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QUAKE 1: AN EARTHQUAKE LOCATION PROGRAMME
FOR A HAND-HELD CALCULATOR

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INTRODUCTION

There can be few seismological observatories today which do not have access to digital computers, and numerous earthquake location programmes have been published over the past 20 years. However, particularly in less-developed countries, there are few observatories which have in-house computers. Computing facilities are generally shared with other users so that even when the computer is up and running there may be long waiting periods for computing time. In addition the computing centres often close down at night, over weekends and public holidays, and during regular maintenance periods. As a result, the computer is often not available when it is most needed - that is, immediately after a felt earthquake. A similar situation arises when a temporary network is installed in a remote region to study an aftershock series or earthquake swarm. In such a case computer facilities may not be available at all.

It is, of course, possible to locate earthquakes without using a computer; Gutenberg and Richter (1953) compiled "The Seismicity of the Earth" and Jeffrey and Bullen (1939) constructed their travel-time tables without the use of digital computers, but the calculations involved are tedious, repetitive and error-prone. Efficient hand-held calculators can remove much of the tedium from the calculations and the increasing sophistication of calculators such as the Texas Instruments TI59 and the Hewlett-Packard HP-67 and HP-41 series has introduced the possibility of loading a complete hypocentre location programme on to a hand-held calculator. This report describes one such programme for the HP 41CV calculator. The HP-41CV is particularly well-suited to the task because of its large basic memory (319 7-byte RAM locations) and the fact that three of the most frequently used routines in iterative hypocentral location distance and azimuth calculations and matrix inversion - are available in ROM. The calculator also handles strings of alphabetic characters easily so that

station codes can be used in their familiar alphabetic forms, and prompts for input data written in straightforward English.

METHOD

The method used is based on the familiar iterative non-linear least-squares method described by, for example, Bullen (1956 section 10.4.1.). Following Bullen we adopt a trial focus for the earthquake and let x , y , z and τ be east, west, vertical and time corrections needed to correct the trial focus. If t_i is the observed arrival time of P-waves at the i^{th} station and T_i is the calculated arrival time then putting $\mu = t_i - T_i$ the equations connecting all these quantities are:

$$\mu_i = - (x \sin Z_i + y \cos Z_i) \left(\frac{\partial T}{\partial \Delta} \right) + (z + \tau) \frac{\partial T}{\partial h} \quad (1)$$

In deriving (1) we ignore any corrections which might be required to the travel time tables. $(dT/d\Delta)$ and (dT/dh) are the derivatives of the travel time with respect to distance Δ and depth h , and in deriving (1) we have assumed that the corrections are sufficiently small that terms involving squares and higher powers of them can be neglected in comparison with the other quantities.

If the number of stations, n , which record the event is greater than or equal to 4 then equations (1) can be solved for the required corrections. Since this formulation is a linear approximation to a non-linear problem a single solution of (1) will rarely, if ever, lead to the best solution. We therefore take the corrected solution as a new trial focus and repeat the process iteratively until either the sum of the squares of the residuals falls below some acceptable small value or the sum fails to decrease with further iteration.

This process is well-known, as are its limitations. An important limitation is caused by the fact that, for most earth models, (dT/dh) varies only very slowly with distance. As a result, origin time and depth are not truly independent parameters - changes in one quantity can be exactly counteracted by changes in the other, a phenomenon sometimes called the origin time/depth trade-off. This phenomenon can be avoided if one or other of the two parameters can be fixed independently. Focal depth can sometimes be fixed fairly accurately from observations of depth phases but in the case of observations of earthquakes by local or regional networks this is rarely possible since depth phases are not well-observed at short distances. Instead, we chose to fix the origin time of the event from observed S-P intervals. In the eastern Caribbean for many years of observation Poisson's relation $V_p = \sqrt{3} \times V_s$ holds reasonably accurately and in this programme origin time is fixed using this relation. Then in equation (1) $\zeta = 0$ and the equations are:

From which are formed the least squares normal equations

$$\begin{aligned} x \quad a_i^2 + y \quad a_i b_i + Z \quad a_i c_i &= a_i \mu_i \\ x \quad a_i b_i + y \quad b_i^2 + Z \quad b_i c_i &= b_i \mu_i \\ x \quad c_i a_i + y \quad c_i b_i + Z \quad c_i^2 &= c_i \mu_i \end{aligned} \quad (3)$$

where

$$\begin{aligned} a_i &= -\sin Z_i \frac{(dT)}{(dZ)}_{\Delta i} \\ b_i &= -\cos Z_i \frac{(dT)}{(dZ)}_{\Delta i} \\ c_i &= \frac{(dT)}{(dh)}_{\Delta i} \end{aligned} \quad (4)$$

In matrix form $\underline{A} \cdot \underline{x} = \underline{B}$

$$\underline{x} = \underline{A}^{-1} \underline{B} \quad (5)$$

Each iteration for the corrections x, y, h therefore involves n calculations of distance, azimuth and the two derivatives as well as inversion of a three-by-three matrix.

