C A R I B B E A N   E X A M I N A T I O N S   C O U N C I L

REPORT ON CANDIDATES' WORK IN THE
CARIBBEAN SECONDARY EDUCATION CERTIFICATE® EXAMINATION

MAY/JUNE 2013

MATHEMATICS
GENERAL PROFICIENCY EXAMINATION

GENERAL COMMENTS

The General Proficiency Mathematics examination is offered in January and May/June each year. There was a candidate entry of approximately 92 400 in May/June 2013 and 30 per cent of the candidates earned Grades I–III. The mean score for the examination was 65 out of 180 marks.

DETAILED COMMENTS

Paper 01 — Multiple Choice

Paper 01 consisted of 60 multiple–choice items. It was designed to provide adequate coverage of the content with items taken from all sections of the syllabus. Approximately 74 per cent of the candidates earned acceptable grades on this paper; the mean score was 33 out of 60 marks. This year, 142 candidates earned the maximum available score of 60.

Paper 02 — Structured Questions

Paper 02 consisted of two sections. Section I comprised eight compulsory questions for a total of 90 marks. Section II comprised three optional questions: one each from Algebra, Relations, Functions and Graphs; Measurement, Trigonometry and Geometry; and Vectors and Matrices. Candidates were required to answer any two out of three questions from this section. Each question in this section was worth 15 marks. The mean score for this paper was 31 out of 120 marks.

Compulsory Section

Question 1

This question tested candidates' ability to

- add, subtract, multiply and divide fractions involving mixed numbers and decimals
- use the calculator to evaluate the square and the square root of rational numbers
- solve problems involving compound interest.

The question was attempted by 99 per cent of the candidates, 3.7 per cent of whom earned the maximum available mark. The mean mark was 5.28 out of 11.

In Part (a), candidates generally provided unsatisfactory responses. While they were able to follow through on the algorithm for subtracting the fractions, many of them were confused as to which of the improper fractions should be inverted when performing division. They demonstrated competence in using the calculator to compute the square and square root of rational numbers. However, two common errors were $(0.32)^2 = 0.32 + 0.32$ and $(0.32)^2 = 2 \times 0.32$.

In Part (b), many candidates did not attempt to calculate either the cost of the same quantity of juice or the volume of juice for the same cost; and in several instances when the correct approach was taken, they experienced difficulty comparing small values such as 0.012 cents and 0.0114 cents.

In Part (c), candidates were able to identify the interest at the end of one year as 8 per cent of the principal. However, those who used the compound interest formula, experienced difficulty in separating the interest from the aggregate.

Solutions

(a) **(i)** $\frac{11}{18}$ **(ii)** **1.3524**

(b) **1 ml of the 350 ml package costs 1.20 cents**
1 ml of the 450 ml package costs 1.14 cents
∴ the 350 ml package is the more cost — effective buy

(c) **(i)** **$768** **(ii)** **$6 000** **(iii)** **$480**

Recommendations

Teachers should provide students with opportunities to work with scientific calculators in performing basic arithmetic operations on rational numbers. They should also incorporate real-life situations into the mathematics lesson so that students can become competent applying the content to authentic situations.

Question 2

This question tested candidates' ability to

- factorize algebraic expressions involving the difference of squares and a quadratic function in three terms
- change the subject of the formula from Fahrenheit to Celsius
- evaluate an algebraic expression by substituting a real number value
- solve problems involving a simple linear equation in one unknown.

The question was attempted by 98 per cent of the candidates, less than 1 per cent of whom earned the maximum available mark. The mean mark was 2.33 out of 12.

The performance of candidates on this question was unsatisfactory. In Part (a), candidates were generally able to factor out $2x$ from $2x^2 - 8x$ but could not proceed further with the difference of two squares. Factorizing $3x^2 - 5x - 2$ proved even more challenging, as candidates could not determine the factors of $-6x^2$ which could be added to produce $-5x$.

In Part (b), candidates experienced difficulty with transposing the formula. A common incorrect strategy is given in the following example: $F - 32 = \frac{9}{5}C$ therefore $\frac{5}{9}F - 32 = C$ or $\frac{F-160}{9} = C$.

In Part (c), many candidates were unable to translate verbal phrases into algebraic expressions or to formulate an equation.

