C A R I B B E A N   E X A M I N A T I O N S   C O U N C I L

REPORT ON CANDIDATES' WORK IN THE SECONDARY EDUCATION CERTIFICATE EXAMINATION

MAY/JUNE 2011

MATHEMATICS
GENERAL PROFICIENCY EXAMINATION

GENERAL COMMENTS

The General Proficiency Mathematics examination is offered in January and May/June each year. There was a candidate entry of approximately 90,000 in May/June 2011. Thirty-five per cent of the candidates earned Grades I–III. The mean score for the examination was 71.43 out of 180 marks.

DETAILED COMMENTS

Paper 01 – Multiple-Choice

Paper 01 consisted of 60 multiple choice items. This year, 180 candidates earned the maximum available score of 60. The mean mark for this paper was 32.32 out of 60 marks.

Paper 02 – Structured Questions

Paper 02 comprised two sections. Section I consisted of eight compulsory questions totalling 90 marks. Section II consisted of three optional questions: one each from Algebra, Relations, Functions and Graphs; Measurement, Trigonometry and Geometry; and Vectors and Matrices. Candidates were required to answer any two questions from this section. Each question in this section was worth 15 marks.

Thirty-seven candidates earned the maximum available score of 120 marks on Paper 02. Approximately 19 per cent of the candidates earned at least half of the maximum mark on this paper.

Compulsory Section

Question 1

This question tested candidates' ability to

- add, subtract, multiply and divide fractions involving mixed numbers and decimals
- express a value to a given number of significant figures
- express one quantity as a fraction of another
- solve problems involving invoices and shopping bills

The question was attempted by 99 per cent of the candidates, 18.8 per cent of whom earned the maximum available mark. The mean mark was 7.45 out of 11.

The responses to this question were generally good. Most candidates demonstrated a high level of proficiency in computing the common denominator and one correct numerator when adding fractions although some of them could not convert a mixed number to an improper fraction. The candidates demonstrated competence with the the application of the order of operations although many could not divide one fraction by another. For example, $\frac{27}{8} \div \frac{9}{2}$ was often written as $\frac{8}{27} \times \frac{9}{2}$ or $\frac{9}{2} \times \frac{8}{27}$. Several candidates interpreted $\sqrt{0.0256}$ as 0.0256^2.

About ten per cent of the candidates made no attempt at calculating the per cent VAT which is an indication that they could not interpret the relevant information given in the table. For those who attempted this part of the question, the major error was dividing the amount paid as VAT by the total amount of the bill rather than by the sub-total before VAT.

Solutions

(a) (i) $\frac{3}{4}$ (ii) 0.83

(b) \$15.60; \$13.20; 6; 12%

Recommendations

Teachers should provide more opportunities for students to practise solving problems with operations involving fractions. They should encourage and teach the use of the calculator in simplifying expressions related to computations with fractions, squares and square roots.

Question 2

This question tested candidates' ability to

- simplify simple algebraic fractions
- perform binary operations
- factorize algebraic expressions
- solve problems involving direct variation

The question was attempted by 98 per cent of the candidates, 6.6 per cent of whom earned the maximum available mark. The mean mark was 4.63 out of 12.

Performance on this question was generally unsatisfactory. Although a large percentage of candidates were able to apply the LCM to the algebraic fraction, they were unable to follow through with expanding, collecting and simplifying the terms. Substituting the numbers into the binary operation and factorizing the expression $xy^3 + x^2y$ were well done; but many candidates were unable to successfully complete the task of factorizing by grouping the expression $2mh – 2nh – 3mk + 3nk$.

Solutions

(a) (i) $\frac{7x - 5}{12}$

(b) 25

(c) (i) $xy(y^2 + x)$ (ii) $(2h – 3k)(m – n)$

(d) $a = 30$; $b = 8$

Recommendations

Teachers should ensure that the algebra of directed numbers is adequately mastered by students. The basic principles of algebra including understanding algebraic terms, expanding brackets, factorizing and collecting terms should be reinforced and tested regularly.