The earth model used in this program is almost the simplest possible, that is a uniform velocity spherical earth. Angular distance Δ and azimuths Z for any two points of known latitude and longitude are calculated using the HP-41-CV ROM routine *GC and the linear distance D from a point at a depth H below one of the points to the other is then given by:

$$D^2 = 4R_\Sigma (R_\Sigma - h) \sin^2 \frac{\Delta}{2} + H^2 \quad (6)$$

Travel times and derivatives are obtained directly from (4), and the matrix is inverted using HP 41-CV ROM routine PVT.

If the matrix A in equation (5) is inverted directly then difficulties may arise because of non-convergence. In expanding the Taylor series from which (1) are derived, second and higher order terms in the corrections have been ignored, but if the initial trial focus is not sufficiently close to the "true" focus this assumption is unjustified and the sum of the squares of the residuals corresponding to the corrected solution may be larger than that corresponding to the initial solution. Generally this problem can be avoided by using damped least squares (Levenberg 1944) in which a small positive constant is added to each of the diagonal terms of matrix A immediately before inversion. I.e instead of $\underline{x} = \underline{A}^{-1} \underline{B}$

$$\text{we calculate } \underline{x} = (A + \lambda I)^{-1} \underline{B} \quad (7)$$

Choice of the value of λ present a problem. If λ is chosen sufficiently large then the damped least squares procedure, always converges but may do so agonizingly slowly. If λ is too small non-convergence will result. Numerous attempts have been made to develop algorithms for optimal choice of λ (e.g. Marquardt 1963, Hoerl and Kennard 1970) but none of these algorithms is

suitable for application in a simple program such as this. Instead we use a semi-empirical approach based on the observation that we wish λ to be large when the trial focus is poor and to reduce λ successively as the quality of the trial focus improves. We therefore make λ proportional to the current value of the mean square error. The constant of proportionality is 10^{-7} , chosen entirely empirically to ensure initial convergence for all reasonable first trials whilst maintaining reasonably rapid convergence thereafter.

PERFORMANCE AND LIMITATIONS

The programme accepts P and S arrival times for up to nine stations and will produce an answer provided that the arrivals include at least 3 P-phases and one S-phase. Instructions for preparation of the calculator and operation of the programme are given in the appendices. The iteration process always converges if the hypocentre is at least as close to the first station as the last station is - i.e if the average distance of the hypocentre from the array is no larger than the aperture of the array. If this condition does not hold then convergence depends on the quality of the initial trial hypocentre. The main limitation on the quality of the final hypocentre results from the simplicity of the assumed earth model. Clearly, complicated travel-time tables cannot adequately be described by a single phase velocity but for each combination of hypocentre and recording stations there exists an optimum velocity which most closely approximates to the true travel times. This optimum velocity is chosen by trial-and-error. QUAKE 1 solutions obtained using the full travel-time tables and the velocity which gives the best solution is adopted for future solution. Generally a velocity can be chosen for which QUAKE 1 epicentres are consistently within 10 km of the "true" epicenter.

Depths are less precisely determined but are usually within 20 km of the true value. These figures refer to regional hypocentre determinations where inter-station distances and hypocentre - to - station distances are of order tens to hundreds of kilometres. For local hypocentres where the corresponding distances are kilometres to tens of kilometres the uncertainties are correspondingly less.

The programme includes no provision for automatic rejection of poor observations. If grossly erroneous arrival times are included the programme will generally converge to a solution with an unacceptably high RMSE and, because of the method of damping used, will do so painfully slowly. This condition can be detected by inspection of the output from successive iterations and corrected by the method described in the appendix. Provided that no poor observations are included, the programme will converge to the best solution within five iterations or less.

APPENDIX 1 : Preparing the calculator for QUAKE 1.

It is assumed that the following are available

- 1) HP 41-CV
- 2) Math 1 ROM
- 3) Navigation ROM
- 4) Magnetic Card Reader
- 5) Printer
- 6) Pre-written magnetic records containing QUAKE 1 and station co-ordinates.

