well done. A few candidates did not recognise that $\log_2 2 = 1$.

(ii) This question was popular with most candidates. Some candidates were able to derive the quadratic equation $y^2 - 2y - 8 = 0$ but could not factorise the equation correctly.

Answers: (a)(ii) $-x^2e^{-x} - 2xe^{-x} - 2e^{-x} + k$

(c)(i) a) $\log_2 x = y$ b) $\log_{x^2} = \frac{1}{y}$

(ii) $x = 16, x = ¼$

Question 2

Specific Objective(s): (b) 6, 8; (c) 1 – 4; 6, 7

(a) This was a question on partial fractions. Several candidates obtained full marks on this part. Those who did not often experienced difficulty with the basic algebraic manipulation. A few candidates incorrectly used B instead of Bx + C as numerator for the factor $x^2 + 1$.

(b) Many candidates seemed familiar with the derivative of the basic trigonometric function cos x, but a few candidates forgot the minus sign. Another group of candidates did not convert $\cos^2 x$ to $1 - \sin^2 x$, and hence could not complete the integration process.

(c) This part was fairly well done. Some candidates could not identify the proportional increases for t =1, t = 2, while some others had a general idea of differential equations but failed to come to grips with the details of the question.

Answers: (a) $\frac{1}{x} - \frac{2x}{x^2+1}$ (b) $\frac{1}{3}\cos^3 x - \cos x + \mathrm{k}$

(c)(ii) (a) $2^{1/3}$ or $\sqrt[3]{2}$

(b) $2^{2/3}$ or $\left(\sqrt[3]{2}\right)^2$

SECTION B (Module 2.2: Sequences, Series and Approximations)

Question 3

Specific Objective(s): (a) 1 – 5; (b) 2

(a) There were several attempts at this question. Candidates readily recognised the first series as an arithmetic progression, but they went on to assume that since one series was on A.P., then a G.P. must also be present. This assumption led to a search for a common ratio in the two latter sequences by some candidates. Specific details to justify the type of sequence were often omitted.

(b) Many candidates attempted this question, but there were weaknesses in the method of finding the n^{th} term in Part (i). However, a good grasp of the principle of Mathematical Induction was exhibited in Part (ii).

Answers: (a) First sequence – divergent since terms increase with n. It is an A.P. with common difference +3.

Second sequence – convergent to zero as $n \to \infty$.

Third sequence – periodic; terms are –1, 1, –1, 1, … that is, pattern is repetitive with each term having absolute value 1 but alternating in sign.

(b)(i) n^{th} term is n(3n – 1).

Question 4

Specific Objective(s): (b) 1, 3, 4, 5, 6; (c) 1, 2

(a) Many candidates attempted this question. Those who did not perform well on this part had difficulty in

(i) establishing a common difference, and

(ii) applying the rules of logarithms appropriately.

(b) This question attracted a high degree of response. Many candidates obtained 5 – 8 marks (maximum 8 marks) on this part, nevertheless, the following deficiencies were noted:

(i) Method of obtaining the common ratio
(ii) Use of the incorrect formula for the sum
(iii) Poor algebraic manipulation
(iv) The direct use of limits was not applied in Part (iii)

(c) There were many responses to this part. The expansion was well done but simplification presented difficulties to some candidates through faulty groupings of the terms.

Answers: (a) (i) $S_n = 21$

(b) (ii) $\frac{4}{7}\left(1 - 2^{-3n}\right)$

(iii) AS $n \to \infty$, $2^{-3n} \to 0$, hence sum $\to \frac{4}{7}$.

(c) (i) The first three terms are: $16 + (32 - 16u)\,x + (24 - 32u)\,x^2$
(ii) a) $u = \frac{3}{4}$

b) 20

SECTION C (Module 2.3: Probability and Mathematical Modelling)

Question 5

Specific Objective(s): (a) 1 – 5, 7, 9 – 12

This question was generally well done. Candidates were proficient in dealing with the tree-diagram and with the concept of selection without replacement. A few errors were made in arithmetic calculation. The use of the table was well understood.

