C A R I B B E A N   E X A M I N A T I O N S   C O U N C I L

REPORT ON CANDIDATES' WORK IN THE ADVANCED PROFICIENCY EXAMINATION

MAY/JUNE 2011

PURE MATHEMATICS

GENERAL COMMENTS

In 2011, approximately 5,855 and 2,970 candidates wrote the Units 1 and 2 examinations respectively. Performances continued in the usual pattern across the total range of candidates with some candidates obtaining excellent grades while some candidates seemed unprepared to write the examinations at this level, particularly in Unit 1.

The overall performance in Unit 1 was satisfactory, with several candidates displaying a sound grasp of the subject matter. Excellent scores were registered with specific topics such as Trigonometry, Functions and Calculus. Nevertheless, candidates continue to show weaknesses in areas such as Modulus, Indices and Logarithms. Other aspects that need attention are manipulation of simple algebraic expressions, substitution and pattern recognition as effective tools in problem solving.

In general, the performance of candidates on Unit 2 was satisfactory. It was heartening to note the increasing number of candidates who reached an outstanding level of proficiency in the topics examined. However, there was evidence of significant unpreparedness by some candidates.

Candidates continue to show marked weakness in algebraic manipulation. In addition, reasoning skills must be sharpened and analytical approaches to problem solving must be emphasized. Too many candidates demonstrated a favour for problem solving by using memorized formulae.

DETAILED COMMENTS

UNIT 1

Paper 01 – Multiple Choice

Paper 01 comprised 45 multiple-choice items. Candidates performed satisfactorily, with a mean score of 55.10 and a standard deviation of 20.02.

Paper 02 – Structured Questions

Section A

Module 1: Basic Algebra and Functions

<u>Question 1</u>

Specific Objectives: (a) 5; (b) 1, 3, 4; (c) 1−3(iii); (d) 5; (e) 2; (f) 4; (g) 1

The topics examined in this question covered the Remainder and Factor Theorems, simultaneous equations, logarithms, inequalities, cubic functions and indices.

In general, both sections of the problem in Part (a) were handled well by the candidates. However, there were many cases in which the requisite skills for manipulation of the expressions were lacking.

Part (a) was attempted by almost all candidates and approximately 30 per cent scored between zero and five marks. Approximately 10 per cent of candidates scored between 21 and 25 marks.

In Part (a) (i) where manipulation of surds was the focus, some candidates did not recognize $(\sqrt{75} + \sqrt{12})^2 - (\sqrt{75} - \sqrt{12})^2$ as a difference of two squares and therefore missed out on a more efficient solution. However, those who saw this expression as a difference of two squares were also able to complete the subsequent manipulation required to arrive at the correct answers. Most candidates expanded the two bracketed terms then proceeded to further manipulation.

For Part (a) (ii), the manipulation of indices and the bases of the indices were the foci. Many candidates recognized that 3 was the smallest common base of the terms in the expression $27^{\frac{1}{4}} \times 9^{\frac{1}{8}} \times 81^{\frac{1}{8}}$, but several could not, complete the solution.

Approximately 90 per cent of the candidates responded well to Part (b) and were able to arrive at the correct answer. Identification from the graph was well done.

Many candidates were able to form the two simultaneous equations in Part (b) (ii). However, some of them were not able to solve the simultaneous equations.

Approximately 80 per cent of candidates who attempted Part (b) (iii) got the correct solution. Although most candidates were not able to factorize the polynomial, they were able to identify the x-value from the given graph.

Part (c) (i) focused on the solution of a quadratic equation involving logarithms. The equation which the candidates were required to solve was $\sqrt{\log_2 x} = \log_2 \sqrt{x}$ and the substitution $y = \log_2 x$ was provided as a possible means of solving the equation. In general, performance on this item was poor and candidates' attempt at solving the problem faltered at the substitution phase due to improper application of the laws of logarithms to transform the right hand side of the equation into a form that allowed the appropriate substitution. These candidates were unable to recognize that $\log_2 \sqrt{x} = \frac{1}{2} \log_2 x$ and hence failed to substitute $\frac{1}{2}y$ in its place. Candidates who completed the substitution correctly divided throughout by y instead of factorizing, thus losing one of the solutions.

Candidates found Part (c) (ii) the most difficult phase of the entire question. This item required solutions to the quadratic inequality $x^2 - |x| - 2 < 0$. Candidates performed poorly on this item due to a lack of understanding of how to deal with the modulus (absolute value) in such a context. In many cases, candidates just dropped both the modulus and the inequality signs and proceeded to solve $x^2 - x - 12 = 0$. In other cases, the inequality sign was retained after the removal of the modulus but only one resulting inequality was recognized ($x^2 - x - 12 < 0$). The candidates did not realize that another valid inequality was $x^2 + x - 12 < 0$, which also contributed to the overall solution.

Solutions:

(a) (i) 120 (ii) $3^2 = 9$

(b) (i) $p = 4$ (ii) $m = -1, n = -4$ (iii) $x = -2at$ Q

(c) (i) $x = 1, x = 16$ (ii) $1x\ 1 < 4$ $(-4 < x < 4)$

Question 2

Specific Objectives: (a) 6, 8; (d) 7; (f) 3, 5 (i)

This question tested knowledge of the roots of quadratic equations, the evaluation of a function at discrete points and mathematical induction.

Apart from Part (a), performance on this question was weak due, in a large number of cases, to faulty basic algebraic manipulation.

Part (a) dealt with the sum and the product of roots of a quadratic equation. The equation given was $x^2 - px + 24 = 0$ for $p \in \mathbf{R}$ and the problem was divided into two major parts.

Part (a)(i) required that candidates express

a) $\alpha + \beta$ and b) $d^2 + \beta^2$ in terms of p.

