C A R I B B E A N   E X A M I N A T I O N S   C O U N C I L

REPORT ON CANDIDATES' WORK IN THE CARIBBEAN ADVANCED PROFICIENCY EXAMINATION®

MAY/JUNE 2012

PURE MATHEMATICS

GENERAL COMMENTS

In 2012, approximately 5500 and 2800 candidates wrote the Units 1 and 2 examinations respectively. Performance continued in the usual pattern across the total range of candidates with some candidates obtaining excellent grades, while some candidates seemed unprepared to write the examinations at this level, particularly in Unit 1.

The overall performance in Unit 1 was satisfactory, with several candidates displaying a sound grasp of the subject matter. Excellent scores were registered with specific topics such as Trigonometry, Functions and Calculus. Nevertheless, candidates continue to show weaknesses in areas such as Modulus, Indices and Logarithms. Other aspects that need attention are manipulation of simple algebraic expressions, substitution and pattern recognition as effective tools in problem solving.

In general, the performance of candidates on Unit 2 was satisfactory. It was heartening to note the increasing number of candidates who reached an outstanding level of proficiency in the topics examined. However, there was evidence of significant unpreparedness by some candidates.

Candidates continue to show marked weaknesses in algebraic manipulation. In addition, reasoning skills must be sharpened and analytical approaches to problem solving must be emphasized. Too many candidates demonstrated a favour for problem solving by using memorized formulae.

DETAILED COMMENTS

UNIT 1

Paper 01 – Multiple Choice

Paper 01 comprised 45 multiple-choice items. Candidates performed satisfactorily, with a mean score of 29.5 and a standard deviation of 20.55.

Paper 02 – Structured Questions

Section A

Module 1: Basic Algebra and Functions

Question 1

Specific Objectives: (a) 5; (b) 1, 2, 3, 4; (g) 1, 4

The topics covered in this question were the Remainder and Factor Theorems, Operations on Surds and Modulus Inequalities.

The majority of candidates showed good performance in Parts (a) (i) and (ii). Some instances of errors in substitution were observed. These included f (1) = –6, f (–1) = 6 and f (–1) = 0. A small number of candidates failed to complete the factorization of f (x).

Part (b) was generally well done. Some computational errors included incorrectly expanding $\left(\sqrt{x} + \sqrt{y}\right)^2$ which resulted in the loss of marks by some candidates, failure to form two equations in x and y and failure to equate the terms to solve x and y. General algebraic weaknesses were also evident.

The majority of candidates removed the modulus in Part (c) (i) by squaring both sides of the inequality obtaining a quadratic inequality in x. Some candidates were able to use a graphical approach to identify the range of values of x for which the inequality was satisfied. Common errors included, (i) stating the critical values of $|3x - 7| = 5$ and $(3x - 2)(x - 4) \le 0 \Rightarrow x \le \frac{2}{3},\ x \le 4.$

In Part (c) (ii), a few candidates used the quadratic inequality obtained by squaring both sides of the inequality $|3x - 7| + 5 \le 0$ and used the characteristic of the discriminant, $b^2 - 4ac < 0$, to prove the desired result. A number of candidates deduced that $|3x - 7| \ge 0$ for all real values of x.

Solutions

(a) (i) $p = -7$, $q = 1$ (ii) $f(x) = (x - 1)(x + 2)(2x + 5)$

(b) $x = 10$, $y = 6$ or $x = 6$, $y = 10$

(c) (i) $\frac{2}{3} \le x \le 4$

<u>Question 2</u>

Specific Objectives: (c) 1, 3; (d) 1, 7; (f) 3

This question tested applications of a composite quadratic function, solution of the resulting quartic equation equal to a linear equation, the relationships between the sum and product of the roots of a quadratic equation, applications of the laws of logarithms in base 10 to simplify a sum of quotients without the use of calculators or tables and the finite sum of a quotient of logarithms in base 10.

Part (a) (i) required a composite function ff (x). This was generally well done. However, many candidates failed to simplify the resulting expression. This resulted in a number of candidates being unable to successfully complete Part (a) (ii). Poor algebraic skills did not allow for the correct solution of $x^4 - 7x^2 - 6x = 0$. Some candidates seemed perplexed that the equation did not contain a term in x^3.

Parts (b) (i) and (ii) were generally well done. A small number of candidates were unable to state $\alpha^2 + \beta^2$ correctly in terms of $\alpha + \beta$ and $\alpha\beta$. Performance on Part (b) (iii) was generally good with the exception of a small number of candidates who found the algebra for $\frac{2}{\alpha^2} + \frac{2}{\beta^2}$ expressed in terms of $\alpha + \beta$ and $\alpha\beta$ beyond their capabilities. Too many candidates did not write the required quadratic equation but instead stated the expression $x^2 - 2x + 64$.

Part (c) (i) was generally well done. Candidates recognized the required laws of logarithms to use and were able to simplify the logarithmic expression.

In Part (c) (ii), a significant number of candidates demonstrated their inability to use the sigma notation. The answer $\log_{10}\left(\frac{99}{100}\right)$ was seen in many cases. Some candidates were able to express $\sum_{r=1}^{99}\left(\frac{r}{r+1}\right)$ as $\sum_{r=1}^{99}(\log_{10} r - \log_{10}(r+1))$ but could not apply the concept of the sigma notation. Generally this part of the question had limited successes.

