CARIBBEAN EXAMINATIONS COUNCIL

REPORT ON CANDIDATES' WORK IN THE SECONDARY EDUCATION CERTIFICATE EXAMINATION

JANUARY 2011

MATHEMATICS GENERAL PROFICIENCY EXAMINATION

GENERAL COMMENTS

The General Proficiency Mathematics examination is offered in January and May/June each year. This was the first January examination for the revised Mathematics syllabus effective from May/June 2010.

There was a candidate entry of approximately 13,760 in January 2011. Thirty-seven per cent of the candidates earned Grades I–III. The mean score for the examination was 74.97 out of 180 marks.

DETAILED COMMENTS

Paper 01 – Multiple Choice

Paper 01 consisted of 60 multiple-choice items. This year, 24 candidates each earned the maximum available score of 60. Sixty-two per cent of the candidates scored 30 marks or more.

Paper 02 – Structured Questions

Paper 02 consisted of two sections. Section I comprised eight compulsory questions totalling 90 marks. Section II comprised three optional questions one each from (i) Algebra, Relation, Functions and Graphs; (ii) Measurement, Trigonometry and Geometry and (iii) Vectors and Matrices. Candidates were required to answer any two questions from this section. Each question in this section was worth 15 marks.

This year, no candidate earned the maximum available mark of 120 on Paper 02. There were five candidates who each scored 119 marks. Approximately 16 per cent of the candidates earned at least half of the maximum marks on this paper.

Compulsory Section

<u>Question 1</u>

This question tested candidates’ ability to

- square, add and multiply decimal numbers
- subtract and divide fractions involving mixed numbers
- solve problems involving wages and overtime

The question was attempted by 99 per cent of the candidates, 15.5 per cent of whom earned the maximum available mark. The mean mark was 6.98 out of 11.

In general, candidates provided satisfactory responses to this question. However, there were a number of incorrect strategies employed in an attempt to solve the problem. These included for Part (a):

(i) $(5.8^2 + 1.02) \times 2.5 = 5.8 \times 5.8 + 1.02 \times 2.5 = 36.19$ (Incorrect application of the distributive property)

(ii) $(5.8^2 + 1.02) \times 2.5 = (5.8 \times 2 + 1.02) \times 2.5 = (11.6 + 1.02) \times 2.5 = 31.55$ (Multiplying by 2 instead of squaring)

(iii) $(5.8^2 + 1.02) \times 2.5 = (5.64 + 1.02) \times 2.5$ (Squaring 0.8 and then adding 5)

(iv) $(5.8^2 + 1.02) \times 2.5 = (25.64 + 1.02) \times 2.5$ (Squaring 5 and 0.8 separately and adding the result)

In Part (b), most candidates were able to calculate the basic weekly wage for one employee, but very few candidates were able to calculate the overtime wages and the number of hours worked overtime.

Solutions:

(a) **(i) 86.65** **(ii)** $\frac{2}{21}$

(b) **(i) \$380** **(ii) \$85.50** **(iii) \$684** **(iv) 48 hours**

Recommendations

Teachers should ensure that students master the use of the scientific calculator in performing basic arithmetic operations on rational numbers. Attention should also be given to the order in which operations are performed.

Problems related to wages and overtime should be approached from a conceptual rather than an algorithmic viewpoint. In addition, a distinction should be made between rates and wages.

Question 2

This question tested candidates' ability to

- simplify algebraic fractions
- factorize algebraic expressions involving HCF
- change the subject of the formula
- solve worded problems involving simple linear equations

The question was attempted by 99 per cent of the candidates, 6.51 per cent of whom earned the maximum available mark. The mean mark was 4.40 out of 12.

The performance of candidates on this question was generally unsatisfactory. In Part (a), candidates were generally able to simplify the algebraic fractions, although there was a high incidence of $\frac{6x-5x}{15} = \frac{1}{15}$.

For Part (b), the majority of candidates were able to obtain a common factor even when they could not follow through to obtain the second factor. In some cases, candidates attempted to group and wrote $a^2b + 2ab = a(a+b) + 2(a+b) = (a+2)(a+b)$.