Most candidates were able to solve these problems which indicated that they understood the roots of quadratic equations. However, some candidates encountered difficulties in expressing $\alpha^2 + \beta^2$ in terms of sums and products of the roots of quadratic equations. Specifically these candidates did not recognize that αβ had to be subtracted from $(\alpha + \beta)^2$ to give the desired $\alpha^2 + \beta^2$ and instead attempted to use only $(\alpha + \beta)^2$.

In Part (a)(ii) candidates were generally able to solve the equation $\alpha^2 + \beta^2 = 33$ to obtain the value for p.

For this problem, the candidates were given the equation $f(2x + 3) = 2f(x) + 3$ along with a stipulation that $f(0) = 6$ and they were then asked to evaluate $f(x)$ at three specific points $f(3)$, $f(9)$ and $f(-3)$.

Although some candidates were able to provide correct solutions, this item was very poorly done in general. The main difficulty was non-recognition of the need to first solve $2x + 3 = a$, where a is the given point at which $f(x)$ was to be evaluated in each of the three cases, then substitute the value of a obtained into the right hand side of the equation as the value of the variable x. Generally, candidates tended to substitute the point at which the function was to be evaluated into $2x + 3$, then substituted the result into the right hand side of the equation. The values of a required to calculate (i) $f(3)$, (ii) $f(-3)$ were respectively $a = 0$, $a = -3$ emphasizing the power of substitution in this question.

The third part of this problem required that candidates solve $f(-3) = 2f(-3) + 3$. Candidates who were able to solve the first two parts of the problem were also able to so1ve this final part.

Part (c) of this question posed significant difficulties for candidates. Candidates needed to prove that the product of any two consecutive integers k and $k + 1$ is an even integer, which merely required candidates to state that for k and $k + 1$ as consecutive integers, one is even the other is odd so that the product $k(k + 1)$ must be even.

Part (d) was also poorly done by candidates primarily because they did not know, or did not understand, how to apply the steps required for a proof by induction. A small percentage of candidates who performed well on this part of the question also recognized the relevance of Part (c) to the solution of Part (d).

Solutions:

(a) (i) $\alpha + \beta = p$ (ii) $\alpha^2 + \beta^2 = p^2 - 48$

(b) (i) $f(3) = 15$ (ii) $f(9) = 33$ (iii) $f(-3) = -3$.

Section B

Module 2: Trigonometry and Plane Geometry

Question 3

Specific Objectives: (b) 1–7; C 1, 2, 9

This question examined vectors and the properties of the circle, the intersection of a straight line and a circle, and the parametric representation.

This question was generally not well done. The main errors encountered in 3 (a) (i) as highlighted below;

- Candidates substituted $|\boldsymbol{a}|$ and $|\boldsymbol{b}|$ into the vector expression $(\boldsymbol{a} + \boldsymbol{b}) \cdot (\boldsymbol{a} - \boldsymbol{b})$ rather than the vectors $a_1\boldsymbol{i} + a_2\boldsymbol{j}$ and $b_1\boldsymbol{i} + b_2\boldsymbol{j}$.

- Candidates who were able to find the dot product $a_1^2 + a_2^2 - (b_1^2 + b_2^2)$ correctly were in many cases unable to make the final substitution of $a_1^2 + a_2^2 = 169$ and $b_1^2 + b_2^2 = 100$ to obtain the final answer. In fact, $a^2 = 169$ and $b^2 = 100$ rather than $a_1^2 + a_2^2 = 169$ and $b_1^2 + b_2^2 = 100$ were frequently seen.

- Candidates who found the dot product by expanding

  $(a_1\boldsymbol{i} + a_2\boldsymbol{j} + b_1\boldsymbol{i} + b_2\boldsymbol{j}) \cdot (a_1\boldsymbol{i} + a_2\boldsymbol{j} - b_1\boldsymbol{i} - b_2\boldsymbol{j})$ were generally unable to simplify the resulting expression by using the fact that $\boldsymbol{i} \cdot \boldsymbol{i} = 1$ and $\boldsymbol{i} \cdot \boldsymbol{j} = 0$.

Part (a)(ii) was poorly done. Many candidates were able to correctly equate the coefficients of $\boldsymbol{i}$, $\boldsymbol{j}$ to obtain $2b_1 - a_1 = 11$ and $2b_2 - a_2 = 0$. They, however, did not recognize the need to use the previous results $a_1^2 + a_2^2 = 169$ and $b_1^2 + b_2^2 = 100$ to solve for a_1, a_2, b_1 and b_2.

Part (b) was not generally well done.

For Part (b)(i), many instances, candidates were unable to correctly identify the centre of the circle.

In Part (b)(ii), though the majority of candidates realized that a substitution was required to find the points of intersection of the line and the circle, many of them were unable to follow through to the correct answers. Errors frequently seen were

- Incorrect transposition of the linear equation resulting in an invalid substitution.
- Incorrect simplification after substitution leading to an invalid quadratic equation.
- Inability to correctly evaluate the roots using the quadratic formula.
- $x^2 = 8 \Rightarrow x = \sqrt{8}$ thereby omitting $x = -\sqrt{8}$.

For Part (b)(iii), though many candidates were able to put the Parametric equations in a valid Cartesian form $\left(\frac{x-b}{a}\right)^2 + \left(\frac{y-c}{a}\right)^2 = 1$, not all of them were able to follow through by comparing coefficients with the original equation given to determine a, b and c.

In Part (b)(iv), the majority of candidates were unable to determine the equations of C_2. The main error seen was that candidates misinterpreted the question because they did not appreciate the difference between the *line intersecting* the circle and the *line touching* the circle. As a result, many candidates used P and Q from Part (b)(ii) as the centres of possible equations of C_2. Very few candidates were able to recognize the need to use the equation of the perpendicular line through (0, 1), $y = -x + 1$, rather than the original line $y = x + 1$. Even in cases where candidates acknowledged the new line $y = -x + 1$, they often could not follow through to the final equation required. In some cases, candidates correctly identified the equation of the new circle C_2 as of the form $(x - a)^2 + (y - b)^2 = 16$. However, this information was rarely used to complete the question.