Solutions

(a) (i) $\mathrm{ff}(x) = x^4 - 6x^2 + 6$ (ii) $x = -2, -1, 0, 3$

(b) (i) $\alpha + \beta = \frac{3}{4},\ \alpha\beta = \frac{1}{4}$ (ii) $\alpha^2 + \beta^2 = \frac{1}{16}$

(iii) $x^2 - 2x + 64 = 0$

(c) (i) -1 (ii) -2

Section B

Module 2: Trigonometry and Plane Geometry

<u>Question 3</u>

Specific Objectives: (a) 4, 5, 9, 10, 12, 13

This question tested trigonometric identities, compound and multiple angle formulae, the factor formulae and solutions of trigonometric equations, use of trigonometric identities, compound and multiple angles formulae and the factor formulae.

Most candidates attempted Part (a) (i). Many of them failed to show the desired result due to poor manipulation of the identities given and subsequent simplification. A number of candidates failed to deduce that the factor formula was required to show the desired result in Part (a) (ii). Those who attempted other approaches were not able to complete the result. In Part (a) (iii), many candidates did not heed the directive ‘Hence…’ thus enabling them to use the result at Part (a) (ii). The factor formula could have been easily used heeding the directive ‘or otherwise’. A small number of candidates attempted the expansions of sin 6θ and sin 2θ in terms of sin θ but experienced algebraic difficulties to complete the solution.

In Part (b), the majority of candidates recognized the need to substitute $\dfrac{\cos^2\theta}{\sin^2\theta}$ for $\cot^2\theta$. In some cases candidates divided the equation $2\cos^2\theta + \cos\theta\sin^2\theta = 0$ by $\cos\theta$ thus losing the result $\cos\theta = 0$. A number of candidates solved the quadratic equation in $\cos\theta$ to obtain $\cos\theta = 1 \pm \sqrt{2}$. However, candidates were penalized for stating $\cos\theta = 1 + \sqrt{2}$. Generally, only a small number of candidates were able to successfully complete this part of the question.

Solutions

(a) (iii) $\theta = \dfrac{\pi}{8}, \dfrac{3\pi}{8}$

(b) $\cos\theta = 0 \quad \cos\theta = 1 - \sqrt{2}$.

Question 4

Specific Objectives: (b) 8, 9; (c) 1, 3, 7, 9, 10

This question tested candidates' ability to give the Cartesian equation of a curve defined in trigonometric parameters, the intersection of a curve with a straight line, expressions of coordinate points in vector form and finding the angle between two vectors using the dot product method.

Part (a) (i) was generally well done by the majority of candidates. There were a few cases of candidates being unable to use the appropriate trigonometric identities to eliminate the parameter. Algebraic errors were evident in Part (a) (ii) where many candidates wrote $\left(\sqrt{10x}\right)^2 = 10x^2$ and $\left(\sqrt{10x}\right)^2 = \sqrt{10}x$. Attempts at subsequent solution of $\dfrac{y^2}{9} - \dfrac{x^2}{9} - 1 = \sqrt{10x}$ resulted in confusion and were often abandoned. The question required candidates to find the points of intersection. Many candidates solved the quadratic equation in *x* and stopped short of finding the corresponding values of *y*. In addition, a number of candidates did not state the coordinates of the points of intersection.

The majority of candidates successfully completed Parts (b) (i) to (iv). Some arithmetic errors were seen in computing the angle in degrees.

Solutions

(a)	(i)	$\frac{y^2}{9} - \frac{x^2}{9} = 1 \Rightarrow y^2 = x^2 + 9$	(ii)	$(1, \sqrt{10}), \ (9, 3\sqrt{10})$
(b)	(i)	$p = -3i + 4j, \ \ q = -i + 6j$	(ii)	$-2i - 2j$
(iii)	27		(iv)	27.41^0

Section C

Module 3: Calculus 1

<u>Question 5</u>

Specific Objectives: (a) 3–5, 7, 10; (b) 5, 11

This question tested the concepts of limit of a function, limit theorems, differentiation of simple functions, continuity and discontinuity and rate of change.

For Part (a) (i), the majority of candidates understood that it was necessary to show that for discontinuity the denominator must be zero. A few candidates solved $x^2 - 4 = 0$ as $x = 2$ only. Generally this part of the question was well done. The majority of candidates had no difficulties successfully completing Part (a) (ii) by using the result at Part (a) (i) and making the relevant cancellation. A few candidates were unable to factorize $x^3 + 8$ as a sum of cubes. The composite function $\frac{2x^3 + 4x}{\sin 2x}$ in Part (a) (iii) seemed to have left candidates in a state of confusion, apparently being drilled in limits involving either rational functions involving factors that cancel or trigonometric functions that are variations of $\frac{\sin x}{x}$. Poor algebraic skills prevented many candidates from simplifying the composite function and using the theorems of limits of sums, differences and quotients. A small number of candidates used L'Hopital's rule and successfully found the correct limit. In Part (b) (i) a), the majority of candidates found the correct limit as 2. It was relatively simple to find the value of p for continuity having found the correct limit for $x > 1$ in b). The answer to Part (b) (ii) was merely deduced from Part (b) (i) a).

