C A R I B B E A N   E X A M I N A T I O N S   C O U N C I L

REPORT ON CANDIDATES' WORK IN THE SECONDARY EDUCATION CERTIFICATE EXAMINATION

JANUARY 2012

MATHEMATICS
GENERAL PROFICIENCY EXAMINATION

GENERAL COMMENTS

The General Proficiency Mathematics examination is offered in January and May/June each year. There was a candidate entry of approximately 14 200 in January 2012. Forty per cent of the candidates earned Grades I–III. The mean score for the examination was 77.14 out of 180 marks.

DETAILED COMMENTS

Paper 01 – Multiple Choice

Paper 01 consisted of 60 multiple-choice items. This year, 24 candidates each earned the maximum available score of 60. Sixty-one per cent of the candidates scored 30 marks or more.

Paper 02 – Structured Questions

Paper 02 consisted of two sections. Section I comprised eight compulsory questions totalling 90 marks. Section II comprised three optional questions: one each from (i) Algebra, Relation, Functions and Graphs; (ii) Measurement, Trigonometry and Geometry; and (iii) Vectors and Matrices. Candidates were required to answer any two questions from this section. Each question in this section was worth 15 marks.

This year, three candidates earned the maximum available mark of 120 on Paper 02. Approximately 21 per cent of the candidates earned at least half the maximum marks on this paper.

Compulsory Section

<u>Question 1</u>

This question tested candidates’ ability to

- square and divide fractions involving mixed numbers
- determine the square root of and add decimal numbers
- write a decimal number in standard form
- solve problems involving wages and overtime.

The question was attempted by 99 per cent of the candidates, 9.6 per cent of whom earned the maximum available mark. The mean mark was 7.89 out of 12.

In general, candidates provided satisfactory responses to this question. However, some incorrect responses provided useful insights into some misconceptions held by candidates.

These included for Part (a):

(i) $\left(1\frac{3}{4}\right)^2 = 2\frac{6}{8}$ (Each digit is multiplied by 2 instead of squaring the numerator and denominator in the equivalent improper fraction)

(ii) $\left(1\frac{3}{4}\right)^2 = 1\frac{9}{16}$ (Each digit in the mixed number is squared)

In Part (b), while most candidates were able to calculate the basic weekly wage, a large proportion of candidates experienced difficulty in determining the overtime wage for one hour. Incorrect attempts included either the multiplication of the basic hourly wage by 2.5 or by 0.5 instead of by 1.5.

Solutions

(a) (i) $\frac{7}{8}$ (ii) 0.446, 4.46×10^{-1}

(b) (i) $900 (ii) $33.75 (iii) $405 (iv) 16 hours

Recommendations

Teachers should provide students with opportunities to use the calculator to perform basic arithmetic operations on rational numbers. Attention should also be given to finding the square and square root of improper fractions and decimals.

<u>Question 2</u>

This question tested candidates' ability to

- solve a pair of simultaneous linear equations
- factorize quadratic functions involving the difference of squares and grouping
- solve worded problems involving simple linear equations.

The question was attempted by 99 per cent of the candidates, 2.5 per cent of whom earned the maximum available mark. The mean mark was 3.54 out of 12. The performance of candidates on this question was generally unsatisfactory.

In Part (a), which required candidates to solve simultaneous equations in two unknowns, a large proportion of candidates showed some proficiency on this objective but some were unable to correctly complete the process. Many of them made errors in attempting to eliminate one of the variables as they experienced some difficulty with directed numbers.

For Part (b), which involved the factorization of quadratic expressions, few candidates demonstrated an awareness of the difference of squares and, as a consequence, the factors were not obtained for $x^2 - 16$. Factorizing $2x^2 - 3x + 8x - 12$ was more competently done. Nevertheless, even when candidates were able to identify the common factors, they encountered difficulty with the signs.

In Part (c), few candidates were able to show that the total amount spent on the 28 tickets was $\$(15x + 420)$. Further, they did not combine the separate expressions $(28 - x) \times 15$ and $30x$ to obtain the stated result.