Solutions:

(a) (i) 69 (ii) $\boldsymbol{a} = 5\boldsymbol{i} + 12\boldsymbol{j}$, $\boldsymbol{b} = 8\boldsymbol{i} \pm 6\boldsymbol{j}$

(b) (ii) $(2\sqrt{2},\ 1 + 2\sqrt{2})$, $(-2\sqrt{2},\ 1 - 2\sqrt{2})$

(iii) $a = 4$, $b = 0$, $c = 1$ (iv) $[(x + 2\sqrt{2})]^2 + [y - (1 + 2\sqrt{2})]^2 = 16$

<u>Question 4</u>

Specific Objectives: (a) 4, 5, 10, 11

This question tested candidates' ability to use and apply Trigonometric Functions, Identities and Equations.

In most cases for Part (a), candidates were able to correctly deduce the correct quadratic equation $8x^2 - 10x + 3 = 0$. However, very few of were able to follow through to obtain full marks because they

- Incorrect factorization leading to invalid roots.
- Did not recognize that these roots represented values of $\cos^2\theta$ and therefore it was required to find the square root to determine $\cos\theta$.
- Candidates worked in degrees rather than in radians as specified.
- Candidates neglected to find the second quadrant angle corresponding to the negative value of $\cos\theta$.
- Candidates changed $8\cos^4\theta$ to $8x^4$ instead of $8x^2$.

Part b (i) was generally well done. Candidates were able to obtain $BC = 8\sin\theta + 6\sin\theta$. However, there were many instances of candidates incorrectly giving BR as $6\sin\theta$ or
$6\sin(90° + \theta)$.

In Part (b)(ii) the majority of candidates were able to correctly equate the answer from Part (i) to 7 to obtain $8\sin\theta + 6\sin\theta = 7$. However, many candidates were unable to follow through to the correct value of $\theta = 7.6°$. Common errors seen included;

- taking $\dfrac{1-\cos 4\theta}{\sin 4\theta} = \tan 2\theta$ to mean $2 \times \dfrac{1-\cos 2\theta}{\sin 2\theta} = 2\tan\theta = \tan\theta$
- using $\dfrac{1-\cos 6\theta}{\sin 6\theta} = \tan 3\theta$ to mean $3 \times \dfrac{1-\cos 2\theta}{\sin 2\theta} = 3\tan\theta = \tan\theta$
- $\dfrac{1-\cos 4\theta}{\sin 4\theta} = \dfrac{1-1+4\sin^2 2\theta}{4\sin 2\theta\cos 2\theta}$
- $\dfrac{1-\cos 6\theta}{\sin 6\theta} = \dfrac{1-1+6\sin^2 3\theta}{6\sin 3\theta\cos 3\theta}$
- squaring both sides of the equation in an attempt to solve rather than using the form $R\cos(\theta - \alpha)$ or $R\sin(\theta + \alpha)$
- in some cases choosing to use the form $10\cos(\theta - 53.13) = 7$, candidates failed to realize that they had to use –45.6 rather than 45.6 as the value of $\cos^{-1}(0.7)$ obtaining $\theta = 53.13 + 45.57 = 98.7$ rather than $\theta = 53.13 - 45.57 = 7.56$

For Part (b)(iii), the majority of candidates were able to identify that 15 was *not* a possible value for $|BC|$. However, candidates did not always justify their answers with a valid reason.

Although in most cases candidates were able to substitute the correct identities $\cos 2\theta = 1 - 2\sin^2\theta$ or $\cos 2\theta = 2\cos^2\theta - 1$ and $\sin 2\theta = 2\sin\theta\cos\theta$, failure to use brackets in substituting resulted in incorrect simplification of the numerator in Part (c)(i).

The majority of candidates did Part (ii) as a ‘hence or otherwise’ rather than as a ‘hence’ opting to basically redo Part (c) (i) rather than deduce the correct results.

For Part (c), very few candidates were able to follow through to get the correct answer '*n*'. Common errors seen included substituting $r = 1$ before using the previous results to reduce the summation to $\sum_{r=1}^{n} 1$. In fact, many candidates using this method, gave the final answer as 1 not recognizing that they had in fact done the summation.

Solutions:

(a) (ii) $\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{\pi}{4}, \frac{3\pi}{4}$

(b) (i) $|BC| = 8\sin\theta + 6\cos\theta$ or $|BC| = 8\sin\theta + 6\sin(90 - \theta)$, (ii) $\theta \approx 8°$,

(iii) No

(c) (iii) n

Section C

Module 3: Calculus 1

<u>Question 5</u>

Specific Objectives: (a) 3, 5, 7, 9; (b) 7, 21

This question tested candidates' knowledge of limits, continuity and basic elements of calculus.

In Part (a), both the L'Hopital and factorization methods were seen. Quite a significant number of candidates substituted positive (+ve) 2 rather than negative (–ve) –2, however, and were penalized for this.

Some candidates divided both the numerator and the denominator by x^2 and therefore lost direction.

The majority of candidates were able to score at least 5 out of 11 for Part (b).

For Part (b)(i), a significant number of candidates seemed not to know how to use a piece-wise function and determine the relevant part of the function for the given domain value. Many candidates substituted into both parts of the function.

For Part (b)(ii), elementary approaches to limits were seen where candidates used a table of values rather than direct substitution. Again some candidates simply substituted in both parts of the function.

Again in Part (b)(iii), some candidates substituted in both parts of the function. Several candidates gave the answer as $-2b + 1$.

For Part (b)(iv), few candidates were able to use the condition for continuity at a point to correctly find the value of b. Many candidates did not respond to this part of the question.

Teachers must reinforce, that

$$\lim_{x\to a^-} f(x) = \lim_{x\to a^+} f(x) = f(a)$$

is the necessary condition for continuity at the point a.