The majority of candidates was able to substitute $t = 1$ in the equation for M in order to find the first equation in terms of u and v. A number of candidates made errors in differentiating $\frac{v}{t^2}$ correctly and as a result found incorrect values for u and v. Some candidates did not deduce that a rate of change meant to find $\frac{dM}{dt}$. Generally, a significant number of candidates failed to find the correct values of u and v.

Solutions

(a) (i) $x = \pm 2$ (ii) -3

(iii) 2

(b) (i) a) 2 b) $p = -2$

(ii) f (1) = 2

(c) $u = 2$ $v = -3$

<u>Question 6</u>

Specific Objectives: (b) 5, 10, 13 – 16; (c) 8 (i)

This question tested first and second derivatives using the chain rule and the product/quotient rules, evaluation of a definite integral, location of stationary points, nature of stationary points and sketching a cubic curve.

Part (a) (i) was satisfactorily done. However, some candidates were not able to show the desired result because of poor algebraic skills and subsequent simplification. Due to poor use of the unsimplified result of Part (a) (i), several candidates were not able to show the required result for Part (b) (ii). The continued applications of the chain rule and the product/quotient were beyond the majority of candidates. A very small number of knowledgeable candidates used implicit differentiation and successfully completed this part of the question.

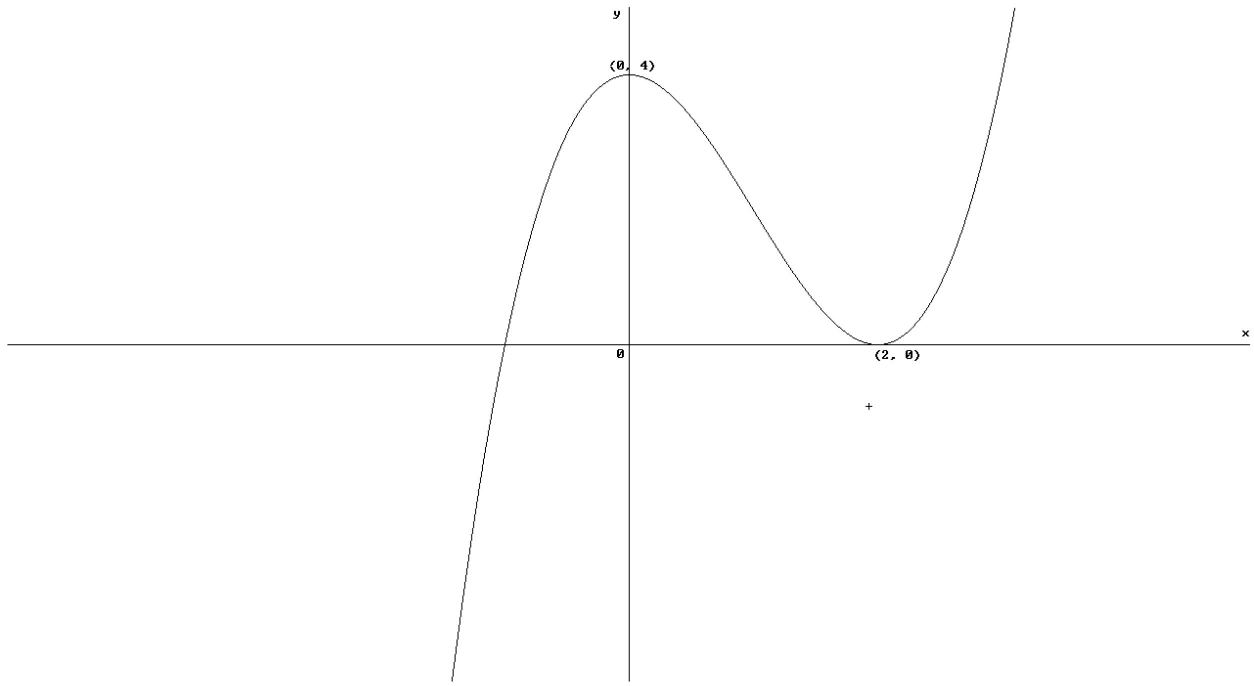
Part (b) (i) required a definite integration. The majority of candidates performed well. Successful solution of $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$ in (ii) assisted the majority of candidates in being awarded maximum marks. Too many candidates substituted the values of x found for $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$ in the equation for $\frac{\mathrm{d}y}{\mathrm{d}x} = 3x^2 - 6x$ to find the y-ordinates of the stationary points. Most candidates knew the concept of using the sign of the second derivative to determine the nature of the stationary points in Part (b) (iii). A number of candidates failed to factorize $x^3 - 3x^2 + 4 = 0$ correctly. Consequently, those candidates were unable to find the correct x-intercepts of the curve in Part (b) (iv). Following the failures at Parts (b) (ii) and (iv), a number of candidates were unable to sketch the curve *C* showing the critical points correctly.

Solutions

(b) (i) $y = x^3 - 3x^2 + 4$ (ii) (0, 4) and (2, 0)

(iii) $(0, 4)_{\text{maximum}}$ $(2, 0)_{\text{minimum}}$ (iv) $(-1, 0)$, $(2, 0)$

(v) see plot



Paper 032 – Alternative to School-Based Assessment

Section A

Module 1: Basic Algebra and Functions

Question 1

Specific Objectives: (a) 8; (c) 1, 3, 5; (f) 4, 5 (ii)

This question tested the roots of a cubic equation, mathematical induction for divisibility, applications of the laws of indices and the laws of logarithms and the solution of a logarithmic equation involving a change of base.