Solutions

(a) $x = 3, y = 2$
(b) (i) $(x - 4)(x + 4)$ (ii) $(2x - 3)(x + 4)$
(c) (i) a) $(28 - x)$ b) $30x$ c) $15(28 - x)$ (ii) $15x + 420$ (iii) 16

Recommendations

Teachers should engage students in translating worded phrases into algebraic expressions and equations. The difference between squares is a useful tool in factorization and needs special attention by teachers and students.

<u>Question 3</u>

This question tested candidates' ability to

- identify and list odd and prime numbers
- draw a Venn diagram to illustrate the relationship between two sets
- construct a triangle which included an angle of $45°$
- construct a perpendicular to a line from a point outside that line
- determine the measure of an angle.

The question was attempted by 99 per cent of the candidates, 1.4 per cent of whom earned the maximum available mark. The mean mark was 5.58 out of 12.

In Part (a), most candidates were able to list the odd numbers from the elements in the set but were however unable to identify the prime numbers. Generally, candidates recognized that the subsets are enclosed by the universal set and were able to represent on the Venn diagram the subsets and the members of $(A \cup B)^I$ correctly.

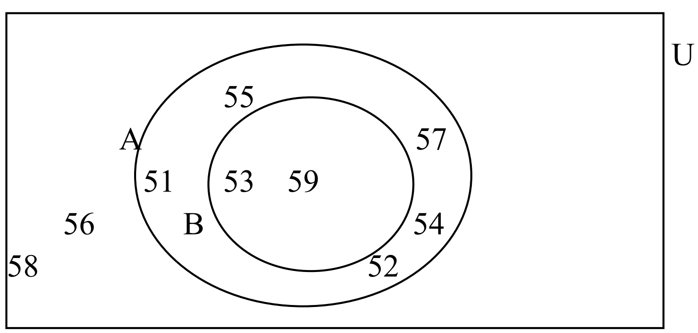
In Part (b), candidates were able to use the ruler to draw at least one line accurately and to complete and label the triangle CDE with the 45° angle at the point D. They, however, were unable to construct the 90° angle, and resorted to the use of either the set square or the protractor to draw the 90° angle, following which they used a pair of compasses to bisect that angle to obtain $\angle$CDE = 45°. However, most candidates simply used their protractor to draw the angle CDE = 45°.

Constructing the perpendicular from C to DE was outside of the level of competence for most candidates. Many of them resorted to constructing the perpendicular bisector of DE, which does not pass through C. A large proportion of candidates drew the perpendicular line from C to DE but located the point F other than at the point of intersection of the two lines.

Solutions

(a) (i) $A = \{51, 53, 55, 57, 59\}$ (ii) $B = \{53, 59\}$

(iii)



Recommendations

Teachers are advised to review sets of numbers such as prime, composite, odd, even, square, natural, integer, whole, rational, irrational and real, as well as the divisibility laws as they pertain to identifying odd and prime numbers. In addition, the appropriate representation of sets in Venn diagrams needs to be taught. Instruction in constructing polygons should also include the construction of angles and perpendicular lines.

Question 4

This question tested candidates' ability to

- determine the time interval between two times written in the 24-hour clock notation
- calculate distance in km given time in minutes and speed in km/h
- convert from litres to cubic centimetres
- calculate the height of a cuboid given the length and width of its base
- derive and solve a simple linear equation in one unknown using concepts of area and volume of a cuboid.

The question was attempted by 98 per cent of the candidates, 5.0 per cent of whom earned the maximum available mark. The mean mark was 3.91 out of 11.

Candidates were proficient at evaluating the time in minutes between 7:35 and 7:45 on the 24−hour clock. They however experienced difficulty in determining the time elapsed between 6:40 and 7:35, with the most common result being 95 minutes instead of 55 minutes.

The formula, $distance = speed \times time$, was well known but attention was not paid to the different units used in the question. The inconsistency with the use of units was again evident in Part (b). Here, candidates knew that $h = \frac{V}{A}$, but did not know how to convert litres to cubic centimetres. A frequent incorrect conversion was $4.8\, l = 480\, cm^3$. The greatest challenge candidates encountered was in expressing area in terms of a variable h, and in establishing the equation connecting the volume of the cuboid to 286.