In Part(c)(i), some candidates had difficulty translating the given information into mathematical statements. Many of the more successful candidates were not able to solve both simultaneous equations to find the solutions for $\boldsymbol{p}$ and $\boldsymbol{q}$.

For Part (c)(ii), the incorrect gradient of the normal was seen. A few candidates applied the formula $y - y_1 = m(x - x_1)$ incorrectly.

Candidates who were able to do Parts (c)(i) and (ii), in most cases calculated the length of MN correctly for Part (c)(iii).

Solutions:

(a) $-\frac{1}{5}$ (b) (i) 5 (ii) 5 (iii) $2b + 1$ (iv) $b = 2$

(c) (i) $p = 10, q = -13$ (ii) $7y + x = 15$ (iii) MN = 14

Question 6

Specific Objectives: (b) 4, 7, 8, 9; (i), 10

This question tested differentiation, integration and calculus.

Several candidates attempted the question with varying degrees of success. The principles involved seemed to be familiar to most, yet some candidates did not receive maximum reward because of weaknesses in simple algebraic manipulations. More practice is recommended in applying the principle involved in part (b) of this question.

In Part (a)(i), some candidates did not know that they should have differentiated to find the stationary points, while a few candidates were unable to differentiate correctly. Some candidates were unable to solve the equation x2 = 4, although many got only ‘2’ as the solution and others got ± 4. A number of candidates were unable to substitute correctly.

For Part (a) (ii), to find the gradient, many candidates treated the function as a straight line.

In Part (a)(iii), a number of candidates chose the wrong function to integrate.

Many candidates did not use the correct limits of integration, several of them used 2 or 4 as the upper limit. Some candidates failed to recognize that area cannot be negative.

Some candidates did not understand the concept of proving, hence, they simply rewrote the question in Part (b)(i). This was also done for Part (b)(ii). Some candidates chose to integrate the product $x\sin x$ in the same manner you would integrate $x + \sin x$ would be integrated.

A few candidates chose to use the method of integration by parts.

Solutions:

(a) (i) A ≡ (–2, 16), B ≡ (2, –16)

(ii) $12y = x$ is the equation of the normal

(iii) Area = 36 sq. units

Paper 032 – Alternative to School-Based Assessment

Section A

Module 1: Basic Algebra and Functions

Question 1

Specific Objectives: (c) 1–4; (d) 1, 7

This question examined the theory of logarithms, functions and exponentials.

Among the small number of candidates there were a few good attempts at Part (a). However, the term 2^{2-x} was not correctly interpreted in the majority of cases.

In Part (b)(i), the notion of one-on-one functions was not properly understood.

There were a few encouraging attempts in Part (b)(ii) but poor algebraic manipulation spoiled some of the efforts at completing the solutions correctly.

Poor or inappropriate substitution was evidenced in the few attempts at Part (c). More practice is recommended in this area.

Solutions:

(a) $x = 0, x = 2$ (b) (ii) $x = -4$ (c) (i) $35 million (ii) $4 million

Section B

Module 2: Trigonometry and Plane Geometry

Question 2

Specific Objectives: (b) 1, 2, 3; Content: (a) (ii), (b) (iii)

This question examined the intersection of lines, equations of straight lines, circles and tangents to circles, using basic concepts of coordinate geometry. Vectors and trigonometric identities were also included.

There were some good attempts at Part (a) although some weaknesses were evident in finding the coordinates of the point P in Part (a)(i) and the point Q in Part (a)(ii).

Part (b) was well done by using established formulae for $2A$.

Not many candidates who attempted Part (b)(ii) completed it correctly. The main source of difficulty was the incorrect manipulation of trigonometric formulae.

Solutions:

(a) (i) P ≡ (3, 1) (ii) Q ≡ (1, –3) (iii) $4y = 3x - 5$

(b) (ii) $\theta = \pi/3,\ 2\pi/3,\ \pi$.

Section C

Module 3: Calculus 1

Question 3

Specific Objectives: (a) 3, 5, 7; (b) 8, 9 (i), 15, 16, 18; (c) 1, 5 (i), 3

This question examined limits, differentiation and the reverse process of integration and applications of differentiation to maxima/minima situations. There was also some mathematical modeling included.

In Part (a) several of the few candidates who made an attempt did not factorize $x^3 - 4x$ correctly, the consequence of which was an incorrect limit.

Some candidates had difficulty differentiating $\frac{x}{3x+4}$ as a quotient in Part (b)(i). However, a few candidates did it quite competently as the product $x\,(3 + 4x)^{-1}$.

A few candidates saw the connection between Part (b)(ii) and Part (b)(i). Most of those who were able to make the connection were able to complete the solution competently.

There were some good attempts at Part (c)(i).

Not many of the candidates who did Part (c)(i) were able to complete part (c)(ii).

Solutions:

(a) 8

(b) (i) $\frac{4}{(3x+4)^2}$ (ii) $\frac{4x}{3x+4}$ + constant

(c) (ii) S_{min} at $r = \left(\frac{5}{\pi}\right)^{\frac{1}{3}}$.

UNIT 2

Paper 01 – Multiple Choice

Paper 01 comprised 45 multiple-choice items. Candidates performed fairly well with a mean score of 60.73 per cent and a standard deviation of 9.29.

Paper 02 – Structured Questions

Section A

Module 1: Calculus II

<u>Question 1</u>

Specific Objectives: (b) 1, 3, 4, 5

This question examined implicit differentiation, differentiation of combinations of polynomials, exponentials, trigonometric functions, application of the chain rule to obtain the tangent of a curve given by its parametric equations, and the second derivative.

Part (a)(i), was well done by the majority of candidates. Full marks were obtained by almost all candidates. Common errors included incorrect transposition of $\frac{dy}{dx} = \ldots$

Full marks were obtained for Part (a)(ii) by approximately 99 per cent of the candidates.