Question 3

This question tested candidates' ability to

- determine elements in the intersection, union and complement of sets
- solve problems involving the use of Venn diagrams
- draw and measure angles and line segments accurately using appropriate geometrical instruments
- construct lines, angles and polygons using appropriate geometrical instruments

The question was attempted by 97 per cent of the candidates, 9.4 per cent of whom earned the maximum available mark. The mean mark was 5.21 out of 11.

The performance of candidates on this question was generally satisfactory. Candidates easily identified the number of students who studied neither Art nor Music, and the number who studied Music only. Nevertheless, many of them encountered difficulty identifying the universal set as representing all the elements in the Venn diagram. Hence, the 4 students who studied neither Art nor Music were excluded from the sum of the elements. In some cases, candidates formulated the equation to be solved for x but were unable to solve the equation.

In Part (b), candidates were able to draw and measure the required straight lines, but demonstrated little proficiency in using the protractor to measure the required angles. Moreover, they seemed not to recognize that 125˚ is an obtuse angle.

Solutions

(a) (i) 4 (ii) 9 (iii) 14

(b) (ii) GH = 8.5 ± 0.2 cm.

Recommendations

Teachers should provide students with more practice in writing and solving algebraic equations in one unknown; they should encourage the use of authentic situations that would involve displaying information in a Venn diagram. The correct use of all geometrical instruments must be taught and reinforced.

Question 4

This question tested candidates' ability to

- solve a simple linear inequation
- determine the length and perimeter of a square
- determine the radius and area of a circle

The question was attempted by 92 per cent of the candidates, 2.1 per cent of whom earned the maximum available mark. The mean mark was 2.54 out of 10.

Generally, the performance of candidates on this question was unsatisfactory. In Part (a), few candidates were able to get the sign of the inequality correct when dividing by the negative coefficient of x. This posed some difficulty in answering Part (a) (ii) especially if $x < -2$ was offered as a solution for Part (a) (i).

A few candidates were able to determine the perimeter of the square from the length calculated for the side of a square. Similarly, the area of the circle was easily computed from the radius.

Solutions

(a) (i) $x > -2$ (ii) $x = -1$

(b) (i) a) 11 cm b) 44 cm.

(ii) a) 7 cm b) 154 cm^2.

Recommendations

Candidates need to be exposed to more practical sessions on transposition and on the division by a negative value across an inequality sign. In addition, they would benefit from more practical work in aspects of measurement such as area and perimeter.

<u>Question 5</u>

This question tested candidates' ability to

- identify the relationship between an object and its image after an enlargement
- use Pythagoras' Theorem to solve problems
- use trigonometric ratios in the solution of right-angled triangles in the physical world
- calculate the area of a triangle

The question was attempted by 83 per cent of the candidates, 4.2 per cent of whom scored the maximum available mark. The mean mark was 2.72 out of 12.

The performance of candidates on this question was unsatisfactory. A large number of candidates were unable to determine the scale factor given the dimensions of the two similar triangles. However, they were able to multiply the value of the scale factor by the length of OM to calculate the length of its image.

Many of the candidates had some knowledge of Pythagoras' Theorem and when to use it, although some candidates did not apply it correctly. Most candidates realized that a trigonometric ratio was to be used to calculate the length QS and the measure of the angle θ, but were unable to identify the correct ratio in these two instances. Some candidates used the sine and cosine rules and invariably applied them incorrectly. In Part (b) (iii), the majority of candidates attempted to find the area of the ΔPQR using ½ base x height, but were unable to find the length of PR. Since this meant using trigonometric ratios to find PS and then SR and adding, many candidates did not carry through the calculations to the end. Candidates resorted to using the dimensions given on the diagram, for example: Area = ½ x 12.6 × 8.4 and Area = ½ × 12.6 x QS. Heron's formula was also used with little success in most cases. Candidates who attempted to use the formula, *Area of triangle = ½absinC*, could not determine the value of angle PQR and used the angle 15˚ instead.

