

**Title:** The Entanglement of Bosonic and Fermionic Systems

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The Einstein, Podolsky and Rosen (EPR) paradox, a thought experiment, initiated research in quantum theory that led to the showcasing of properties of entanglement. Entanglement, which has many applications in quantum teleportation, quantum communication and quantum computation, was investigated to explain its effects and implication of its use in superconductivity.

In non-relativistic second quantum formalism, the properties of entangled systems are studied. Second quantization is a mathematical tool which deals with "many-particle" problems in quantum theory and is employed in the study of entanglement. The properties of entanglement were observed in interacting and non-interacting harmonic oscillators whose properties were applied to interacting and non-interacting fermions, bosons and electron gas. Non interacting fermions and bosons were looked at and the resulting theory applied to two-electron entangled systems.

Entanglement was observed in two-particle sub-systems of an ensemble in which the particles were spatially separated but their spin and momenta showed correlation. Entanglement was quantified using the space-spin density matrix.

A collection of non-interacting fermions at zero temperature can be entangled in spin, once their differences do not exceed the inverse Fermi wave number. It was found that beyond this required distance, the entanglement between the two parties vanished.

However, Classical Correlation may still be valid beyond the inverse Fermi wave number distance.

At zero and finite low temperatures, Green's function was applied to show that change in the entanglement measures, such as the concurrence, was proportional to  $T^2$  where  $T$  is a finite temperature. The BCS ground state was used to obtain the two-electron space-spin density matrix via the Green's function for a Cooper pair in an  $s$ -wave superconductor. The two-spin states were given by a Werner state and were not of a spin singlet state.

In the BCS states with opposite spin (up and down), momentum entanglement of particles at finite temperatures was studied. The structure of the partial transposition of the two-particle density matrix in momentum space, along with the corresponding negativity, was analyzed using homogeneous states. The eigenvalues of a partially transposed matrix showed that for particles with momenta slightly above a chemical potential related to the Fermi energy, momentum entanglement was maximized.

Entanglement was shown for many-body fermionic systems due to its spin singlet state which was non-existent for bosonic systems.