Parts (a) (i) and (ii) were satisfactorily done. Candidates were vague on Part (b). Apart from proving that the statement is true for $n = 1$ and hence the assumption that the statement is also true for $n = k$, $k > 1$, poor algebraic skills prevented a significant number of candidates from proving that the statement is true for $n = k + 1$. In the majority of cases, this part of the question was badly done. The majority of candidates was unable to express a logarithm in index form and use that form to change the base of a logarithm, thus (c) (i) was poorly done. Consequently, Part (c) (ii) was not done by the majority of candidates.

Solutions:

(a) (i) $\alpha = -2$ (ii) $p = 28$

(c) (ii) $x = 2$ or $x = 4$

Section B

Module 2: Trigonometry and Plane Geometry

<u>Question 2</u>

Specific Objectives: (a) 11, 14; (b) 2, 5, 6, 7; (c) 4, 5, 8

This question tested the properties of a circle, a tangent to a circle at a given point, intersection of a curve with a straight line, expression for $a \cos \theta + b \sin \theta = r \cos (\theta - \alpha)$, maximum value of $a \cos \theta + b \sin \theta$, unit vector and a displacement vector.

Candidates performed with limited successes in Parts (a) (i) to (iii). Algebraic and arithmetic errors resulted in loss of marks by some candidates.

Most candidates demonstrated a lack of knowledge of the trigonometric forms in Parts (b) (i). Consequently, performance on Parts (b) (ii) was poor.

Most candidates in attempting Part (c) seemed unaware that it was required to find the unit vector to $\overrightarrow{\mathbf{PQ}}$ before multiplying $\overrightarrow{\mathbf{OR}}$ by $\sqrt{5}$. Generally this part of the question was done poorly.

Solutions

(a) (i) centre (3, –1) radius = 5 units (ii) $\text{tangent}_{(7,\ 2)}\ 4x + 3y - 34 = 0$

(iii) Q (–1, –4)

(b) (i) $\text{f}(\theta) = 6 \cos\left(\theta + 30^0\right)$ (ii) $\text{f}(\theta)_{\text{maximum}} = 6$

(c) $a = \frac{1}{\sqrt{2}}$ $b = \frac{3}{\sqrt{2}}$

Section C

Module 3: Calculus 1

Question 3

Specific Objectives: (a) 3 to 6; (b) 1, 5, 7, 11

This question tested limits using limit theorems, gradient at a point and rate of change.

Overall, candidates performed satisfactorily on this question. More than 50 per cent of the candidates scored at least seven of the 20 available marks and more than 30 per cent scored at least 10 of the 20 available marks. Most candidates were able to use the algebraic expression given and to correctly substitute the given value of x to obtain the limit in Part (a) (i). Using the result from Part (a) (i) the majority of candidates was able to obtain the correct limit in Part (a) (ii).

Candidates demonstrated a fair level of understanding of a gradient function at a point. Good performance was shown in Part (b).

Part (c) posed severe challenges for many candidates. The algebraic expressions for V in terms of t only and V in terms of x only in Parts (b) (i) and (ii) respectively were poorly done by many candidates. The majority of them could not see a relationship between the height of the water and the radius at that instant.

Solutions

(a)(i) $\frac{1}{4}$ (ii) $\frac{1}{12}$

(b) 24

(c) (i) $V = 10t$ (ii) $V = \frac{1}{3}\pi x^3$

(iii) ≈ 0.24 cm/s

UNIT 2

Paper 01 – Multiple Choice

Paper 01 comprised 45 multiple-choice items. Candidates performed satisfactorily, with a mean score of 32.11 and a standard deviation of 18.27.

Paper 02 – Structured Questions

Section A

Module 1: Calculus II

<u>Question 1</u>

Specific Objectives: (b) 1, 3 to 7

This question tested first and second derivatives of a product of a polynomial and an exponential function, stationary points, differentiation of parametric equations including an inverse trigonometric function, gradient at a point and equation of a tangent at a point to a curve defined by parametric equations.

In Part (a) (i) a), candidates demonstrated a sound understanding of the first and second derivatives using the product rule although many of the answers were left unsimplified.

Candidates were able to determine the values of *x*-coordinates correctly for which $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$ and $\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} = 0$ respectively in Parts (a) (i) b) and c). Using the second derivative and the significance of the sign change, the majority of candidates was able to distinguish the maxima and minima points as required in Part (a) (ii). However, a significant number of candidates seemed not to know the conditions for points of inflection, using the properties of the second derivative to identify these points. This part of the question had very limited successes.

Candidates demonstrated a sound understanding of the chain rule to obtain the first derivative of parametric equations required in Part (b) (i). However, the majority of candidates opted to find $\frac{\mathrm{d}x}{\mathrm{d}t}$ which involved differentiating an inverse trigonometric function along with differentiating a variable involving a fractional index. Many errors in the evaluation of this differential resulted in the incorrect expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of *t*. A number of candidates used the fact that $\sin^{-1}\left(\frac{1}{2}\right) = 30^0$ instead of $\frac{\pi}{6}$.

As a result of the inability by many candidates to complete Part (b) (i) successfully, it was not possible to derive the correct equation of the tangent as required in Part (b) (ii).

