

ABSTRACT

On Tree Polynomials of Graphs

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This thesis continues the accelerated work in the area of graph theory and its association with polynomials. It provides an expansion on previous work done on the tree polynomial and introduces new results. The tree polynomial is one of the few F -polynomials, that has not been extensively investigated.

The *tree polynomial* of a graph G is defined as follows. Let G be a given graph. Let us associate an indeterminate $w(\alpha)$, called the weight of α , with every tree α in G . Let F be a *spanning forest* or "*cover*" of G . Let us assign a weight $w(F) = \prod_{\alpha} w(\alpha)$ to F , where the product is taken over all components of F . The *tree polynomial* of G is $\sum w(F)$, where the summation is taken over all covers F in G .

The characterizing properties of the tree polynomial have been investigated in this thesis. We have been able to establish that the tree polynomial is a characterizing polynomial, for some families of graphs. Also, we have established the existence of co-tree graphs. We have shown that some trees can be characterized by the terms of its polynomial and that the graph of the tree can be constructed from the coefficients of these terms.

Extending results on the linear and long linear polygonal chains have allowed us to develop general formulae for each. We have also extended results on the forest decomposition of graphs with cyclomatic number, up to 4.

Research on the tree polynomial has not been as extensive as other more common types of polynomials. This may be so because of the general nature of a tree. While more extensive work has been done on some special types of trees, not enough comparative work has been done on trees. It is hoped that the work presented here will help to reduce the disparity between tree polynomial and some of the more common polynomials.

Keywords: Derek V. West; F-polynomials; tree polynomial; spanning forest; co-tree graphs; linear and long linear polygonal chains; cyclomatic number.