Solutions

(a) **(i)** $2x\,(x-2)\,(x+2)$ **(ii)** $(3x+1)\,(x-2)$
(b) **(i)** $C = \frac{5}{9}(F-32)$ **(ii)** $C = 45$
(c) **(i)** **a)** $(500 - x)$ **b)** $10(500 - x) + 6x$ **(ii)** $x = 223$

Recommendations

Teachers should reinforce the following skills:

(i) Selecting appropriate numerical factors from a quadratic expression
(ii) Clearing fractions in an equation
(iii) Differentiating between factorizing an expression and solving an equation

Question 3

This question tested candidates' ability to

- determine elements in intersections, unions and complements of sets
- solve problems involving the use of Venn diagrams
- calculate lengths of line segments using properties of similar triangles
- explain why two given triangles are similar.

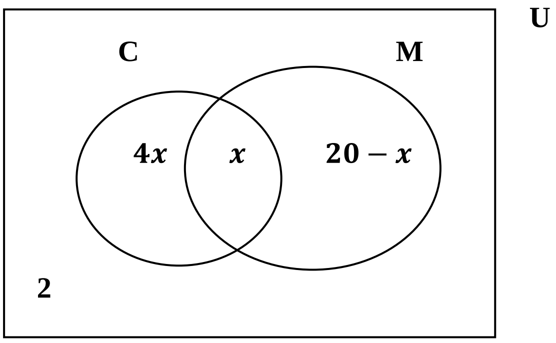
The question was attempted by 98 per cent of the candidates, 1.3 per cent of whom earned the maximum available mark. The mean mark was 4.19 out of 12.

In Part (a), most candidates identified the intersection of the two sets and were able to insert x in the correct region. A large number of candidates also knew the region which represented $C \cap M'$ and correctly inserted $4x$. However, a common incorrect response was to place $4x - x$ in this region. Most candidates incorrectly entered 20 instead of $20 - x$ in the region representing $C' \cap M$. In general, candidates were unable to formulate the correct expression for members in the universal set from information entered on their diagrams.

In Part (b) (i), while candidates correctly chose to apply Pythagoras' theorem, a large number of candidates could not correctly formulate it, and it was common to see $B^2 = 10^2 + 8^2$ instead of $BC^2 = 10^2 - 8^2$. In Part (b) (ii), many candidates wrote that the triangles were similar simply because they were both right–angled triangles.

Solutions

(a) **(i)**



(ii) **4*x* + 22** **(iii)** **8 students**

(b) **(i)** ***BC* = 6 cm** **(ii)** **The corresponding angles are equal.**

(iii) ***DE* = 3.6 m**

Recommendations

Students must be made aware that they should not assume that diagrams given on an examination paper are drawn according to scale. Teachers should encourage students to use mathematical jargon to explain similarity of plane geometrical figures. They need to assist students in differentiating between figures that are similar and those that are congruent. Attention should be given to the difference between an algebraic expression and an algebraic equation. Candidates need to give greater attention to solving problems involving the use of similar triangles.

Question 4

This question tested candidates' ability to

- use a ruler to measure the length of a line segment
- use a protractor to determine the measure of an angle
- determine by measurement the perimeter of a triangle
- determine the area of a triangle
- determine the gradient of a line, given the coordinates of two points on the line
- determine the coordinates of the midpoint of a line segment
- determine the equation of the perpendicular bisector of a line, given two points on the line.

The question was attempted by 87 per cent of the candidates, 1.6 per cent of whom earned the maximum available mark. The mean mark was 3.53 out of 12.

In Part (a), candidates demonstrated some proficiency in measuring the length of a line segment although there were those who stated their measured values with a constant error of 1 cm. A large number of candidates were unable to state the measure of angle EOD. Most candidates appeared to be familiar with the concept of perimeter even when their measurements were inaccurate. They generally applied the correct algorithm for finding the area of a triangle and a variety of formulae given in the rubrics were applied.

In Part (b), many candidates could not determine the gradient of a line that is perpendicular to a given line. In some situations, candidates who knew the gradient of the new line could not complete the solution to the problem because they did not use the coordinates of the midpoint of the given line. In most cases, the coordinates of one of the end points of the given line segment were substituted.