Solutions

(a) (i) a) $\dfrac{\mathrm{d}y}{\mathrm{d}x} = x\mathrm{e}^{x}(2 + x)$ and $\dfrac{\mathrm{d}^2 y}{\mathrm{d}x^2} = e^{x}\left(2 + 4x + x^2\right)$

b) $x = 0$ or $x = -2$ c) $x = -2 \pm \sqrt{2}$

(ii) $x = 0$ gives a relative minimum point, $x = -2$ gives a relative minimum point,

$x = -2 \pm \sqrt{2}$ gives relative inflection points

(b) (i) $\dfrac{\mathrm{d}y}{\mathrm{d}x} = 4(t-1)\sqrt{t(1-t)}$

(ii) $4y + 4x = \pi - 3$

<u>Question 2</u>

Specific Objectives: (c) 1 (iii), 3, 4, 6, 8, 10

This question tested partial fractions, indefinite integral using partial fractions, the reduction formula and definite integration of a trigonometric function.

Some candidates did not recognize the partial fractions of the form $\dfrac{A}{x-1} + \dfrac{Bx + C}{x^2 + 1}$ in Part (a) (i). However, this part of the question was generally satisfactorily done despite some arithmetic errors which resulted in the wrong values for the constants A, B and C.

In Part (a) (ii) many candidates did not express $\displaystyle\int\left(-\frac{1}{x-1} + \frac{2x-1}{x^2+1}\right)\mathrm{d}x$ as $\displaystyle\int -\frac{1}{x-1}\,\mathrm{d}x + \int \frac{2x}{x^2+1}\,\mathrm{d}x - \int \frac{1}{x^2+1}\,\mathrm{d}x$ which would have facilitated simple integration. Approximately 20 per cent of the candidates evaluated $\displaystyle\int \frac{\mathrm{f}'(x)}{\mathrm{f}(x)}\,\mathrm{d}x$ correctly and approximately 10 per cent of the candidates evaluated $\displaystyle\int \frac{1}{x^2+1}\,\mathrm{d}x$ correctly. A number of candidates did not show evidence of good integration skills. A common error was the omission of the constant of integration for indefinite integrals.

There were no difficulties with Part (b) (i). However, Part (b) (ii) was very challenging to most candidates. Most candidates understood that application of integration by parts was required to find the required result. However, the sequence of the integration techniques and the resulting algebra were beyond the capabilities of most candidates. The candidates who attempted this part of the question failed to recognize and use the link at Part (b) (i). This part of the question was poorly done.

Given the substitution $m = 1$ allowed approximately 10 per cent of the candidates to show the desired result in Part (b) (iii). A number of candidates did not include the limits of integration for $-\cos x \cos 3x$.

Approximately 60 per cent of the candidates evaluated Part (b) (iv) correctly, without any reference to the preceding results.

Solutions

(a) (i) $-\dfrac{1}{x-1} + \dfrac{2x-1}{x^2+1}$ (ii) $\ln\left(\dfrac{x^2+1}{x-1}\right) - \tan^{-1}(x) + C$

(b) (iv) $\dfrac{1}{2}$

Section B

Module 2: Sequences, Series and Approximation

<u>Question 3</u>

Specific Objectives: (a) 2, (b) 1, 4, 6, 9, 13

This question tested geometric progression, sequences, proof by mathematical induction and Maclaurin's series expansion.

The majority of candidates did not find it difficult to complete Part (a) (i) correctly. A few candidates made errors evaluating indices.

Successful completion of Part (a) (ii) by those candidates who found the correct values for a and r followed easily from Part (a) (i). However, a number of candidates made errors in evaluating $177146 = \dfrac{2(3^n - 1)}{3-1}$. Common errors included $2(3^n - 1) = 6^n - 2$ or $2 \times 3^{n-1}$.

A number of candidates were unable to express the rth term of the sequence in Part (b) (i). Proof by mathematical induction was poorly done in Part (b) (ii) by the majority of candidates. A number of candidates showed some evidence of knowing how to begin the proof but lacked the algebraic skills and the sophistication necessary to complete the proof. Apart from proving the statement true for $n = 1$ and attempting to show that it is true for $n = k + 1$, the resulting algebraic substitutions were poorly done. As a result, a number of candidates were not able to complete the proof.

Some candidates who probably memorized Maclaurin's expansion for cos x simply substituted $2x$ for x and obtained the result. A number of candidates however used differentiation and Maclaurin's theorem to obtain the result for Part (c) (i).

Candidates who attempted to express $\sin^2 x$ in terms of $\cos 2x$ for Part (c) (ii) invariably used the wrong identity and could not obtain the expansion of $\sin^2 x$ correctly.

Solutions

(a) (i) $a = 2, \quad r = 3$ (ii) $n = 11$

(b) (i) $u_r = r(r+2),\ r \in \mathrm{N}$

(c) (i) $1 - 2x^2 + \frac{2}{3}x^4 + \ldots$ (ii) $x^2 - \frac{1}{3}x^4$

Question 4

Specific Objectives: (c) 1, 2, 3, 4; (e) 1, 4

This question tested the concepts of factorials, binomial expansion, location of roots and the Newton-Raphson iterative method.

Parts (a) (i) and (ii) were well done by the majority of candidates.