Solutions

(a) (i) k = 2 (ii) 10 cm (iii) 20 cm

(b) (i) 3.26 m (ii) 67.2° (iii) 32.4 m^2

Recommendations

Students should be engaged in activities to help them determine the most efficient strategies for finding the solutions to problems. Mathematical terms should be used consistently when teaching a topic and in solving problems. Further, instruction in the use of trigonometric ratios should include authentic tasks where possible.

Question 6

This question tested candidates' ability to

- derive a composite function
- derive the inverse of a function
- determine the intercept of the graph of a linear function
- determine the gradient and equation of a straight line

The question was attempted by 90 per cent of the candidates, 4.3 per cent of whom earned the maximum available mark. The mean mark was 4.56 out of 12.

The majority of candidates faced significant challenges while attempting to simplify the algebraic fractions in Part (a). They knew that they needed to interchange *x* and *y* in the equation to obtain the inverse of the function *f*(*x*), but having done this, they were unable to make *y* the subject of the equation.

Some candidates demonstrated a limited understanding of the concept of gradient, using the ratio $\Delta x \div \Delta y$ instead of $\Delta y \div \Delta x$. In addition, many candidates were unable to determine the equation of a line, resulting in many calculating the length of the line or giving a range of values between which the length of the line lies.

Solutions

(a) (i) $g\left(\frac{1}{2}\right) = -\frac{1}{2}$ (ii) $2x + 2$ (iii) $f^{-1}(x) = \frac{x-8}{6}$

(b) (i) A (-2, 3), B(4, 6) (ii) $\frac{1}{2}$ (iii) $y = \frac{1}{2}x + 4$

Recommendations

Teachers should give special attention to Mathematical terminology and symbolism since many concepts, definitions and symbols appear to be misinterpreted by students. Special attention should also be given to simplifying algebraic expressions and transposing equations.

Question 7

This question tested candidates' ability to

- complete a cumulative frequency table for grouped data
- draw a cumulative frequency graph
- estimate the median of a data set by using a cumulative frequency graph
- determine simple probability using the cumulative frequency graph

The question was attempted by 88 per cent of the candidates, 3.8 per cent of whom earned the maximum available mark. The mean mark was 4.18 out of 12.

Generally, the performance of candidates on this question was unsatisfactory. Some candidates demonstrated competence in correctly completing the cumulative frequency table. In addition, they were able to use the given scales correctly and to plot the vertical coordinates of the points on the graph, although many candidates plotted the frequency instead of the cumulative frequency. Some candidates attempted to calculate the median mass of the packages from the table rather than estimating it from the graph.

Solutions

(a)

Mass (kg)	**No. of Packages**	**Cumulative Frequency**
1–10	12	12
11–20	28	40
21–30	30	70
31–40	22	92
41–50	8	100

(c) (i) Median = 24 kg (ii) $\frac{80}{100}$

Recommendations

Teachers should expose students to the drawing and interpretation of cumulative frequency curves. Authentic tasks are a useful tool to use in this regard. Attention must be given to the calculation of simple probability in a variety of situations.

<u>Question 8</u>

This question tested candidates' ability to

- generate a term of a sequence
- derive a general rule given the terms of a sequence
- solve problems involving concepts in number theory

The question was attempted by 96 per cent of the candidates, 6.6 per cent of whom earned the maximum available mark. The mean mark was 6.03 out of 10.

The performance of candidates on this question was satisfactory. Almost all of the candidates who attempted this question were able to draw the fourth diagram in the sequence, calculate the number of sticks in the sixth diagram and the number of thumb tacks in the seventh diagram. In addition, most of the candidates were able to determine the pattern connecting the number of sticks and the number of thumb tacks, and hence complete the given table. However, some candidates were unable to use the order of operations to write the rule to correctly show the relationship between *t* and *s*.

Solutions

(b) (i) 24 (ii) 22

(c)

	No of Sticks *s*	**Rule Connecting** *t* **and** *s*	**No. of Thumb Tacks** *t*
(i)	52	1 + (¾ × 52)	40
(ii)	72	1 + (¾ × 72)	55

(c) $t = 1 + (¾)\, s$

Recommendations

Teachers should expose students to more concrete experiences in discovering number patterns and sequences, including pictorial representations. They should give students more opportunities to practise forming generalizations from number patterns.