Solutions

(a) (i) (4.9 ± 0.1) cm (ii) $36^{o} \pm 1^{o}$ (iii) (18 ± 0.3) cm (iv) (12 ± 0.2) cm^2
(b) (i) $-\frac{1}{2}$ (ii) (1,3) (iii) y = 2*x* + 1

Recommendations

Teachers should give greater attention to the use of the protractor in measuring angles. They should employ different methods in calculating the gradient of a line and the relationship between the gradient of a line and the gradient of the perpendicular bisector of the line should be reinforced.

Question 5

This question tested candidates' ability to

- write an equation to represent the direct proportion between two variables
- determine the value of the constant of proportionality
- determine the composite of a function
- determine the inverse of a function.

The question was attempted by 86 per cent of the candidates, 4.4 per cent of whom earned the maximum available mark. The mean mark was 3.00 out of 11.

The performance of candidates on this item was generally unsatisfactory. In Part (a), candidates were unable to derive the equation $A = kR^2$ from the information given. Some common incorrect representations were: $A = kR$; $R = \frac{\sqrt{A}}{k}$; $A = k + R^2$ and $A = (kR)^2$. In many situations where candidates obtained the correct equation, they encountered difficulty in transposing the equation to determine the value of k. For example: $36 = k \times 9 \therefore 36 - 9 = k$; $36 = k \times 9 \therefore \frac{9}{36} = k$.

In Part (b), candidates encountered difficulty in evaluating $f\,g$ (2). The majority of candidates calculated the value of g (2), but many did not proceed to evaluate f (13). Some popular incorrect responses were:

$f\,g\,(2) = f\,(2) + g\,(2)$; $f\,g(2) = f\,(2) \times g\,(2)$; $f\,g(x) = 4 \times \left(\frac{2x+1}{3}\right) + 5$ which actually is $g\,f\,(x)$; $f\,g\,(x) = \frac{2x(4x+5)+1}{3}$ which is an attempt at $f\,g\,(x)$ without replacing the x in $f\,(x)$.

Also, most candidates wrote the inverse function of $f\,(x)$ correctly but were unable to evaluate f^{-1} (3).

Solutions

(a) **(i)** $\boldsymbol{A = kR^2}$ **(ii)** $\boldsymbol{k = 4}$ **(iii)** **For $\boldsymbol{A}$ = 196, $\boldsymbol{R}$ = 7; $\boldsymbol{R}$ = 5, $\boldsymbol{A}$ = 100**
(b) **(i)** **9** **(ii)** **4**

Recommendations

Teachers are advised to encourage students to read the entire question carefully since some candidates appeared to focus only on the numerical aspects of the question and as a consequence missed important information. They should assist students in recognizing linkages between parts of a question. The use of flow charts might prove useful in helping students to better understand composition of functions and the determining the inverse.

<u>Question 6</u>

This question tested candidates' ability to

- convert from $km\,h^{-1}$ to $m\,s^{-1}$
- calculate the distance travelled, given speed and time
- describe completely a reflection in the plane
- draw the image of a triangle after undergoing a translation in the plane
- describe a transformation as the combination of two simple transformations.

The question was attempted by 87 per cent of the candidates, 1.1 per cent of whom earned the maximum available mark. The mean mark was 2.03 out of 11.

In Part (a), the candidates knew the formula relating distance, speed and time and were able to apply it correctly in the majority of cases, even when they could not convert from kmh^{-1} to ms^{-1}.

The performance on this question was generally unsatisfactory. Most candidates were able to identify the first transformation as a reflection but were unable to state the equation of the mirror line. They experienced difficulty with the use of the given translation vector and in many cases, when they correctly drew the image of LMN under the translation, they could not describe the combination of transformations that would map this image onto $L'M'N'$.

Solutions

(a) **(i)** **15 $\boldsymbol{ms^{-1}}$** **(ii)** **300 m**

(b) **(i)** **a reflection in the line $\boldsymbol{x = 7}$**

(iii) **a reflection in the line $\boldsymbol{x = 7}$ and a translation by the vector $\boldsymbol{\binom{0}{3}}$**

Recommendations

Teachers should place greater emphasis on the conversion of units within measurement. They should reinforce terminology used to describe translations, reflections, rotations and enlargements. Technology could be used to demonstrate to students the processes and effects of these basic geometrical transformations on plane figures.