In Part (a) (iii), a small number of candidates used the complete expansion of $\left(x^2 - \frac{3}{x}\right)^8$ before extracting the term in x^4. Some arithmetic errors with the negative sign in the terms involving $\left(-\frac{3}{x}\right)^n$ were evident. Generally, this part of the question was well done, with the majority of candidates demonstrating a good understanding of the binomial theorem.

The majority of candidates who attempted Part (a) (iv) stopped at the expansion of $(1+x)^{2n}$ up to and including the 4th term. No candidate recognized that ${}^{2n}C_n$ is the coefficient of the term in x^n. Consequently, candidates could not proceed to make the link with Part (a) (ii) and hence were unable to show the desired result. Beyond the expansion of $(1+x)^{2n}$ no candidate obtained marks other than those awarded for the partial expansion. The analysis and algebra beyond this point was beyond the grasp of all the candidates who attempted this part of the question.

For Part (b) (i), the majority of candidates concluded that a root exists in the given interval, using the concept of a sign change. Only a very small number of candidates stated that the function is continuous over the given interval. This failure resulted in the majority of candidates being penalized.

The majority of candidates who attempted Part (b) (ii) carried out the required number of iterations with some cases of arithmetic errors. In the absence of the final answer specified to a given number of decimal places or significant figures, most candidates used varying approximate values for x_2, x_3, x_4 and x_5. In many cases the value of the approximation required was not consistent.

Solutions

(a) (i) $\dfrac{n!}{(n-r)!\,r!}$ (iii) 5670

(b) (ii) $T \approx 1.002$

Section C

Module 3: Counting, Matrices and Complex Numbers

Question 5

Specific Objectives: (a) 2, 4, 7; (b) 1, 7, 8

This question tested arrangements with and without repetitions, selections with restrictions, probability, matrix multiplication and subtraction and the solution of a system of linear equations.

Many candidates demonstrated a penchant for using the formula for permutations in Parts (a) (i) and (ii) rather than analysing the problem with particular regard to the restrictions. Satisfactory performance was duly awarded.

In Parts (b) (i) and (ii), candidates demonstrated a sound grasp of determining selections with or without restrictions. This part of the question was well done.

Apart from some arithmetic errors candidates performed well in Part (c) (i). Those candidates who were successful in Part (c) (i) were able to show the desired result in Part (c) (ii).

Candidates who failed to show the result in Part (c) (ii) made attempts to find the inverse of A as required in Part (c) (iii) using the cofactor method. A small number of candidates attempted the row-reduction method. Poor algebraic skills prevented a number of candidates from obtaining the specified result using the result at Part (c) (ii). Generally, this part of the question had limited successes.

Except for those candidates who were successful in Parts (c) (ii) and (iii), a number of candidates attempted to solve the system of linear equations with 3 unknowns by the elimination method. Arithmetic errors apart, some correct solutions were obtained for Part (c) (iv). Candidates who failed to show the result at Part (c) (i) did not reason that they could obtain the solutions at Part (c) (iv) using B^{-1} in terms of A from the equation at Part (c) (ii).

Solutions

(a) (i) 480 ways (ii) 1372 ways

(b) (i) $\frac{1}{77}$ (ii) 144 ways

(c) (i) $B = \begin{pmatrix} 0 & -3 & -3 \\ -3 & -2 & 7 \\ -3 & 1 & 1 \end{pmatrix}$ (iii) $A^{-1} = -\frac{1}{9}\begin{pmatrix} 0 & -3 & -3 \\ -3 & -2 & 7 \\ -3 & 1 & 1 \end{pmatrix}$

(iv) $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -1 \\ -\frac{1}{3} \\ -\frac{2}{3} \end{pmatrix}$

<u>Question 6</u>

Specific Objectives: (c) 1, 2, 3, 5, 6, 9, 11

This question tested representation of complex numbers on the Argand diagram, the principal argument of a complex number, the square roots of a complex number, finding the complex roots of a quadratic equation and application of de Moivre's theorem to prove a trigonometric identity.

Part (a) (i) was well done by the majority of candidates. Severe constraints due to poor algebraic skills resulted in poor performance on Part (a) (ii). Candidates failed to use the relationship between the complex numbers A and B and the representation of these points on the Argand diagram to find the argument of A + B. A number of candidates attempted to rationalize $\frac{1+\sqrt{2}+\mathrm{i}}{1-\mathrm{i}}$ and find $\tan^{-1}\left(1+\sqrt{2}\right)$ approximating the answer to $\frac{3\pi}{8}$.

The majority of candidates who attempted Part (b) (i) obtained full marks. However, a number of candidates could not link the result at Part (b) (i) to the solutions of the quadratic equation in Part (b) (ii). Nevertheless, most candidates demonstrated a sound grasp of the techniques involved to find complex roots of a quadratic equation.

The majority of candidates who attempted Part (c) understood de Moivre's theorem and its application to proof of trigonometric identities. This part of the question was well done.

Solutions

(a) (i) Points $(0, 1)$ and $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

(b) (i) $z = \pm\frac{1}{\sqrt{2}}(1 + i)$ (ii) $z = 2 + 3i,\ 1 + 2i$

Paper 032 – Alternative to School-Based Assessment

Section A

Module 1: Calculus II

<u>Question 1</u>

Specific Objectives: (b) 2, 4, 5; (c) 8, 13

This question tested logarithmic and implicit differentiation involving polynomials and trigonometric functions, the trapezium rule and definite integration by parts to find the area under a given curve.