In fact, operation of the programme is possible without 4, 5 or 6 but the whole programme must then be entered through the keyboard, which is extremely tedious.

- A) Execute master clean (see HP 41-CV handbook), switch off.
- B) Ensure that the Math 1 and Navigation modules are attached and attach the magnetic-card reader.
- C) Switch calculator on.
- D) Load card marked "QUAKE 1 STATUS" (Tracks 1 & 2).
- E) Load 5 cards labelled QUAKE 1 1 and 2 etc. (See card reader handbook for correct loading).
- F) Press XEQ "EDTAX" (Quote marks mean press key marked ALPHA)
Calculator will prompt CARD.
- G) Load relevant data card containing station coordinates.
- H) Switch calculator off, attach printer, switch on.

APPENDIX 2 : Running QUAKE 1

- A) Either XEQ "QUAKE 1" or press R↵ in USER mode. The calculator will prompt for all subsequent input as follows:

<u>PROMPT</u>	<u>ACTION</u>
STATION ?	Enter station code as three letter string. Press R/S. If the station is not recognized the calculator signals its disapproval and prompts again.
P = ?	Enter arrival time of P-waves in seconds and decimals after some convenient reference time such as the previous minute. All subsequent times are referred to this reference.
S = ?	If an S arrival time is available enter it as for P. If there is no S-arrival, press zero, R/S.

<u>PROMPT</u>	<u>ACTION</u>
O.K. ?	If further data are to be entered, press zero, R/S. If not, press any non-zero digit, R/S.
O.T = XXXX SEC	If the origin time is acceptable press R/S. If not, enter a new origin time and press R/S.
LAT = ?	Enter the latitude of the first trial focus in degrees and decimals. R/S.
L ON = ?	Enter the longitude of the first trial focus in degrees and decimals. R/S.
Z = ?	Enter the trial depth in kilometres. R/S.
RMSE = XX.XX	The current RMSE is displayed and the calculator pauses for about three seconds. In the (unlikely) event that the RMSE is precisely zero the calculator will branch to its output routine without operator intervention. If the RMSE is non-zero, three actions are possible. a) No action. The calculator proceeds to a new iteration. b) Press R/S during the pause period. The calculator halts operation. To branch to the output routine press zero, R/S. To continue, simply press R/S.

- c) Press R/S during the pause, GTO 145, wait 5 seconds. Press R/S. The calculator will prompt for a new start point.

The output routine displays (and prints) latitude, longitude and depth of the current hypocentre together with the RMSE. Station codes of contributing stations with the associated station residuals are also displayed. The calculator then stops. Several actions are possible:

- a) To continue iteration, GTO 43, R/S.
- b) To enter new start point GTO 145, R/S.
- c) To delete a station 0

STO NN

GTO 43

R/S

(NN = 60 + n)

where n is the number of the station to be deleted. The number of the station is written on the appropriate data card.

After each iteration the calculator prints the values (in kilometres) of the corrections it is about to apply to the current trial hypocentre. No action is required but the magnitude of the corrections is an indication of how close the algorithm is to termination. Generally five iterations or less are required.

TO ADD A NEW STATION TO THE DATA FILE

If there are less than nine stations in the file use the lowest unused location. If there are already nine stations on file overwrite an unwanted station. Let n = position of station in file:

```
"SSS (SSS = station code)
STO (Display says ASTO)
NN" (NN = FO + n)
XX.XXX (Latitude of station in degrees and decimals)
STO NN (NN = 80 + n)
XX.XXX (Longitude)
STO NN (NN = 90 + n)
```

TO CHANGE THE VELOCITY USED

The velocity is stored in location 100 which must be addressed indirectly, e.g.-

```
100
STO 00
XX.XX (New velocity)
STO ( ) (Calculator says STO IND----)
00
```

To restart on a new event go back to A.

TO PREPARE A NEW STATION DATA CARD

- 1) Store station codes in locations 71-79. Store alphabetically (i.e use ASTO)
- 2) Store station latitudes in locations 81-89 and longitudes in 91-99.
- 3) Store velocity in location 100

- 4) Attach card-reader and enter F1.1.
- 5) XEQ "WDTAX", calculator will prompt CARD.
- 6) Insert a new unclipped card. Write data on both tracks.

Station coordinates must be in degrees and decimals. If coordinates are in degrees minutes and seconds the calculator will convert for you. E.g. $18^{\circ} 04' 11''$. Enter 18.0411. XEQ "HR". Display says 18.08.

