CARIBBEAN EXAMINATIONS COUNCIL

REPORT ON CANDIDATES' WORK IN THE
CARIBBEAN ADVANCED PROFICIENCY EXAMINATION
MAY/JUNE 2005

APPLIED MATHEMATICS

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INTRODUCTION

The revised Applied Mathematics syllabus was followed this year for the first time. Six candidates registered for this examination and they sat all the required papers of Option C.

This is a one-Unit course comprising three papers and three Options. However a candidate is required to take only ONE Option. Papers 01 and 02 were examined externally while Paper 03 was examined internally by the teachers and moderated by CXC. Contributions from Papers 01, 02 and 03 to the unit were 40 per cent, 40 per cent and 20 per cent respectively.

The three Options are Option A, Option B and Option C.

Option A consists of Discrete Mathematics, Probability and Distribution and Statistical Inferences. Option B consists of Discrete Mathematics, Particle Mathematics and Rigid Bodies, Elasticity, Circular and Harmonic Motion. Option C consists of Discrete Mathematics, Probability and Distribution and Particle Mechanics.

GENERAL COMMENTS

All of the candidates obtained acceptable grades, Grades I – V. The average standard of work seen from most of the candidates in this examination was satisfactory. Candidates appeared to be well-prepared in the Discrete Mathematics and generally answered the questions well. All of the candidates attempted all the questions. Only two candidates displayed areas of weakness in Probability and Distribution and Mechanics. The other four candidates' work was of a satisfactory standard in both of these modules.

Areas of strength displayed by most candidates on this paper were as follows:

- Graphical solution of a Linear Programming problem
- Construction of truth tables
- Express symbolic proposition in words
- Determine Boolean expression from a given logic circuit
- Calculate P(A) given P(A)
- Use conditional probability
- Select *n* distinct objects taken *r* at a time with or without restrictions
- Calculate the expected value of a linear combination of two independent variables
- Draw and use a Venn diagram
- Construct and use a probability distribution table
- Calculate the resultant of a force
- Draw and use velocity-time graphs
- Apply the principle of conservation of linear momentum

The areas of the course which candidates found difficult were as follows:

- Calculations of variance of a linear combination of 2 independent variables
- Recall of definitions associated with propositions
- Translation of worded problem into symbols and vice versa

- Identification of a distribution to model a particular situation
- Use of continuous random variable to find probability and the cumulative function
- Correct direction of a force acting on a particle
- Calculation of Power

Comments on Candidates' Performance on each Question

Paper 01

SECTION A (Module 1: Discrete Mathematics)

Question 1

The question teste the candidates' understanding of graph theory terminology. Most candidates did well on this question. Where there was an error, it usually came in misinterpreting the degree of a vertex as an angle.

Question 2

This question tested the candidates' ability to determine a Boolean expression and construct the corresponding truth table from a logic circuit.

Candidates either received full marks or no marks for this question.

Question 3

In this question, candidates were required to state the converse, contrapositive an inverse of a proposition and to formulate a compound proposition in words.

Only two of the six candidates scored more than half the minimum mark available for the question. It was evident that the remaining candidates were unfamiliar with the terms "converse", "contrapositive" and "inverse" in relation to a proposition.

Question 4

This question tested the candidates' ability to formulate a linear programming model in two variables from real world data.

Candidates experienced some difficulty with this modeling question, since it required the translation of words to symbols.

Question 5

This question which required the candidates to solve a linear problem graphically, was the best one done in this section. Four candidates scored full marks (8) while two scored 7.

SECTION B

Question 6

This question tested the candidates' ability to

(i) calculate the probability of an event given its compliment
(ii) find the intersection of events
(iii) calculate the probability and an event given the union of the two events

Of the six candidates writing the paper, only one was unable to calculate P(A) given P(A). The calculation of $P(A \cup B)$ posed a greater problem to some candidates who were able to state correctly

$P \cup B) = P(A) + P(B) – P(A \cap B)$

but did not relate P(A) to the given P(A|B)

Most candidates were able to calculate P(B) using

$P(B) = P(A \cap B)/ P(A/B)$

as the value for $P(A \cap B)$ was stated in part b.