Question 7

This question tested candidates' ability to

- complete the cumulative frequency in a grouped-frequency table
- draw a cumulative frequency graph, using given values in a table
- use a cumulative frequency graph to estimate the median of a data set
- use the cumulative frequency graph to estimate probabilities.

The question was attempted by 91 per cent of the candidates, 4.1 per cent of whom earned the maximum available mark. The mean mark was 4.57 out of 11.

Candidates performed moderately well on this question. The majority of them were able to complete the cumulative frequency column from the given frequency distribution and to use the given scales for the axes. Generally, they knew that a cumulative frequency curve should produce an ogive, although their attempts did not always produce a smooth curve and there were instances where candidates interchanged the axes when plotting the points. Drawing lines on the graph to show how the median was estimated and for the proportion of students who spent less than $23 posed a significant challenge for some of the candidates. It was common to see lines drawn vertically at $30 and horizontally at 18 or 19 students. This suggested that some candidates thought that the median was located at the midpoint of the amount of money spent. Further, there were candidates who correctly determined the number of students who spent less than $23 but did not proceed to state the probability.

Solutions

(a)

Amount Spent ($)	No. of Students	Cumulative Frequency
31 - 40	**11**	**30**
41 - 50	**8**	**38**
51 - 60	**2**	**40**

(c) **(i)** **$31** **(ii)** $\frac{12}{40}$

Recommendations

Teachers need to provide students with more practice in drawing cumulative frequency graphs and in using them to estimate measures of position and simple probabilities. Attention should be given to the plotting of points at the boundaries rather than at the limits of class intervals.

Question 8

This question tested candidates' ability to

- draw the fourth diagram in a sequence of diagrams in which the first three diagrams in the sequence are given
- complete a table to show the values in a sequence of numbers
- derive the general rule representing the patterns shown.

The question was attempted by 93 per cent of the candidates, 3.0 per cent of whom earned the maximum available mark. The mean mark was 4.08 out of 10.

Candidates' responses to this question revealed a number of weaknesses in their basic mathematical knowledge. There was a general disregard for the fact that each diagram after the first consisted of cubes linked together. Many candidates ignored the number 20 in the first column, which represented the 20^{th} diagram, and instead completed the table for the 5^{th} diagram. Very few candidates were able to write the algebraic expression for the nth diagram in the sequence and many opted for a description in words rather than symbols.

Solutions

(b) **(i)** $\boldsymbol{W = 36}$; $\boldsymbol{B = 20}$ **(ii)** $\boldsymbol{W = 164}$; $\boldsymbol{B = 84}$

(c) **(i)** $\boldsymbol{W = 8N + 4}$ **(ii)** $\boldsymbol{B = 4N + 4}$

Recommendations

Teachers should assist students with writing expressions and formulae derived from number sequences. They should devote time to the drawing of prisms on plain paper and graph sheets, pointing out cross-sections, unseen lines, faces, edges and vertices.

Optional Section

Question 9

This question tested candidates' ability to

- write linear inequalities to represent constraints presented in words
- draw graphs to represent linear inequalities in two unknowns
- write in words the relationship between two variables expressed as an inequality
- represent on a graph the region which represents the solution set of a system of inequalities
- complete the square of a quadratic function
- sketch a graph to represent a quadratic function.

The question was attempted by 56 per cent of the candidates, 1.1 per cent of whom earned the maximum available mark. The mean mark was 3.25 out of 15.

Candidates' performance on this question was unsatisfactory. While a large number of candidates were able to derive the inequality $\mathrm{x} + \mathrm{y} \leq 6$, and were proficient at drawing a line on the graph to represent $\mathrm{x} + \mathrm{y} = 6$, they experienced difficulty with drawing the line $y = 2$ and could not explain in words the meaning of $y \leq 2x$. When lines were drawn on the graph, many candidates labelled them using the inequality signs.