For Part (c), a large proportion of candidates attempted to transpose *r* before clearing the fraction, so $q = \frac{p^2-r}{t}$ became $q + r = \frac{p^2}{t}$. In general, candidates were unable to transpose *t* and to find *p* from p^2.

The primary difficulty experienced in Part (d) was writing an expression for the total number of donuts sold. It was common to see $5 \times 2x + 3$ for the number sold in 5 large boxes instead of $5(2x+3)$. A considerable number of candidates used trial and error to arrive at the number of donuts in each type of box.

Solutions:

(a) (i) $\boldsymbol{\frac{x}{15}}$

(b) $\boldsymbol{ab(a+b)}$

(c) $\boldsymbol{p = \sqrt{qt+r}}$

(d) (i) $\boldsymbol{8x + 5(2x+3)}$

(ii) a) $\boldsymbol{number\ in\ small\ box = 10}$ $\boldsymbol{b)}$ $\boldsymbol{number\ in\ large\ box = 23}$

Recommendations

When factorizing, students should practise dividing each of the expressions separately by the common factor.

Teachers should provide students with adequate examples of finding the inverse operations, as well as the operation associated with each quantity or variable to be transposed.

Question 3

This question tested candidates' ability to

- apply the laws of indices to simplify expressions
- calculate the volume of a right rectangular prism
- convert from litres to cubic centimetres
- solve problems involving volume

The question was attempted by 99.3 per cent of the candidates, 2.2 per cent of whom earned the maximum available mark. The mean mark was 3.61 out of 11.

Candidates performed satisfactorily when applying the laws of indices for multiplication, although there were a few candidates who did not multiply the coefficients.

The formula for calculating the volume of the rectangular prism was widely known and correctly applied. However, some candidates omitted the units of volume from their calculations. In general, candidates did not consider the number of cartons to be a discrete quantity. In addition, a significant number of candidates could not distinguish between diameter and radius in their attempt to calculate the volume of the cylinder, and as a consequence, the diameter was substituted in place of the radius.

Solutions:

(a) $\mathbf{14p^7q^4}$

(b) (i) $\mathbf{240\ cm^3}$ **(ii)** $\mathbf{13\ cartons}$ **(iii)** $\mathbf{12.2\ cm}$

Recommendations

Students should be reminded of the importance of writing the units of any measurement derived from calculations. Further, the difference between discrete and continuous variables should be emphasized.

Question 4

This question tested candidates' ability to

- identify numbers which are prime and numbers which are odd
- draw Venn diagrams to show the relationship between sets of numbers
- construct an angle of 60 degrees using ruler and compasses only
- use a protractor to measure an angle
- construct a triangle and a parallelogram

The question was attempted by 98.3 per cent of the candidates, 0.94 per cent of whom earned the maximum available mark. The mean mark was 5.10 out of 12.

Candidates were proficient at listing the odd numbers between 4 and 12. They, however, were generally unaware that 1 is not a prime number and that 2 is a prime number. Some candidates did not understand that the universal set included the sets of prime numbers and the set of odd numbers under consideration, and proceeded to draw a third set to represent the universal set.

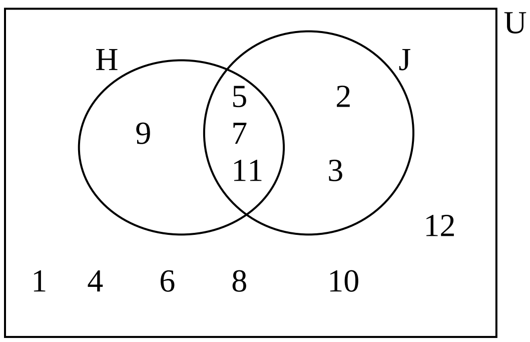
Most candidates were able to draw lines of length 9 cm and 7 cm. However, they experienced difficulty with determining the measure of $M\widehat{N}L$ and some candidates gave the length of LM instead.

Although the majority of candidates who attempted this part of the question demonstrated knowledge of the properties of a parallelogram, many of them failed to arrive at the position of K by construction.