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APPENDIX 3: Programme Listing

01	+LBL "QUAKE1"	51	ASTO X	101	GTO 10
02	0000.00901	52	CLA	102	RCL IND 04
03	STO 00	53	X=Y?	103	X=0?
04	01	54	GTO 03	104	GTO 10
05	STO 01	55	CLA	105	STO 05
06	+LBL a	56	RCL 21	106	RCL IND 03
07	CF IND 01	57	80	107	ST- 05
08	1	58	-	108	1.37
09	ST+ 01	59	X=0?	109	ST* 05
10	ISG 00	60	GTO 04	110	RCL IND 03
11	GTO a	61	1	111	RCL 05
12	0001.06901	62	ST+ 21	112	-
13	STO 80	63	ST+ 22	113	Σ+
14	01	64	ST+ 23	114	+LBL 10
15	STO 00	65	GTO 02	115	1
16	101	66	+LBL 03	116	ST+ 01
17	STO 90	67	"P=?"	117	ST+ 03
18	+LBL 05	68	AVIEW	118	ST+ 04
19	0	69	STOP	119	ISG 00
20	STO IND 00	70	STO IND 22	120	GTO 11
21	STO IND 90	71	RCL 22	121	MEAN
22	1	72	50	122	STO 50
23	ST+ 00	73	-	123	BEEP
24	ST+ 90	74	STO 02	124	"O.T="
25	ISG 80	75	SF IND 02	125	ARCL 50
26	GTO 05	76	"S=?"	126	" SEC"
27	+LBL 01	77	AVIEW	127	AVIEW
28	.05	78	STOP	128	STOP
29	STO 35	79	STO IND 23	129	0001.00901
30	71	80	"O.K?"	130	STO 45
31	STO 21	81	AVIEW	131	61
32	51	82	STOP	132	STO 47
33	STO 22	83	X=0?	133	51
34	61	84	GTO 01	134	STO 48
35	STO 23	85	GTO 07	135	+LBL 13
36	6378	86	+LBL 04	136	RCL IND 48
37	STO 30	87	TONE 0	137	X=0?
38	AON	88	GTO 01	138	GTO 14
39	"STATION?"	89	+LBL 07	139	RCL 50
40	BEEP	90	CLF	140	-
41	AVIEW	91	0001.00901	141	STO IND 47
42	STOP	92	STO 00	142	+LBL 14
43	ASTO 40	93	1	143	1
44	CLA	94r	STO 01	144	ST+ 47
45	AOFF	95	51	145	ST+ 48
46	+LBL 02	96	STO 03	146	ST+ 49
47	ARCL 40	97	61	147	ISG 45
48	ASTO Y	98	STO 04	148	GTO 13
49	CLA	99	+LBL 11	149	"LAT=?"
50	ARCL IND 21	100	FC?C IND 01	150	AVIEW

151	STOP	201	111	251	STO 46
152	STO 80	202	STO 46	252	121
153	"LONG=?"	203	+LBL 17	253	STO 47
154	AVIEW	204	60	254	131
155	STOP	205	ST/ IND 46	255	STO 48
156	STO 90	206	1	256	+LBL 20
157	"Z=?"	207	ST+ 46	257	RCL IND 46
158	AVIEW	208	ISG 45	258	X=0?
159	STOP	209	GTO 17	259	GTO 21
160	STO 70	210	0001.00901	260	RCL IND 47
161	+LBL 43	211	STO 45	261	-
162	0001.00901	212	111	262	STO IND 48
163	STO 45	213	STO 46	263	X12
164	81	214	100	264	Σ+
165	STO 47	215	STO 48	265	+LBL 21
166	91	216	121	266	1
167	STO 48	217	STO 49	267	ST+ 47
168	101	218	+LBL 18	268	ST+ 48
169	STO 49	219	RCL IND 46	269	ST+ 46
170	111	220	X=0?	270	ISG 45
171	STO 60	221	GTO 19	271	GTO 20
172	61	222	2	272	MEAN
173	STO 46	223	/	273	SORT
174	+LBL 15	224	SIN	274	STO 45
175	RCL IND 46	225	X12	275	"RMSE="
176	X=0?	226	RCL 30	276	ARCL 45
177	GTO 16	227	RCL 70	277	BEEP
178	RCL 80	228	-	278	AVIEW
179	STO 07	229	*	279	PSE
180	RCL 90	230	RCL 30	280	PSE
181	STO 08	231	*	281	CLD
182	RCL IND 47	232	4	282	X=0?
183	STO 09	233	*	283	GTO 44
184	RCL IND 48	234	RCL 70	284	0001.00901
185	STO 10	235	X12	285	STO 39
186	XROM "*GC"	236	+	286	101
187	STO IND 60	237	SORT	287	STO 40
188	RDN	238	RCL IND 48	288	111
189	STO IND 49	239	/	289	STO 41
190	+LBL 16	240	STO IND 49	290	141
191	1	241	+LBL 19	291	STO 42
192	ST+ 46	242	1	292	151
193	ST+ 47	243	ST+ 46	293	STO 43
194	ST+ 48	244	ST+ 49	294	161
195	ST+ 49	245	ISG 45	295	STO 44
196	ST+ 60	246	GTO 18	296	100
197	ISG 45	247	CLE	297	STO 45
198	GTO 15	248	0001.00901	298	121
199	0001.00901	249	STO 45	299	STO 46
200	STO 45	250	61	300	+LBL 50