In Part (a)(iii), few instances of errors in using the concept of differentiation of composite functions were seen. Common errors included $\frac{d}{dx}\cos x = \sin x$. Generally, most candidates obtained full marks for this part of the question.

For Part (b)(i), a small percentage of candidates had difficulty applying the concept of differentiation of the composite function $\sin\frac{1}{x}$, particularly $\frac{d}{dx}\left(\frac{1}{x}\right)$, although, the correct application of the product rule was applied. However, only a small percentage of candidates were unable to obtain full marks for this part of the question.

Generally, Part (b)(ii) was well done. A very small percentage of candidates was unable to apply the concept of implicit differentiation correctly, with the result that they were unable to show the final answer as required.

Part (c)(i) was well done. Some arithmetic errors were made in substituting for $t = 4$ resulting in the incorrect value of the gradient of the tangent. However, candidates were able to get follow through marks for Part (c)(ii).

Part (c)(ii) was well done. Candidates who made errors calculating the correct gradient in Part (c)(i) were not penalized having earned follow through marks.

Solutions:

(a) (i) $\frac{dy}{dx} = \frac{1-x}{1+y}$.

(ii) $\frac{dy}{dx} = (-\sin x)e^{\cos x}$

(iii) $\frac{dy}{dx} = 8 \sin 16x - 6 \sin 12x$

(c) (i) $\left(\frac{dy}{dx}\right)_{t=4} = \frac{15}{4}$

(ii) $15x - 4y = 12$

<u>Question 2</u>

Specific Objectives: (c) 1, 3, 4, 8, 10

This question required candidates to derive a reduction formula and use it for a partial expansion of the product of an exponential function of *x* and the derived reduction formula; derive partial fractions and the integration of a rational function involving a trigonometric substitution and an inverse trigonometric function.

Approximately 20 per cent of the candidates were unable to obtain the correct answers to Part (a)(i). Many candidates could not deduce that $\int_0^0 F_n(0)\, dx = 0$. Some difficulty was also experienced in evaluating $\int_0^x F_0(x)\, dx$ correctly, particularly using the limits of integration, obtaining $e^{-x} - 1$ instead of $1 - e^{-x}$.

In Part (a)(ii), poor algebraic skills resulted in many candidates being unable to complete integration by parts and to show the correct reduction formula for $F_n(x)$. In particular, some candidates were unable to simplify $\frac{n}{n!} \equiv \frac{n}{n\,(n-1)!} = \frac{1}{(n-1)!}$ to enable the expression of $F_{n-1}(x)$.

Very few candidates were able to complete Part (a)(iii), having failed to determine $F_0(x)$ and $F_n(0)$ correctly. Partially correct answers were facilitated using the given result in Part (a)(ii).

In Parts (b)(i)(ii), there was evidence of candidates applying the concepts of repeated factors and the form of a linear numerator for a quadratic factor to find the partial fractions required. Some candidates were able to find the required partial fractions and proceeded to integrate the resulting rational functions. A few candidates successfully completed that part of the integration which involved an inverse trigonometric function. *This part of the question was zero-weighted and adjustments were made to the final marks so that candidates were not disadvantaged.*

Solutions:

(a) (i) $F_n(0) = 0$, $F_0(x) = 1 - e^{-x}$

(b) (i) $\frac{2x^2+3}{(x^2+1)} \equiv \frac{2}{x^2+1} + \frac{1}{(x^2+1)^2}$

(ii)

$$\int \frac{2x^2+3}{(x^2 \ +1)^2} dx = \frac{5}{2}\ tan^{-1}(x) + \frac{x}{2(x^{2+1)}} + \ constant$$

Section B

Module 2: Sequences, Series and Approximation

Question 3

Specific Objectives: (a) 5; (b) 2, 4, 5

This question examined candidates' abilities to establish the properties of a sequence by applying Mathematical Induction; expand $(1 + ax)^n$, for $n = -1$; identify that a given series follows an arithmetic progression. Overall performance on this question was unsatisfactory.

A small percentage of candidates obtained full marks for Part (a)(ii). Generally, candidates seemed unfamiliar with proof by induction of sequences. Many of them knew that they had to prove the assertion for $n = 1$ and to proceed to taking arbitrary $k + 1$ for $n + 1$. However, having obtained $x_{k+2} = x_{k+1}^2 x + \frac{1}{4}$, they could not proceed further. Some candidates lost marks by using strict equality signs, ignoring the restriction $x_n < \frac{1}{2}$.

Only a few candidates were successful in obtaining full marks on Part (a)(ii). Those who were able to express $x_{n+1} - x_n$ as a perfect square made progress to completely solve the problem.

Part (b)(i) was well done. The majority of candidates demonstrated a sound knowledge of partial fractions and were able to obtain a correct solution. Those candidates who did not secure the full three marks were mainly faulted by arithmetic and algebraic errors.

Part (b)(ii) was well done by approximately half of the candidates. Generally, candidates used two different methods to solve this problem, namely, the binomial and Maclaurin's expansions. Those who failed to secure the full four marks committed a range of arithmetic and algebraic errors.

Part (b)(ii)a) was poorly done. More than 50 per cent of the candidates merely stated the ranges $|x| < 1$ and $|x| < \frac{1}{2}$ without proceeding to the correct answer $\left(-\frac{1}{2} < x < \frac{1}{2}\right)$.

Part (b)(iii)b) challenged the majority of the candidates' including those who successfully completed the previous parts of the question. They could not make the link to the earlier parts of the question. A significant number of candidates did not respond to this part of the question.