Optional Section

Question 9

This question tested candidates' ability to

- solve a pair of equations in two variables when one is linear and the other non-linear
- determine the maximum or minimum value of a quadratic function expressed in the form $a(x + h)^2 + k$
- interpret a speed-time graph to determine time, speed and distance

The question was attempted by 57 per cent of the candidates, one per cent of whom earned the maximum available mark. The mean mark was 2.44 out of 15.

In an attempt to solve the pair of simultaneous equations, most candidates recognized that some strategy was needed to eliminate one variable, but they lacked the skills needed to simplify the algebraic terms and to transpose these terms. Hence, candidates were unable to obtain an equation in one variable. The majority of the candidates could not complete the square to find the minimum value of the function. Many candidates used the formulae for *h* and *k*, which were very often not stated correctly and therefore produced incorrect answers.

In Part (b), candidates found it difficult to find the distance travelled from the speed-time graph. Several of them used the formula $d = s \times t$ instead of calculating the area under the curve. Many candidates interpreted the shape of the graph in the second stage to be a level road and did not recognize that the gradient was zero.

Solutions

(a) (1, 3) (– 3, 15)

(b) (i) $4(x – 1)^2 – 6$ (ii) Minimum value = – 6 (iii) Minimum at x = 1

(c) (i) 20 seconds (ii) gradient = 0, constant speed (iii) 210 m

Recommendations

Teachers should provide students with adequate practice in solving a pair of equations in two variables when one equation is quadratic and the other linear. Students also need more practice in expressing a quadratic function in the form $a(x + h)^2 + k$, identifying the minimum value of a given function and the value of x for which this minimum occurs.

Question 10

This question tested candidates' ability to

- use the properties of circles and circle theorems to determine the sizes of angles
- use the sine and cosine rules to solve problems involving triangles
- solve problems involving bearings

The question was attempted by 45 per cent of the candidates, one per cent of whom earned the maximum available mark. The mean mark was 3.38 out of 15.

Candidates generally knew that the measure of angle XYZ = 116° but incorrectly stated the reason for this being so. Most of them were unable to interpret and apply the alternate segment theorem to correctly find the value of angle YXZ.

In Part (b), the majority of the candidates correctly calculated the value of x. However, candidates experienced difficulty calculating the distance *RP* and the bearing of R from P.

Solutions

(a) (i) 116° (ii) 23° (iii) 26°

(b) (i) 76° (ii) 299 km (iii) 218.5°

Recommendations

Teachers should use a systematic approach to provide students with sufficient exposure and practice in solving problems based on circles and circle theorems, the use of the sine rule and cosine rule to solve problems and solving practical problems involving bearings.

Question 11

This question tested candidates' ability to

- evaluate the determinant of a 2 × 2 matrix
- derive the inverse of a non-singular 2 × 2 matrix
- perform multiplication of matrices by a scalar
- determine a 2 × 2 matrix associated with a given transformation
- combine vectors
- use vectors to solve problems in geometry

The question was attempted by 48 per cent of the candidates, less than one per cent of whom earned the maximum available mark. The mean mark was 1.93 out of 15.

The performance of candidates on this question was generally unsatisfactory. Very few candidates knew how to determine, by matrix algebra, the transformation matrix which maps two given points onto given images. Several candidates tried to solve the problem by inspection. Many of the candidates did not have the correct order to perform the matrix multiplication. However, the most challenging part of the question was proving the three points to be collinear.

Solutions

(a) $\frac{1}{2}\begin{pmatrix} 4 & -5 \\ -2 & 3 \end{pmatrix}$

(b) (i) $a = 1, b = -1$ (ii) Clockwise Rotation of 90 degrees about (0,0)

(c) (i) a) $-b + a$

b) $\frac{1}{3}(-b + a)$

c) $\frac{1}{3}(b + 2a)$

Recommendations

Teachers should provide students with more guided practice mapping various 2 × 2 matrices with their associated transformations and solving problems involving vectors. The use of vectors to solve various problems in geometry should also be emphasized.