Solutions

(a) (i) 10 minutes (ii) 55 minutes (iii) 49.5 km
(b) 16 cm
(c) (i) $4h\, cm^2$ (ii) $52h\, cm^3$ (iii) 5.5 cm

Recommendations

Teachers should give special attention to conversion of units. Emphasis should be placed on the value of a quantity based on its unit; for example: $4.8l > 300cm^3$. A common-sense approach to the calculation of time elapsed should be encouraged so that when Mathematics is applied, the result of the calculation fits the estimation.

Question 5

This question tested candidates' ability to

- use the properties of parallel lines, transversals, triangles and angles to determine the measure of angles
- state the coordinates of points on a graph
- fully describe a simple transformation given an object and its image
- determine the coordinates of the images of two points after a translation by a given vector.

The question was attempted by 92 per cent of the candidates, 3.1 per cent of whom earned the maximum available mark. The mean mark was 3.44 out of 10.

The performance of candidates on this item was generally unsatisfactory. In Part (a), candidates were able to recognize that the sum of the internal angles of a triangle is $180°$ and that alternate angles are equal. However, they were less familiar with recognizing the equality of corresponding angles. A glaring deficit in their responses was failure to give correct reasons to support their calculations.

In Part (b), candidates recognized the transformation as being a reflection but were unable to give a clear description using appropriate vocabulary. The greatest challenge was identifying the mirror line of the reflection.

Solutions

(a) (i) $72°$ (ii) $22°$ (iii) $72°$ (iv) $86°$
(b) (i) $P(2, 1)$ $Q(4, 3)$ (ii) Reflection in the X-axis (iii) $P^{//}(5, -5)$ $Q^{//}(7, -3)$

Recommendations

Teachers are advised to use the correct mathematical language when explaining concepts in geometry and encourage students to include reasons for the answers when performing calculations in geometry. Students should be taught to test the plausibility of their answers based on information given in the question (for example, $122°$ cannot be the measure of an acute angle). Teachers should give students adequate practice in performing transformations, and in stating the main features needed for describing a transformation.

Question 6

This question tested candidates' ability to

- complete a table of values for a quadratic function over a given domain
- plot points on a pair of axes and draw a smooth curve through the points
- use a quadratic graph to estimate the value of y for a given value of x
- state the equation of the line of symmetry associated with the graph of a quadratic function
- use a quadratic graph to estimate the minimum value of a quadratic function
- determine the roots of a quadratic equation from a quadratic graph.

The question was attempted by 92 per cent of the candidates, 2.7 per cent of whom earned the maximum available mark. The mean mark was 4.80 out of 11.

Candidates' performance on this item was generally satisfactory. The majority of them were able to calculate the missing values of y in the table of values, and even those who encountered difficulties in so doing, proceeded to correctly plot the points given to obtain the graph of the function. There were some challenges estimating, from the graph, the value of y which corresponds to a given value of x. Generally, candidates did not know how to illustrate the method used to obtain their solution by drawing lines on the graph. They demonstrated competence at reading the minimum value of the function, but were unable to determine the equation of the line of symmetry and the roots of the equation $x^2 - 2x - 3 = 0$ from the graph.

Solutions

(a) when $x = -1, y = 0$ and when $x = 2, y = -3$
(b) minimum value quadratic curve with roots at -1 and 3, y-intercept at -3, minimum point at (1, -4)
(c) $y = 2.25$
(d) (i) $x = 1$ (ii) $y = -4$ (iii) $x = -1, x = 3$

Recommendations

Teachers should provide candidates with practice in writing the equations of lines parallel to the coordinate axes. Students should be taught that curves are drawn free hand and not against straight edges. They should be exposed to writing equations and in-equations and to determine the solutions associated with each type.

<u>Question 7</u>

This question tested candidates' ability to

- complete a grouped frequency table to show mid-interval values and frequencies using information provided on a histogram
- identify the median class associated with a histogram
- determine from a histogram the number of items in the data set
- calculate an estimate of the mean of a grouped data set
- estimate what proportion of the data set is above a given value.