The majority of candidates showed a marked weakness in the applications of logarithmic differentiation required in Part (a). A few candidates attempted to complete the differentiation by using the product and quotient rules. However, they failed to apply the chain rule for the composite functions correctly. This part of the question was poorly done.

Some weak attempts were made at sketching the curve in Part (b) (i). Most candidates showed a straight line between the limits stated.

Weak performances were also recorded in Part (b) (ii). A number of candidates failed to use the trapezium rule correctly. In addition, some arithmetic errors resulted in wrong values of the required approximation.

Most candidates demonstrated a fair understanding of integration by parts required in Part (c) (i).

However, many of them appeared at a loss when required to repeat the process for $\int 2x \sin x \, dx$.

Limited successes were recorded for this part of the question.

Solutions

(a) $\frac{dy}{dx} = \frac{x(x-1)^{\frac{1}{3}}}{1+sin^3x}\left[\frac{1}{x} + \frac{1}{3(x-1)} - \frac{3\,sin^2x\cos x}{1+sin^3x}\right]$

(b) (ii) 1.115 square units

(c) (i) $sin^2\,sinx\; +\; 2xcosx\; - 2sinx\; + C$

(ii) $\frac{\pi^2}{4} - \;2$

Section B

Module 2: Sequences, Series and Approximation

<u>Question 2</u>

Specific Objectives: (c) 2, 3, 4; (d) 1; (e) 3, 4

This question tested the binomial theorem, the Newton-Raphson iteration method and errors of measurement.

Most candidates performed well in Part (a) (i). In Part (a) (ii), a number of candidates could not determine that it was necessary to substitute x = 0.1 in the expansion found in Part (a) (i). Arithmetic errors resulted in some incorrect answers.

A number of candidates failed to observe the range of values of x for which the graphs of f (x) and g (x) were to be sketched in Part (b) (i). However, most candidates knew the shapes of the curves to be sketched.

Part (b) (ii) was generally well done, except for some arithmetic errors and failure to observe that the approximation was required to *four decimal places*.

The majority of candidates who attempted Part (b) (iii) knew the concept of errors of measurements and generally this part of the question was done satisfactorily.

Solutions

(a) (i) $1+\frac{5}{4}x + \frac{5}{8}\,x^2 + \frac{5}{32}\,x^3 + \frac{5}{256}x^4 + \frac{1}{1024}x^5$

(ii) 1.131

(b) (ii) 0.3419

(iii) 0.1226

Section C

Module 3: Counting, Matrices and Complex Numbers

Question 3

Specific Objectives: (a) 2, 7; (b) 1, 2, 7

This question tested the number of arrangements with and without repetitions, selections with restrictions, multiplication of conformable matrices and inverting a 3×3 matrix.

Candidates attempting Part (a) (i) did not follow the requirement of repetitions. Rather, most of the candidates obtained distinct permutations of the letters given. Part (a) (ii) was poorly done by the few candidates who attempted it.

Candidates demonstrated a sound understanding of the cofactor method for inverting a matrix required in Part (b) (i). Arithmetic errors resulted in some candidates losing marks due to inaccuracy. Part (b) (ii) was generally well done.

Part (b) (iii) was poorly done. Candidates could not make the links with Parts (b) (i) and (ii) to determine the matrix B. Poor algebraic skills prevented candidates from manipulating the links in Parts (b) (i) and (ii) to solve Part (b) (iii). This part of the question was poorly done.

Solutions

(a) (i) 24 times (ii) 576

(b) (i) $A^{-1} = \begin{pmatrix} 3 & -3 & 1 \\ -3 & 5 & -2 \\ 1 & -2 & 1 \end{pmatrix}$ (ii) $\begin{pmatrix} 4 \\ 7 \\ 12 \end{pmatrix}$

(iii) $B = \begin{pmatrix} 6 & -6 & 2 \\ -6 & 10 & -4 \\ 2 & -4 & 2 \end{pmatrix}$

Paper 031 – School Based Assessment (SBA)

For the year 2012, 191 Unit 1 and 149 Unit 2 SBAs were moderated. It is increasingly difficult to successfully complete the moderation for samples for the SBAs submitted by some centres since many teachers continue to submit solutions *without unitary* mark schemes. In a number of cases, *neither question papers nor detailed worked solutions with unitary mark schemes* were submitted. Mark schemes for questions and their subsequent parts were not broken down into unitary marks. In an increasing number of cases, marks awarded were either too few or far too many. (Example: an entire SBA module test was 20 marks in total and in another case only one syllabus objective assessed within a module test was allocated 40 marks).

The majority of the samples submitted were not of the required standard. Teachers must pay particular attention to the following guidelines and comments to ensure effective and reliable submission of the SBAs.

The SBA is comprised of three module tests.

The main features assessed are:

- Mapping of the items tested to the specific objectives of the syllabus for the relevant unit.
- Content coverage of each module test
- Appropriateness of the items tested for the CAPE level
- Presentation of the sample (module test questions, detailed mark schemes and students' scripts)
- Quality of the teachers' solutions and mark scheme
- Quality of teachers' assessments – consistency of marking using the mark schemes
- Inclusion of mathematical modelling in at least one module test for each unit

FURTHER COMMENTS

1. Too many of the module tests comprised items from CAPE past examination papers.
2. Teachers are reminded that CAPE past examination papers should be used *only* as a guide. They should not constitute any part of or an entire module test.
3. Untidy 'cut and paste' presentations with varying font sizes were common and as a result very unsatisfactory. Module tests must be neatly handwritten or typed.