Candidates responded well to Part (b)(iv). The concepts of the difference between S_{n+1} and S_n resulting in the n^{th} term and subsequently $T_n - T_{n-1} = d$ were known to most candidates. However, poor algebraic manipulations resulted in candidates' unsuccessful efforts to prove the required solution

Solutions:

(b) (i) $A = B = 1$

(ii) $1 + x + x^2 + x^3$; $1 + 2x + 4x^2 + 8x^3$

(iii) a) $-\frac{1}{2} < x < \frac{1}{2}$

b) $1 + 2^n$

(iv) $u_n = S_n - S_{n-1} = 6n - 7$; $d = u_n - u_{n-1} = 6$

Question 4

Specific Objectives: (b) 9, 11, 12, 13

This question examined candidates' ability to manipulate a geometric progression and determine the first term and common ratio; obtain a series expansion of a fraction involving a denominator of $e^x + e^{-x}$; find the sum and limit of a finite series using the method of differences. Overall, candidates' performance on this question was very unsatisfactory. Approximately 40 per cent of the candidates either offered no responses or scored no marks.

For Part (a)(i) a large percentage of the candidates obtained 2 of the 4 marks available by establishing the equations $a + ar + ar^2 = \frac{26}{3}$ and $a^3r^3 = 8$. Some candidates used the equations $\frac{a(1-r^3)}{1-r} = \frac{26}{3}$ and $a^3r^3 = 8$. Poor algebraic skills prevented the majority of these candidates from eliminating a and thus finding the required equation in terms of r.

For Part (a)(ii) a), a significant number of candidates could not simplify the equation given in Part (a)(i) to solve for r. A few of the candidates who found two values for r, ($r = 3$) or ($r = \frac{1}{3}$), did not follow the constraint $0 < r < 1$.

Candidates who did not use the constraint for r abandoned Part (a)(ii)b).

A small percentage of candidates obtained full marks for this Part (a)(ii)(c).

Part (b) required candidates to find a series expansion for a fraction involving the denominator $e^x + e^{-x}$. Although a few candidates were able to recall and use the expansion for e^x and e^{-x}, the majority of them employed Maclaurin's theorem for the expansion of the denominator without success. Successive differentiation proved problematic and the exercise was abandoned. Approximately 20 per cent of the candidates were able to obtain the result $\frac{2}{e^x + e^{-x}} = \frac{1}{1 + \frac{x^2}{2} + \frac{x^4}{24}} + \ldots$ The required expansion of the denominator, using the binomial expansion with $\left(\frac{x^2}{2} + \frac{x^4}{24}\right) = X$ in the expansion of $(1 + X)^{-1}$ was beyond the ability of a majority of the candidates.

Part (c)(i) was well done.

In Part (c)(ii), a large percentage of the candidates incorrectly found

$$3\sum_{r=1}^{n}\left(\frac{1}{r(r+1)} - \frac{1}{(r+1)(r+2)}\right)$$ giving the

resulting incorrect sum of $3\left(\frac{1}{2} - \frac{1}{(n+1)(n+2)}\right)$.

Following the error made in Part (c) (ii), those candidates obtained the wrong limiting sum of the series in Part (c)(iii).

Solutions:

(a) (ii) a) $r = \frac{1}{3}$

b) $a = 6$

c) $S_\infty = 9$

(b) $1 - \frac{x^2}{2} + \frac{5x^4}{24} + \ldots$

(c) (i) $\frac{2}{r(r+1)(r+2)}$

(ii) $\frac{3}{2}\left(\frac{1}{2} - \frac{1}{(n+1)(n+2)}\right)$

(iii) $S_\infty = \frac{3}{4}$

Section C

Module 3: Counting, Matrices and Complex Numbers

Question 5

Specific Objectives: (a) 2, 4, 7; (c) 2, 3, 5

This question examined the concepts of arrangements of n distinct objects; the selection of r distinct objects from n distinct objects; the probability of an event occurring; the complex roots of a quadratic equation and the square roots of a complex number.

The overall performance by most of the candidates was satisfactory in parts of the question. As evident in previous problems which required algebraic manipulation, most candidates were at a severe disadvantage in using algebra to show required results. Algebraic simplification continues to prove problematic to most candidates.

Approximately 75 per cent of the candidates was able to obtain full marks for Part (a)(i). Some candidates substituted numbers to show the required result.

For Part (a)(ii), the algebra required to show the required result was beyond most of the candidates. Half-hearted attempts were made to simplify the initial definitions of the left hand and right hand sides of the equations. Many candidates resorted to substituting numbers to balance the equation.

In Part (a)(iii), candidates simply used the numbers given in the equations and calculated the arithmetic results. No attempts were made to use the results of Parts (a)(i) and (ii).

Part (b)(i) was well done. Some arithmetic errors resulted in some candidates being unable to obtain full marks.

Part (b)(ii) was well done by approximately half of the candidates. Arithmetic errors and some loss of reasoning resulted in many candidates not obtaining full marks.

In Part (c)(i) a), a significant number of candidates found the square roots of $-2i$ using the approach $(x + iy)^2 = -2i$. This resulted in some of these candidates making algebraic errors and subsequently obtaining incorrect roots. Some candidates misunderstood the question and attempted to show that $(1 - i) \times (1 + i) = -2i$.

For Part (c) (i) candidates who found the square roots of $-2i$ using the method described in Part (c)(i) a) were able to get the correct answer. There was no evidence that candidates simply applied the concept that the square root of a complex number $(x + iy)$ is $\pm (a + ib)$.

Part (c)(ii), most candidates who used the quadratic formula to solve this equation could not establish the link with $b^2 - 4ac = -2i$ and use the results of Part (c)(i). Very few candidates were able to obtain full marks for this part of the question.

Solutions:

(b) (i) 96

(ii) $\frac{3}{8}$

(c) (i) b) $-(1 - i)$

(ii) $2 + 2i$, $1 + 3i$

Question 6

Specific Objectives: (b) 1, 2, 6, 8

This question examined matrices and systems of linear equations. Particularly tested were operations with conformable matrices and manipulation of matrices using their properties; evaluation of determinants for 3×3 matrices; solutions of a consistent system; solution of a 3×3 system of linear equations.

Overall this question was well done. A notable number of candidates obtained marks ranging from 15 to 20.