In Part (b), some candidates who attempted to write $3x^2 - 12x + 8$ in the form $a(x + h)^2 + k,$ using the traditional method of completing the square, demonstrated little proficiency in this skill. Many of them attempted to factorize the expression instead. When asked to sketch the graph of $y = 3x^2 - 12x + 8,$ many candidates resorted to constructing a table of values, plotting points, and drawing a curve through these points. Furthermore, some candidates, even having sketched the curve, could not determine the y-intercept on the sketch.

Solutions

(a) **(i)** $\boldsymbol{x + y \leq 6}$ **(ii)** $\boldsymbol{y \geq 2}$

(iii) $\boldsymbol{The\ number\ of\ mangoes\ must\ not\ exceed\ twice\ the\ number\ of\ oranges}.$

(b) (i) $\boldsymbol{3(x-2)^2 - 4}$

Recommendations

Teachers should assist students in differentiating between sketching a graph and drawing a graph of a given function. They should expose students to the traditional method of completing the square. Some emphasis should be placed on determining the coordinates of minimum or maximum point from the result of completing the square.

<u>Question 10</u>

This question tested candidates' ability to

- use the properties of circles and circle theorems to determine the measures of angles
- use trigonometric ratios to solve problems related to bearings and angles of elevation in a three-dimensional figure.

The question was attempted by 44 per cent of the candidates, less than 1 per cent of whom earned the maximum available mark. The mean mark was 2.98 out of 15.

Candidates' performance on this question was generally unsatisfactory. There was evidence that candidates were exposed to the angle properties of triangles and circles but demonstrated misconceptions in their applications. For example, some candidates doubled the angle at the centre of the circle to obtain a value for the angle at the circumference.

Also, in Part (b) candidates encountered much difficulty in extracting plane shapes from the three-dimensional diagram, and even when they succeeded, they invariably labelled the right–angle incorrectly. Most of them could not identify the angle of elevation of T from S. Moreover, some candidates proceeded to use trigonometric rules instead of ratios in their attempt at solving the right–angled triangles.

Solutions

(a) **(i)** **55^o** **(ii)** **100^o** **(iii)** **50^o** **(iv)** **15^o**

(b) **(iii)** $\boldsymbol{65.5\ m}$ **(iv)** **30^o**

Recommendations

Teachers should employ a more practical approach in teaching trigonometry involving three-dimensional figures. They should assist students in making the determination as to when to use trigonometric ratios and when to use trigonometric formulae.

Question 11

This question tested candidates' ability to

- use vector geometry to determine the resultant of two or more vectors
- use vectors to solve problems in geometry
- evaluate the determinant of a 2 × 2 matrix
- obtain the inverse of a non-singular 2 × 2 matrix
- prove that the product of a matrix and its inverse is the identity matrix
- solve for four unknowns in a matrix equation.

The question was attempted by 45 per cent of the candidates, 1.1 per cent of whom earned the maximum available mark. The mean mark was 3.01 out of 15.

Candidates performed unsatisfactorily on this question. They were able to identify OB and PQ as being parallel even when their algebraic expressions for the vectors did not match their observation. In most instances, they were unable to arrive at the correct routes for the resultant vectors.

In Part (b), candidates encountered difficulty with finding the inverse of a two by two matrix, matrix multiplication and with solving the matrix equation. In the latter case, they resorted to writing two pairs of simultaneous equations and proceeded to solve them.

Solutions

(a) **(i)** **a)** $\overrightarrow{\boldsymbol{AB}} = -2\boldsymbol{a} + 2\boldsymbol{b}$ **b)** $\overrightarrow{\boldsymbol{PQ}} = \boldsymbol{b}$

(ii) $\boldsymbol{OB = 2PQ}$; $\boldsymbol{OB\ is\ parallel\ to\ PQ}$

(b) **(i)** $\frac{1}{2}\begin{pmatrix} 3 & -1 \\ -4 & 2 \end{pmatrix}$ **(iii)** $\boldsymbol{r = 1}$; $\boldsymbol{s = 2}$; $\boldsymbol{t = 0}$; $\boldsymbol{u = -3}$

Recommendations

Teachers should introduce vectors sufficiently early in the curriculum so that students have enough time to be familiar with the content. In teaching vector geometry, they should give attention to identification of a convenient route for determining the resultant vector, and to the skill of substituting and simplifying values for the route chosen.

Attention should also be given to the use of inverse matrices in the solution of systems of two linear equations.