301 RCL IND 46
302 X=0?
303 GTO 22
304 RCL IND 45
305 X12
306 *
307 1/X
308 STO 31
309 RCL 70
310 RCL 30
311 -
312 RCL IND 41
313 SIN
314 *
315 RCL IND 40
316 SIN
317 *
318 RCL 31
319 *
320 STO IND 42
321 RCL IND 40
322 TAN
323 /
324 STO IND 43
325 RCL IND 41
326 .5
327 *
328 SIN
329 X12
330 CHS
331 RCL 30
332 *
333 2
334 *
335 RCL 70
336 +
337 RCL 31
338 *
339 STO IND 44
340 +LBL 22
341 1
342 ST+ 40
343 ST+ 41
344 ST+ 42
345 ST+ 43
346 ST+ 44
347 ST+ 46
348 ISG 39
349 GTO 50
350 0001.01501

351 STO 38
352 15
353 STO 39
354 *LBL 41
355 0
356 STO IND 39
357 1
358 ST+ 39
359 ISG 38
360 GTO 41
361 SF 05
362 SF 04
363 3
364 STO 14
365 0001.00901
366 STO 38
367 141
368 STO 39
369 151
370 STO 40
371 161
372 STO 41
373 131
374 STO 42
375 +LBL 25
376 RCL IND 39
377 X=0?
378 GTO 40
379 X12
380 ST+ 15
381 LASTX
382 RCL IND 40
383 *
384 ST+ 16
385 ST+ 18
386 LASTX
387 X12
388 ST+ 19
389 LASTX
390 RCL IND 41
391 *
392 ST+ 20
393 ST+ 22
394 LASTX
395 X12
396 ST+ 23
397 LASTX
398 RCL IND 39
399 *
400 ST+ 17

401 ST+ 21
402 RCL IND 39
403 RCL IND 42
404 *
405 ST+ 27
406 RCL IND 40
407 RCL IND 42
408 *
409 ST+ 28
410 RCL IND 41
411 RCL IND 42
412 *
413 ST+ 29
414 +LBL 40
415 1
416 ST+ 39
417 ST+ 40
418 ST+ 41
419 ST+ 42
420 ISG 38
421 GTO 25
422 RCL 45
423 X12
424 1 E-7
425 *
426 ST+ 15
427 ST+ 19
428 ST+ 23
429 XROM "PVT"
430 CLD
431 RCL 29
432 ST+ 70
433 RCL 28
434 111
435 /
436 ST+ 80
437 RCL 27
438 111
439 /
440 ST- 90
441 RCL 70
442 X 0?
443 0
444 STO 70
445 CF 05
446 CF 03
447 GTO 43
448 +LBL 44
449 CLA
450 "LAT="

451 ARCL
452 " N"
453 PRA
454 "LONG="
455 ARCL 90
456 " W"
457 PRA
458 "DEPTH="
459 ARCL 70
460 " KM."
461 PRA
462 "RMSE="
463 ARCL 45
464 " SEC."
465 PRA
466 CLA
467 0001.00901
468 STO 39
469 131
470 STO 40
471 71
472 STO 41
473 +LBL 51
474 RCL IND 40
475 X=0?
476 GTO 52
477 ARCL IND 41
478 " "
479 ARCL IND 40
480 PRA
481 CLA
482 +LBL 52
483 1
484 ST+ 40
485 ST+ 41
486 ISG 39
487 GTO 51
488 STOP
489 .END.

STATUS:
SIZE= 170
E = 11
DEG
FIX 4

USER KEYS:

12 XROM 30, 03
22 "QUAKE1"