The question was attempted by 92 per cent of the candidates, 3.0 per cent of whom earned the maximum available mark. The mean mark was 4.27 out of 12.

Candidates performed satisfactorily on this question. A number of candidates were unfamiliar with the term 'modal class' and proceeded, in many instances, to state the highest frequency (25) instead of the interval (11–20) which had the highest frequency. When calculating the mean, many candidates divided by the number of intervals (5) or by 50 which is the largest limit in the table. A large number of candidates did not equate the total number of seedlings with the total frequency. For example, they added the upper class limits to obtain 150, or they summed the mid-interval values to obtain 127.5. Candidates in general did not use the sum of products to estimate the mean for the grouped frequency distribution. In many instances, the 5 mid-values were added and in several other cases the 5 frequencies were summed.

Solutions

(a)

Height in cm	**Mid-point**	**Frequency**
21–30	25.5	23
31–40	35.5	20
41–50	45.5	14

(b) (i) 11–20 (ii) 100 (iii) 24.2 cm (iv) $\frac{34}{100}$

Recommendations

Teachers need to emphasize the difference between the measures — mode, median and mean — and to distinguish between median class and modal class. They should use more practical examples to assist students in understanding the concept of probability. Further, greater emphasis should be placed on the extraction and interpretation of information from statistical graphs.

Question 8

This question tested candidates' ability to

- draw the fourth figure in a sequence of shapes given the first three figures in the sequence
- complete a table to show the terms in a sequence of numbers
- determine the term of a sequence
- derive the general equation connecting the value of the function with the term of the sequence.

The question was attempted by 95 per cent of the candidates, 10.1 per cent of whom earned the maximum available mark. The mean mark was 6.60 out of 10.

Generally, the performance of candidates on this question was satisfactory, with a large proportion of candidates recognizing the pattern of squares in the sequence. Various strategies were used to compute the number of straws used in the fourth figure. These included counting, use of formulae and following the pattern. However, some candidates were unable to deduce the correct formula and ended up with the incorrect number of straws used for the fourth figure. Trial and error was commonly used by candidates in an attempt to deduce the value of *n* for which the number of straws totalled 106. In addition, a large number of candidates omitted Part (d) of the question which required that they derive a formula to connect the number of the figure to the number of straws used.

Solutions

(b)

Figure	**Total Number of Straws**	
	Formula	**Number**
4	4(6) - 3	21
10	10(6) - 9	51

(c) 21

(d) 6n-(n-1) = 5n+1

Recommendation

Teachers should assist students with writing formulae and the expressions derived from number sequences.

Optional Section

Question 9

This question tested candidates' ability to

- change the subject of the formula for a rational function
- determine the inverse of a rational function
- determine, for a rational function, the value in the domain that is mapped onto zero in the range
- solve problems using linear techniques.

The question was attempted by 49 per cent of the candidates, 1.0 per cent of whom earned the maximum available mark. The mean mark was 2.40 out of 15.

Candidates' performance on this question was generally unsatisfactory. While some candidates were able to correctly write down the coordinates of at least one vertex and to substitute their values in the profit function, most of the candidates did recognize the relationship between transposing the formula and finding the inverse of the function. In fact, a few got Part (a) (i) incorrect but were able to use the same technique required to find the inverse of the function. In Part a (iii), $f(0)$ was often seen instead of $f^{-1}(0)$. In Part (b), stating the inequalities proved to be problematic as seen from the following examples where instead of $x \geq 6$ and $x + y \leq 40$ respectively, many candidates wrote $x > 6$, $x = 6$, $x \leq 6$, $x < 6$ and $x + y = 40$, $x + y > 40$, $x + y < 40$, $x + y \geq 40$.