Part (a)(i) was well done and, arithmetic errors apart, candidates obtained full marks. Candidates generally answered Parts (a), (b) and (c) of the question by making the required changes and using the algorithmic approach. No evidence was seen that any candidate used the properties of matrices to obtain their answers.

All parts of Part (b) were well done and full marks were obtained by almost all candidates.

All parts of Part (c) were well done and full marks were obtained by almost all candidates. In Part (c)(iv), some candidates having shown that (1, 1, 1) was a solution for the system of equations, were unable to find the general solution for the system of equations, despite some attempts. It was not recognized that the system represented parallel planes thus resulting in infinitely many solutions.

Solutions:

(a) (i) $|\mathbf{A}| = 5$

(ii) a) $|\mathbf{B}| = |\mathbf{A}| = 5$, The value of a determinant is unaltered when the columns and rows are completely interchanged.

b) $|\mathbf{C}| = |\mathbf{A}| = 5$, The value of the determinant is not changed if any row (column) is added or subtracted from any other row (column).

c) $|\mathbf{D}| = 5^3\ |\mathbf{A}| = 625$. If all rows are multiplied by λ, the determinant is multiplied by λ^3.

(b) (i) $\mathbf{AM} = \begin{pmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{pmatrix} = 5\mathbf{I}$

(ii) $\mathbf{A}^{-1} = \frac{1}{5}\ \mathbf{M} = \frac{1}{5}\begin{pmatrix} 12 & -1 & 5 \\ 2 & -1 & 0 \\ -9 & 2 & -5 \end{pmatrix}$

(c) (i) $\underset{\mathbf{A}}{\begin{pmatrix} 1 & 1 & 1 \\ 2 & -3 & 2 \\ -1 & -3 & -2 \end{pmatrix}}\underset{\boldsymbol{x}}{\begin{pmatrix} x \\ y \\ z \end{pmatrix}} = \underset{\mathbf{b}}{\begin{pmatrix} 5 \\ -10 \\ -11 \end{pmatrix}}$

(ii) $\mathbf{A}^{-1}\,\mathbf{A}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \mathbf{A}^{-1}\begin{pmatrix} 5 \\ -10 \\ -11 \end{pmatrix} \Rightarrow \boldsymbol{x} = \mathbf{A}^{-1}\ \mathbf{b}$

(iii) $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \\ -2 \end{pmatrix}$

(iv) b) $(x, y, z) = \lambda(1, 0, -1) + \mu(0, 1, -1) + (1, 1, 1)$

Paper 032 – Alternative to School-Based Assessment

Section A

Module 1: Calculus II

<u>Question 1</u>

Specific Objectives: (b) 3; (c) 12

This question examined differentiation of parametric equations, rate(s) of increase/decrease and the general solution of a second order differential equation.

Performance, overall, was generally poor. The majority of candidates seemed to be unprepared for this question.

In Part (a)(i), some measure of successful differentiation of y and x with respect to t was seen. However, candidates could not determine $\frac{dy}{dx}$ in terms of t and to proceeded to equate $\frac{dy}{dx} = \tan\,\theta$.

Candidates did not respond to Part (a)(ii), having not completed Part (a)(i).

Some attempts were made to answer Part (a)(iii). Problems encountered by candidates involved incorrect transpositions of x and y and identifying with the correct trigonometric identities.

Part (b) did not elicit many responses. Those candidates who attempted to solve the auxiliary equation used the wrong roots to express the complementary function. The solution for the particular integral was beyond the abilities of almost all the candidates.

Solutions:

(a) (i) rate of decrease = $\frac{24}{31}$

(ii) radians per second

(iii) $\left(\frac{x-4}{3}\right)^2 + \left(\frac{y-5}{2}\right)^2 = 1$

(b) $y = Ae^{-x} + \mathrm{B}e^{4x} - 2x^2 + 3x - \frac{13}{4}$

Section B

Module 2: Sequences, Series and Approximations

Question 2

Specific Objectives: (b) 3, 13; (e) 1, 2

This question examined the existence of a real root in a given interval, finding an approximation using a given iterative method, expansion of a logarithmic and exponential function using Maclaurin's theorem and determining the n^{th} term of a sequence of terms.

It was evident that candidates were underprepared for most of this question. Overall, performance was poor.

For Part (a)(i), most candidates were able to establish a change of sign over the given interval. Without stating continuity of the function over this interval, candidates concluded that a real root existed over the interval.

Part (a)(ii) was done satisfactorily.

Some candidates showed some understanding of Maclaurin's theorem and were able to obtain full marks for Part (b)(i).

There were no favourable responses to Part (b)(ii).

Part (c)(i) was well done.

In Part (c)(ii), candidates were not able to make a deduction to obtain the equation.

There were no meaningful responses to Part (i)(iii). Candidates appeared to be guessing about a suitable approach to this part of the question.

Solutions:

(a) (ii) 0.904

(b) (i) $\ln(1-x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} - \ldots \ (-1 \le x < 1)$

$e^{-x} = 1 - x + \frac{x^2}{2} - \frac{x^3}{6} + \ldots$ for all real x

(c) (i) $p_1 = 1000(1.20) - 100$ $p_2 = 1.20[1000(1.20) - 100] - 100$

(ii) $p_{n+1} = (1.20)p_n - 100$

Section C

Module 3: Counting, Matrices and Complex Numbers

<u>Question 3</u>

Specific Objectives: (b) 1, 8; (c) 4, 6, 7

This question examined simple operations on a conformable matrix, solutions of a system of equations and operations on a complex number. Overall, performance was poor.

Most candidates were able to obtain marks for Part (a)(i) of the question. The common problems evidenced were arithmetic and in some cases failing to identify I (identity matrix).

In Part (a)(ii), candidates were unable to deduce $\mathbf{A}^{-1}$ in the given form since they could not identify the identity matrix.

For Part (a)(iii), those candidates who attempted to find the solution of the system of equations completely ignored the link from Part (a)(ii). As a result, arithmetic errors inhibited their ability to obtain the correct solutions.