Solutions:

(a) (i) $\boldsymbol{H = \{5, 7, 9, 11\}}$ **(ii)** $\boldsymbol{J = \{2, 3, 5, 7, 11\}}$

$(\mathbf{iii})$



(b) (ii) $\angle \boldsymbol{MNL = 48^\circ \pm 1^\circ}$

Recommendations

Teaching the historical background to the development of our decimal number system should provide candidates with greater motivation to study the system of numbers, identify their various subsets and understand the relationships between the subsets.

Construction of polygons using geometrical tools must be practised if students are to be proficient at constructing the simple geometrical figures including angles of 30, 45, 60, 90 and 120 degrees.

Question 5

This question tested candidates' ability to

- determine the gradient of a line from a given equation
- obtain the equation of the line which is perpendicular to a given line
- solve problems involving a mapping and a given function

The question was attempted by 80.6 per cent of the candidates, 4.42 per cent of whom earned the maximum available mark. The mean mark was 2.87 out of 11.

In Part (a), candidates had difficulty determining the gradient of the line, even though a number of candidates attempted to rearrange the equation to the form $y = mx + c$. Further, candidates were unfamiliar with the concept that the product of the gradients of perpendicular lines is -1.

Candidates demonstrated good proficiency in using the given function and known mappings to calculate the values of k and $f(3)$ using a variety of strategies including trial and error.

<u>Solutions:</u>

(a) **(i)** $\boldsymbol{\frac{2}{3}}$ $\qquad$ $(\boldsymbol{ii})\ \boldsymbol{y = \frac{-3}{2}x + 13}$

(b) $(\boldsymbol{i})\ \boldsymbol{k = 5}$ $\qquad$ $(\boldsymbol{ii})\ \boldsymbol{f(3) = 4}$ $\qquad$ $(\boldsymbol{iii})\ \boldsymbol{x = 10}$

Recommendations

Changing the subject of the formula is an essential skill in algebra and as such should be given appropriate attention.

Students should approach the concept of Coordinate Geometry by actually drawing graphs and be allowed to discover the relationships between parallel and perpendicular lines from this graphical approach.

<u>Question 6</u>

This question tested candidates' ability to

- read and interpret data presented on a line graph
- calculate the mean of a data set
- interpret the trend presented on a line graph

The question was attempted by 98.8 per cent of the candidates, 17 per cent of whom earned the maximum available mark. The mean mark was 7.94 out of 11.

Candidates performed satisfactorily on this question. They showed good proficiency at reading the values from the graph, determining the sales for the month of June and calculating the mean. However, there were some candidates who stated the median instead of the mean and others who divided the total sales by 2 instead of 5 to calculate the mean.

Candidates had difficulty stating the two consecutive months in which the largest decrease in sales occurred.

Solutions:

(i)

Month	**Jan**	**Feb**	**Mar**	**Apr**	**May**
Sales in Thousands $	**38**	**35**	**27**	**15**	**10**

(ii) **March and April** **(iii)** **$ 25000** **(iv)** **$ 25000**

Recommendations

The different measures of central tendency should be thoroughly reviewed by students and they should be encouraged to describe the trends and patterns observed on charts and statistical diagrams.

Question 7

This question tested candidates' ability to

- read and write coordinates from the Cartesian plane
- describe the single transformation which maps one triangle onto another in the plane
- enlarge a triangle given the centre of enlargement and the scale factor
- determine the area of an image given the area of the object
- describe the relationships between the object and the image under an enlargement

The question was attempted by 90.9 per cent of the candidates, less than one per cent of whom earned the maximum available mark. The mean mark was 2.09 out of 12.

Candidates performed unsatisfactorily on this question. They demonstrated some competence in reading the coordinates of R and R' and correctly stated that the transformation which maps triangle RST onto triangle $R'S'T'$ is a reflection. They, however, could not go on to completely describe the reflection.

Candidates recognized that a triangle and its image under a reflection are similar, but arriving at a second geometrical relationship proved difficult.

Very few candidates used the correct point (0, 2) as the centre of enlargement and many of them proceeded to use a matrix method which presumed that (0, 0) was the centre.

