ABSTRACT

Almost Sure Asymptotic Stability of the Equilibria of Discretisations of Stochastic Differential Equations

Peter Cleavland Palmer

This dissertation is an examination of the almost sure (a.s.) asymptotic behaviour of the equilibrium solutions of stochastic difference equations with multiplicative noise. Three variants of a discretised Itˆo formula will be applied to develop sharp conditions for globally a.s. asymptotically stable and unstable equilibria of linear and nonlinear stochastic difference equations, and a variant of the semimartingale convergence theorem to develop a sufficient condition for a globally a.s. asymptotically stable equilibrium solution of a linear scalar stochastic difference equation driven by two independent perturbations. The stochastic difference equations rely on a parameter (scalar) h which is understood to be the stepsize of either an Euler-Maruyama, Euler-Milstein or θ -Milstein discretisation. Whenever the discretised Itˆo formula is applied we require that h be sufficiently small so that the stability/instability conditions will be valid. The versions of the discrete Itˆo formula that are developed for scalar linear stochastic difference equations enable us to construct a computable bound for h which can be expressed explicitly in terms of the scalars. In the case of a nonlinear stochastic difference equation we find that the variant of the formula that is developed does not allow this and so Monte-Carlo methods were used to obtain an estimate of the bound. The a.s. asymptotic stability condition that is obtained from an application of the semimartingale convergence theorem is expressed in terms of the parameters of the stochastic difference equation. Numerical simulations of sample paths of the equation are also presented and these simulations are used specifically to illustrate the usefulness of the stability condition.

Keywords: Peter Cleavland Palmer; stochastic difference equations; a.s. asymptotic stability and instability; discrete Itˆo formula.