4. The stipulated time for module tests $(1\text{hr} - 1\text{hr}\,30\,\text{mins})$ must be strictly adhered to as students may be at an undue disadvantage when the times allotted for module tests are too extensive or insufficient.
5. The following guide can be used: 1 minute per mark. About 75 per cent of the syllabus should be tested and mathematical modelling *must* be included.
6. Multiple choice questions will **NOT** be accepted in the module tests.
7. Cases were noted where teachers were unfamiliar with recent syllabus changes, that is,
   - Complex numbers and the Intermediate Value Theorem are tested in Unit 2.
   - Three dimensional vectors, dividing a line segment internally and externally, systems of linear equations have been *removed* from the CAPE syllabus (2008).
8. The moderation process relies on the validity of teachers' assessment. There were a few cases where tampering of the scripts and subsequent questionable mark changes occurred. There were also instances where the marks on students' scripts did not correspond to the marks on the moderation sheet. There were a few cases where students' solutions were replicas of teachers' solutions — some contained identical errors and full marks were awarded for incorrect solutions. There were also instances where the marks on students' scripts did not correspond to the marks on the moderation sheet. The SBA must be administered under examination conditions at the centre. It is not to be done as a homework assignment or research project.
9. Teachers *must* present evidence of having marked each question on students' scripts before a total is calculated at the top of the script. The corresponding whole number score out of 20 should be placed at the front of students' scripts.
10. Teachers *must* indicate any changes/omissions that were made to question papers, solutions and mark schemes and scripts. Teachers should also inform the examiner about the circumstances regarding missing script(s).
11. Students' names on the computer generated form must correspond to the names on PMATH 1-3 and PMATH 2-3 forms and students' scripts.
12. The maximum number of marks for each assessment should be the same for all students.

To enhance the quality of the design of the module tests, the validity of teachers' assessments and the validity of the moderation process, the SBA guidelines are listed below for emphasis.

Module Tests

- Design a separate test for each module. The module test *must* focus on objectives from that module.
- In cases where several groups in a school are registered, the assessments should be coordinated, common tests should be administered and a common marking scheme used.
- One sample of *five* students will form the sample for the centre. If there are fewer than five students, *all* scripts will form the sample for the centre.
- In 2012, the format of the SBA remains unchanged.

Guidelines for Module Tests and Presentation of Samples

1. <u>Cover Page To Accompany Each Module Test</u>

   The following information is required on the cover of each module test:

   - Name of school and territory, name of teacher, centre number
   - Unit number and module number
   - Date and duration $(1\text{hr} - 1\text{hr}\,30\,\text{mins})$ of module test
   - Clear instructions to students
   - Total marks allotted for module test
   - Sub-marks and total marks for each question *must* be clearly indicated.

2. <u>Coverage Of The Syllabus Content</u>

   - The number of questions in each module test must be appropriate for the stipulated time of $(1\text{hr} - 1\text{hr}\,30\,\text{mins})$.
   - *CAPE past examination papers should be used as a guide ONLY***.**
   - Duplication of specific objectives and questions must be avoided.
   - Specific objectives tested must be from the relevant unit of the syllabus.

3. <u>Mark Scheme</u>

   - Unitary mark schemes *must* be done on the detailed worked solution. (that is, one mark should be allocated per skill assessed, not 2, 3, 4 etc marks per skill)
   - Fractional/decimal marks must not be awarded (that is, do not allocate $\left(\frac{1}{2}\right)$ marks
   - The total marks for module tests *must* be clearly stated on teachers' solution sheets.

- A student’s final mark out of 20 *must* be entered on the front page of the student’s script.
- Hand written mark schemes *must* be *neat* and *legible*. The unitary marks *must* be written on the right side of the page.
- Diagrams *must* be neatly drawn with geometrical/mathematical instruments.

4. Presentation Of Sample

- Students’ responses *must* be written on letter-sized $\left(8\frac{1}{2}\times 11\right)$ or A4$\left(8\frac{1}{2}\times 11.69\right)$ paper.
- Question numbers *must* be written clearly in the left hand margin.
- The total marks for *each* question on students’ scripts MUST be clearly written in the left or right margin.
- Only original students’ scripts *must* be sent for moderation.
- Photocopied scripts *will not be accepted*.
- Typed module tests *must* be *neat* and *legible*.
- The following are required for each module test:
  - ❖ A question paper
  - ❖ Detailed solutions with detailed unitary mark schemes
  - ❖ The question paper, detailed solutions, unitary mark schemes and five students’samples should be batched together for each module.
- Marks recorded on PMATH 1-3 and PMATH 2-3 forms must be rounded off to the nearest whole number. If a student scored zero, then zero must be recorded. If a student was absent, then absent must be recorded.
- Form PMATH 2-4 is for official use *only* and should not be completed by the teacher. However, teachers may complete the relevant information: Centre Code, Name of Centre, Territory, Year of Examination and Name of Teacher(s).
- The guidelines at the bottom of the PMath forms should be observed. (See page 57 of the syllabus, no. (6).