Answer: (a) 1/4 (b) 5/36 (c) 5/9

Question 7

This question tested the candidates' ability to calculate the number of selections of *n* distinct objects taken *r* at a time, with or without restrictions.

Only half of the candidates were able to do this correctly. Errors made included candidates finding the team so as to include at least three men rather than more than three men. One candidate omitted the fact that all five men could be included.

Part (b) of this question was poorly done with some candidates using a Poisson distribution or binomial distribution to calculate the probability.

Answer: (a) 81 (b) 8/15

Question 8

This question tested the candidates' ability to calculate

(i) E(X) given $E(X^2)$ and Var (X)
the expected value and the variance of a linear combination of 2 independent random variables.

Most candidates were able to find

E(X) given $E(X^2)$ and Var (X) as well as

E(5X + 3Y) given E(X) and E(Y)

However, the calculation of Var (2X – Y) posed greater problems to candidates who incorrectly wrote

$\mathrm{Var}(2X – Y) = 4\,\mathrm{Var}(X) – \mathrm{Var}(Y)$ or $2\,\mathrm{Var}(X) – \mathrm{Var}(Y)$ or $E(2X^2 – Y^2) – E(2X – Y)^2$

Answer: (a) 3 (b) 21 (c) 44

Question 9

This question tested the candidates' ability to

(i) construct a probability distribution table
(ii) use it to find the value of an unknown constant
(iii) calculate the mean and standard deviation

This question was generally well done by the candidates. However, a few of them neglected to put a negative sign in the probability distribution table (i.e. -2) or they found the variance instead of the standard deviation.

Answer: (b) 0.56 (c) E(X) = 1.186 million dollars;

Standard deviation = 1.58 million dollars

Question 10

This question tested the candidates' ability to

(i) identify the distribution that may be used to model data
(ii) calculate the mean of the distribution
(iii) calculate $P(X \leq 2)$

This question was not well done. One candidate did not respond. Candidates incorrectly identified the distribution as a binomial or Poisson distribution. Of the two candidates correctly identifying the distribution as geometric one incorrectly stated the formula for $P(X = 2)$ as q^2p.

Answer: (a) $X \sim \text{Geo}(p)$ (b) $p = 1/6$ (c) $P(X \leq 2) = 11/36$

Question 11

Candidates were required to

(a) find the resultant of three forces, each written in the form $a\mathbf{i} + b\mathbf{j}$.

(b) use the principle that when a particle is in equilibrium, the vector sum of its components is zero.

Instead of determining the resultant of the forces acting on the particle, many candidates calculated the magnitude of the resultant of each force.

Few candidates were able to apply the fact that for equilibrium $SF_i = 0$.

Answer: (a) $R = 6\mathbf{i} - 3\mathbf{j}$ (b) $p = -6, q = 3$

Question 12

In this question the candidates were required to resolve forces on an inclined plane, and apply the principle that for a particle in equilibrium, the sum of the components of the forces in any direction is 0.

Diagrams were fairly well drawn, but too many candidates experienced difficulty showing the correct directions in which forces were acting.

Answer: W = 7.26 n

Question 13

In this question candidates were required to sketch a velocity-time graph, and use it to calculate the time taken by a train to cover a given distance.

Candidates had no difficulty sketching the graph. They also knew that the area of the enclosed region represented the total distance. Difficulty was, however, experienced in elementary concepts such as finding the area of the trapezium.

Answer: (b) 380s

Question 14

In this question candidates were required to apply Newton's law of motion for two particles connected by a light, inelastic string passing over a rough inclined plane while the other particle hangs freely.

There was hardly any middle score on the question. Candidates either did it very well or were unable to do it. Those who were unable to do it experienced difficulty from the outset as they were unable to insert the tension in the portion of the string passing over the pulley. Newton's law was then incorrectly applied.

Answer: (b)(i) 2.24 ms^{-2} (ii) 30.2N

Question 15

Candidates were required to

(i) apply the principle of conservation of linear momentum, to a bullet passing through a stationary block of wood

(ii) find the power at which an engine traveling up an inclined hill, was working.