Answers: (a)(i) $\frac{37}{120} = 0.308$ (ii) $\frac{7}{24} = 0.292$

(b)(i) $\frac{82}{150} = 0.547$ (ii) $\frac{114}{150} = 0.760$ (iii) $\frac{94}{150} = 0.627$ (iv) $\frac{27}{82} = 0.329$

(v) $\frac{82}{150} \times \frac{81}{149} = 0.297$

Question 6

Specific Objective(s): (b) 1, 3, 4, 5

(a) This part was poorly done. More practice on such fundamental concepts is needed.

(b) (i) Most candidates found no difficulty in forming the differential equation to model the given situation.

(ii) In general, candidates did not readily recognise that the rational function $\frac{1}{p(1-p)}$ should be resolved into partial fractions before attempting to evaluate $\int \frac{1}{p(1-p)}\,dt = \int k\ dt$. As a consequence, this part presented some difficulty.

(iii) This part was not well done. Of those candidates who were able to solve the differential equation in Part (ii), approximately 70 per cent were able to obtain this result.

(iv) This part merely required substitution into the formula in Part (iii) to obtain a numerical result.

See model solution in Appendix.

UNIT 2

PROJECTS

Although the majority of projects had a slant towards Statistical Analysis, it was pleasing to note that this year, there were more projects which required pure mathematical concepts, (for example, Calculus) for their solution.

Module Tests

The comments regarding Unit 1 are pertinent to Unit 2 as well. ***See page 14.***

Projects

In general, candidates had well-defined project titles and descriptions of the problems. Most gathered relevant data and used appropriate mathematics for the solution of the mathematical model postulated. However, in a number of cases, candidates needed to pay greater attention to the comparison of their results with the behaviour of the phenomenon studied.

APPENDIX

MODEL SOLUTIONS

UNIT 1, PAPER 02

Question 3

(a) *At A,* $x = 0$ *so* $y = 2 \Rightarrow A \equiv (0, 2)$

At B, $y = 0$ *so* $x = 3 \Rightarrow B \equiv (3, 0)$

Let $C \equiv (X, Y)$*. Then* $\frac{X+0}{2} = 3,\ \frac{Y+2}{2} = 0$

$\Rightarrow C \equiv (6, -2)$

(b) $CD \perp ar\ AB \Rightarrow$ *grad of CD* $= \frac{3}{2}$

$\Rightarrow$ *equat*^{n} *of CD is* $y + 2 = \frac{3}{2}(x - 6)$

that is, $3x - 2y = 22$, *(or* $y = \frac{3x}{2} - 11$*).*

$AD \,//\, 5x + y = 7 \quad \Rightarrow$ *grad of AD is* -5

$\Rightarrow$ *equat*^{n} *of AD is* $y - 2 = -5x$

that is, $y + 5x = 2$

(c) *At D,* $3x - 2(2-5x) = 22$

$\Rightarrow \quad 13x = 26$
$\Rightarrow \quad x = 2$
$\Rightarrow \quad y = 2-10 = -8$
$\Rightarrow D \equiv (2, -8)$

(d) $A\hat{C}D = 90^{\circ} \Rightarrow \Omega ACD = \frac{1}{2} AC \times CD$

$AC = \sqrt{6^2 + 4^2} = \sqrt{52}$ *units*

$CD = \sqrt{4^2 + 6^2} = \sqrt{52}$ *units*

Area of $\Omega ACD = \frac{1}{2}\sqrt{52} \times \sqrt{52} = 26$ *units*^{2}.