Solutions

(a) (i) $x = \frac{4y + 3}{y - 2}$ (ii) $f^{-1}(x) = \frac{4x + 3}{x - 2}$ (iii) $x = \frac{-3}{2}$

(b) (i) $x \geq 6,\ x + y \leq 40$ (ii) $(6, 2), (6, 34),\ (30, 10)$ (iii) $(30, 10)$

Recommendations

Teachers should place more emphasis on questions involving linear inequalities. The difference between the signs for different inequalities should be clearly explained when students are in junior school. When teaching the solution of linear equations, more examples on solving literal equations should be done so that students can see the relationship between solving such equations and transposing formulae.

Question 10

This question tested candidates' ability to

- calculate the angle subtended at the centre of a regular hexagon by one of its sides
- determine the area of a regular hexagon given the length of one side
- use trigonometric ratios and formulas to solve problems related to bearings and three-dimensional figures.

The question was attempted by 43 per cent of the candidates, 1.2 per cent of whom earned the maximum available mark. The mean mark was 2.89 out of 15.

Candidates' performance on this question was generally unsatisfactory. In Part (a), a large number of candidates were able to evaluate the measure of angle AOB and quite a few also knew that to find the area of the hexagon they needed to multiply the area of one triangle by 6. However, a number of candidates assumed that triangle AOB was right-angled and used OA as the height and AB as the base of the triangle. Part (b) proved more challenging for candidates as many of them could not identify the angle of elevation and the right angles in the three-dimensional drawing. It was common to see the angle PKM used as the angle of elevation instead of angle PKL. A large number of candidates knew that they could use the cosine formula for calculating the length of KM in triangle KLM. Nevertheless, even when they substituted correctly into the formula, they could not complete the response since they encountered difficulty with the use of directed numbers.

Solutions

(a) (i) 60° (ii) $65\ cm^2$
(b) (ii) a) $7.98\ m$ b) $28.8\ m$ c) 23°

Recommendations

Teachers should provide students with sufficient practice in calculating the areas of triangles using the formula $A = \frac{1}{2}ab\ SinC$. They should employ a more practical approach to teaching trigonometry involving three-dimensional situations, assisting students in differentiating between the perpendicular height and the slant height.

Question 11

This question tested candidates' ability to

- write the coordinates of points as position vectors
- derive a displacement vector
- solve problems in geometry using vectors
- add and multiply matrices
- find the inverse of a two by two matrix
- solve a matrix equation with two unknowns using the inverse method approach.

The question was attempted by 65 per cent of the candidates, 2.3 per cent of whom earned the maximum available mark. The mean mark was 4.33 out of 15.

While a number of candidates were able to write the coordinates of points as position vectors, determine displacement vectors and identify the correct route for summing vectors, they were unable to write the inverse of vectors whose additive inverses were required as part of the route. Ina addition, challenges were experienced with the multiplication of two matrices, inverting a matrix and using the inverse of a matrix to solve a system of linear equations. Many candidates resorted to re–writing the linear equations associated with the matrix equation and solving the system by the process of elimination or by substitution.

Solutions

(a) (i) a) $\overrightarrow{OA} = \begin{pmatrix} -1 \\ 3 \end{pmatrix}$ b) $\overrightarrow{OB} = \begin{pmatrix} 5 \\ 1 \end{pmatrix}$ c) $\overrightarrow{BA} = \begin{pmatrix} -6 \\ 2 \end{pmatrix}$

(ii) a) $\overrightarrow{BG} = \begin{pmatrix} -3 \\ 1 \end{pmatrix}$ b) $\overrightarrow{OG} = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$

(b) (i) $\begin{pmatrix} 1 & 8 \\ 1 & 8 \end{pmatrix}$ (ii) $\begin{pmatrix} -3 & 13 \\ -1 & 11 \end{pmatrix}$

(c) (i) $\frac{1}{2}\begin{pmatrix} 1 & -2 \\ -1 & 4 \end{pmatrix}$ (ii) $x = 1;\ y = 2$

Recommendations

Teachers should utilize diagrams including the Cartesian plane in the teaching of vector algebra. They should emphasize that direction in vectors is important and ought to be stressed in addition to the magnitude. Further, they should provide students with more practice in matrix multiplication and using the matrix method to solve a pair of simultaneous equations in two unknowns.