The majority of candidates attempted to apply the principle of conservation of linear momentum, and did so correctly. Others, however, attempted to use conservation of kinetic energy, which was incorrect.

In part (b) they were unable to derive an expression for the pull of the engine up the hill. This resulted in their inability to calculate the Power as required.

Answer: (a) 0.3ms^{-1} (b) 606kW

Paper 02

SECTION A (Module 1: Discrete Mathematics)

Question 1

This question tested topics in Logic and Boolean Algebra.

With the exception of part (d), which required the use of laws of Boolean algebra to simplify an expression, all parts of this question were adequately answered by most candidates.

Question 2

In this question, candidates were required to formulate and solve a linear programming problem.

Parts (a), (b) and (c) were well done by most candidates. Although candidates had correct graphs drawn, not all of them were able to identify the vertices of the feasible region, which simply required the read-off of coordinates. None of the candidates answered part (d) well, perhaps because a description of a method was required.

SECTION B (Module 2: Statistics)

Question 3

This question tested the candidates' ability to

(i) find the value of a constant for a given continuous random variable

(ii) calculate the probability, expected value, variance, cumulative function and the lower quartile

This question was not well done. Candidates were generally able to find the value of the constant *k* after which they incorrectly set up the continuous random variable as a discrete random variable and proceeded to find $P(X \geq 2)$, $E(X)$, $Var(X)$ and $F(x)$.

Answers: (b)(i) 16/17 (ii) $E(X) = 3.21$ $Var(X) = 0.406$

(c)(1) 0 $x \leq 1$

$\frac{x^4 - 1}{225}$ $\quad 1 \le x \le 4$

$1 \quad x \ge 4$

(ii) Lower quartile, q = 2.84

Question 4

This question tested the candidates' ability to calculate probability using the

(i) binomial distribution
(ii) Poisson distribution as an approximation to the binomial

It also required candidates to calculate a conditional probability $P(A|B)$ given $P(A)$, $P(B)$ and $P(A \cup B)$ as well as draw a Venn diagram to indicate given probabilities in each.

This question was quite well done with candidates getting full marks on (a) (i) and (b) (i) and (ii). Some candidates incorrectly interpreted probability of more than 2 as $P(X \ge 2)$. Part (a)(ii) was poorly done as all candidates who incorrectly used the normal distribution to the binomial instead of the Poisson distribution.

Answers: (a) (i) 0.135 (b) 0.362
(ii) 0.647

(b) (i) 1/3
(ii)

Question 5

Given an equation for the velocity of a particle, in the form v = f(t), candidates were required to find

(i) the acceleration of the particle at a given time
(ii) the distance covered by the particle between two given times

Candidates were generally able to use acceleration = dv/dt, to find the value of the acceleration at a given time. They were also unable to perform the necessary integration to calculate the distance covered between the two given times.

Answer: (i) 30.4 ms^{-2} (ii) 700 m

Given that the resistance to motion of a particle falling through a medium is proportional to the velocity v, candidates were required to

(i) find an expression for *v* in terms of *t*, and
(ii) show that the velocity approaches a given limit as t ! ¥.

The candidates were able to find the resultant force acting on the falling particle and apply F = ma = m dv/dt. They, however, experienced some difficulty in solving the resultant differential equation. Some candidates avoided the solution of the differential equation by interpreting 'approaches a limit', as "becomes constant", thereby, finding *v* when *a* = 0.

Answer: (a) 304 ms^{-2} (ii) 700m (b)(i) $v = mg/k\,(1 - e^{-kt/m})$.

Question 6

This question required candidates to

(i) model the motion of a projectile as a particle moving under constant gravitational force neglecting air resistance, and use the equations of motion of a projectile

(ii) use the principle of conservation of energy.

On the whole, this was a well done question. Candidates showed that they were familiar with solving problems of this type. A weakness was evident when they were required to use the principle of conservation of energy. They were unsure about the application of

Total energy at P = total energy at Q

Or

Potential Energy = Kinetic energy

Answer: (a) 1.5s (b) 11.25m (c) 2s (d) 26.5 m s^{-1}