In Part (b), candidates demonstrated an understanding of the method(s) to be used. However, poor algebra resulted in incorrect answers.

Most candidates did not attempt Part (c). The few candidates who attempted it did not show a fair understanding of the modulus and argument of a complex number.

Solutions:

(a) (iii) $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$

(b) $z + \frac{1}{z} = \frac{1}{10}(7 + 9\text{i})$

(c) $r = \sqrt{\frac{13}{10}}$, $\tan\,\theta = \frac{9}{7}$

Paper 031 – School-Based Assessment (SBA)

This year, 174 Unit 1 and 145 Unit 2 SBAs were moderated. Far too many teachers continue to submit solutions without unitary mark schemes. In some cases, neither question papers with solutions nor mark schemes were submitted. Mark schemes for questions and their subsequent parts were not broken down into unitary marks. In an increasing number of cases, the marks awarded were either too few or far too many for the skills tested. (Example: an entire SBA module test was worth 20 marks and in another case on one test paper a simple probability question was awarded 27 marks and a matrix question was awarded 24 marks).

In Unit 1, the majority of the samples submitted were not of the required standard. Teachers **must** pay particular attention to the following guidelines and comments to ensure effective and reliable submission of SBAs.

The SBA is comprised of three module tests. The main features assessed are:

- Mapping of the items tested to the specific objectives of the syllabus for the relevant Unit
- Content coverage of each module test
- Appropriateness of the items tested for the CAPE level
- Presentation of the sample (module test and students' scripts)
- Quality of the teachers' solutions and mark schemes
- Quality of teachers' assessments — consistency of marking using the mark schemes
- Inclusion of mathematical modelling in at least one module test for each unit

FURTHER COMMENTS

1. Too many of the module tests comprised items from CAPE past examination papers.

2. Untidy ‘cut and paste’ presentations with varying font sizes were commonplace.

3. Teachers are reminded that the CAPE past examination papers should be used *only* as a guide.

4. The stipulated time for module tests *(*1–1 hour 30 minutes) must be strictly adhered to as students may be at an undue disadvantage when module tests are too extensive or too insufficient.

5. The following guide can be used: 1 minute per mark. About 75 per cent of the syllabus should be tested and mathematical modelling *must* be included.

6. Cases were noted where teachers were unfamiliar with recent syllabus changes that is,

    - Complex numbers and the Intermediate Value Theorem are now tested in Unit 2.

    - Three dimensional vectors, dividing a line segment internally and externally, systems of linear equations were removed from the Unit 1 CAPE syllabus (2008).

7. The moderation process relies on validity of the teachers’ assessments. There were few cases where students’ solutions were *replicas* of the teachers’ solutions — some contained identical errors and full marks were awarded for incorrect solutions. There were also instances where the marks on students’ scripts did not correspond to the marks on the moderation sheet. The SBA must be administered under examination conditions at the school. It is not to be done as a homework assignment or research project.

8. Teachers must present evidence of having marked each individual question on students’ scripts before the marks scored out of the possible total is calculated at the top of the script. The corresponding whole number score out of 20 must be placed at the front of students’ scripts.

Module Tests

- Design a separate test for each module. The module test must focus on objectives from that module.

- In cases where several groups in a school are registered, the assessments should be coordinated, common tests should be administered and a common marking scheme used.

- A sample of five students will form the sample for the centre. If there are less than five students, *all* scripts will form the sample for the centre.

- In 2011, the format of the SBA remains unchanged.

To enhance the quality of the design of module tests, the validity of teachers’ assessments and the validity of the moderation process, the SBA guidelines are listed below for emphasis.

GUIDELINES FOR MODULE TESTS AND PRESENTATION OF SAMPLES

1. **COVER PAGE TO ACCOMPANY EACH MODULE TEST**

The following information is required on the cover of each module test.

- Name of school and territory, name of teacher, centre number
- Unit number and module number
- Date and duration of module test
- Clear instructions to candidates
- Total marks allocated for module test
- Sub-marks and total marks for each question **must** be clearly indicated

2. **COVERAGE OF THE SYLLABUS CONTENT**

- The number of questions in each module test must be appropriate for the stipulated time.
- CAPE past examination papers should be used as a guide ONLY.
- Duplication of specific objectives and questions must be avoided.
- Specific objectives tested must be from the relevant unit of the syllabus.

3. **MARK SCHEME**

- Detailed mark schemes MUST be submitted, that is, one mark should be allocated per skill (not 2, 3, 4 marks per skill)
- Fractional or decimal marks MUST NOT be awarded. (that is, do not allocate $\frac{1}{2}$ marks).
- A student's marks MUST be entered on the front page of the student's script.
- Hand written mark schemes MUST be NEAT and LEGIBLE. The **unitary** marks MUST be written on the right side of the page.
- **Diagrams MUST be neatly drawn with geometrical/mathematical instruments**.

PRESENATION OF SAMPLE

- Students' responses MUST be written on letter sized paper (8 ½ x 11) or A4 (8.27 x 11.69).
- Question numbers MUST be written clearly in the left hand margin.
- The total marks for EACH QUESTION on students' scripts MUST be clearly written in the left or right margin.
- ONLY ORIGINAL students' scripts MUST be sent for moderation.
- Photocopied scripts WILL NOT BE ACCEPTED.
- Typed module tests MUST be NEAT and LEGIBLE.
- The following are required for each Module test:
    - ❖ A question paper
    - ❖ Detailed solutions with detailed unitary mark schemes.
    - ❖ The question paper, detailed solutions, mark schemes and five students' samples should be batched together for each module.

- Marks recorded on PMath–3 and PMath2–3 forms must be rounded off to the nearest whole number. If a student scored zero, then zero must be recorded. If a student was absent, then absent must be recorded. The guidelines at the bottom of these forms should be observed. (See page 57 of the syllabus, no. 6.)