UNIT 2, PAPER 1

Question 6

$$\lim_{n\to\infty} x_n = l \Rightarrow \lim_{n\to\infty} x_{n+1} = l$$

Now $$\lim_{n\to\infty} x_{n+1} = \lim_{n\to\infty} \tfrac{1}{2}\left(x_n + \frac{a}{x_n}\right)$$

$$= \frac{1}{2} \lim_{n\to\infty} x_n + \frac{1}{2} \lim_{n\to\infty} \frac{a}{x_n}$$

$$\Rightarrow \quad l = \frac{1}{2} l + \frac{1}{2} \frac{a}{l}$$

$$\Rightarrow \quad 2l^2 = l^2 + a$$

$$\Rightarrow \quad l^2 = a$$

$$\Rightarrow \quad l = \sqrt{a}, \quad a > 0 \text{ since } l > 0$$

UNIT 2, PAPER 02

Question 6

(a) Let $e^{\ln y} = z$.
Then, $\ln y = \ln z$, so $y = z$.

(b) (i) $\frac{dp}{dt} = kp(1-p)$, k is a constant.

(ii)(i) $$\Rightarrow \frac{dp}{p(1-p)} = k\,dt$$

$$\Rightarrow \left(\frac{1}{p} + \frac{1}{1-p}\right) dp = k\,dt$$

$$\Rightarrow \ln p - \ln(1-p) = kt + c, \quad c \text{ is a constant, } p>0$$

$$\Rightarrow \ln\left(\frac{p}{1-p}\right) = kt + c, \quad \text{since } 0<p<1$$

$$\Rightarrow \quad \frac{p}{1-p} = e^{kt+c}$$

$$\Rightarrow \quad \frac{p}{1-p} = a e^{kt}, \quad a = e^c \text{ is a constant}$$

$$\Rightarrow \quad p = a e^{kt}(1-p)$$

$$\Rightarrow \quad p \quad = \quad \frac{ae^{kt}}{1+ae^{kt}}$$

(iii) $\quad t = 0,\ p = \frac{1}{10} \quad \Rightarrow \quad \frac{1}{10} = \frac{a}{1+a}$

$$\Rightarrow \quad a = \frac{1}{9}$$

$t = 2,\ p = \frac{1}{5} \quad \Rightarrow \quad \frac{1}{5} = \frac{ae^{2k}}{1+ae^{2k}}$

$$\Rightarrow \quad 1+ae^{2k} = 5\,ae^{2k}$$

$$\Rightarrow \quad 4\,ae^{2k} \quad = 1$$

$$\Rightarrow \quad e^{2k} = \frac{1}{4a} \quad = \frac{9}{4}$$

$$\Rightarrow \quad 2k = ln\ \frac{9}{4}$$

$$\Rightarrow \quad k = \frac{1}{2}\ ln\ \frac{9}{4} = ln\left(\frac{9}{4}\right)^{1/2} = ln\ \frac{3}{2}$$

From Part (a),* $\quad p \;=\; \dfrac{\frac{1}{9} \times \left(\frac{3}{2}\right)^t}{1+\frac{1}{9}\left(\frac{3}{2}\right)^t} = \dfrac{\left(\frac{3}{2}\right)^t}{9+\left(\frac{3}{2}\right)^t}$

$$\Rightarrow \quad 9p + p\left(\frac{3}{2}\right)^t = \left(\frac{3}{2}\right)^t$$

$$\Rightarrow \quad 9p = \left(\frac{3}{2}\right)^t (1-p)$$

$$\Rightarrow \quad \frac{9p}{1-p} = \left(\frac{3}{2}\right)^t,\ since\ p < 1$$

(iv) $\quad t = 4 \quad \Rightarrow \quad \dfrac{9p}{1-p} = \left(\dfrac{3}{2}\right)^4 = \dfrac{81}{16}$

$$\Rightarrow \quad \frac{p}{1-p} = \frac{9}{16}$$

$$\Rightarrow \quad 16p = 9 - 9p$$

$$\Rightarrow \quad p = \frac{9}{25}$$