Most candidates could not relate the area of the object to its image based on the scale factor.

Solutions:

(i) $\boldsymbol{R(2,4)}$ $\boldsymbol{and}$ $\boldsymbol{R'}$ $\boldsymbol{(2,0)}$ **(ii) Reflection in the line** $\boldsymbol{y = 2}$

(iii) b) $\boldsymbol{Area = 36\ square\ units}$

Recommendations

Students are encouraged to practise enlarging objects from any point in the plane using different scale factors. It is also useful to employ concrete objects in the teaching and learning of transformations, paying attention to the relationships between object and image in each case.

Further, it might be beneficial to integrate the teaching of transformations with the properties of similar and congruent figures.

Question 8

This question tested candidates' ability to

- determine the possible dimensions of a rectangle given its area
- determine the dimensions of a rectangle with the maximum area for a given perimeter

The question was attempted by 90 per cent of the candidates, 2.41 per cent of whom earned the maximum available mark. The mean mark was 5.35 out of 10.

Candidates performed satisfactorily on this question. They generally knew how to draw the rectangles for the given areas and how to complete the table to show the dimensions of the given rectangles. They were also able to determine the dimensions of length and width which would produce a certain perimeter even though, in several cases, it was not the desired perimeter of 24 units.

The major challenge to candidates was determining the dimensions of a rectangle that would give the maximum area for a perimeter of 36 cm. Candidates generally did not recognize that a square is a special rectangle.

Solutions:

Rectangle	Length	Width	Area (square units)	Perimeter (units)
A	**10**	**2**	**20**	**24**
B	**9**	**3**	**27**	**24**
C	**8**	**4**	**32**	**24**
D	**6**	**6**	**36**	**24**
E	**9**	**9**	**81**	**36**

Recommendations

Teachers should emphasize the properties of plane shapes with reference to similarities and differences between them. The fact that a square is a rectangle should also be reinforced.

Students should be provided with practice exercises involving drawing figures and calculating the respective areas and perimeters.

Optional Section

Question 9

This question tested candidates' ability to

- calculate the value of a function for a given value of its domain
- determine the inverse of a rational function
- write an expression for the composite of two functions
- use the method of completing the square to write an expression for a quadratic function expressed as $ax^2 + bx + c$
- determine the maximum value, the axis of symmetry and the x-intercepts of a quadratic function

The question was attempted by 73 per cent of the candidates, less than one per cent of whom earned the maximum available mark. The mean mark was 3.01 out of 15.

Candidates correctly substituted 5 into the function $f(x)$ but were sometimes unable to simplify to obtain the answer of $\frac{3}{5}$. Finding the inverse of the rational function proved difficult primarily because candidates lacked the algebraic skills to make y the subject of the formula after having interchanged x and y in the equation. They were also generally unable to write the composite function in the correct order.

In attempting to write $ax^2 + bx + c$ in the form $k - a(x+h)^2$, the formulae $h = \frac{-b}{2a}$ and $k = \frac{4ac-b^2}{2a}$ were widely used to good effect. However, among those who chose to complete the square, difficulties were encountered when dealing with the negative sign. Many candidates extracted the -1 coefficient of x^2 but were unable to correctly reinsert it. As a consequence, the expressions $-10 - 4(x+3)^2$ and $-10 - 1(x-3)^2$ resulted.

Having arrived at an expression in the form required, candidates in general could not identify the maximum value nor state the equation of the axis of symmetry. They, for the most part, attempted the solution of the quadratic equation by applying the quadratic formula but experienced challenges inserting the correct sign for each root of the equation.

Solutions:

(a) (i) $\mathbf{\frac{3}{5}}$ **(ii) a)** $\boldsymbol{f^{-1}(x) = \frac{7}{2-x}}$ **b)** $\boldsymbol{gf(x) = \sqrt{\frac{5x-7}{x}}}$

(b) (i) $\mathbf{10 - (x+3)^2}$ **(ii) a) 10 b)** $\boldsymbol{x = -3}$ **(iii)** $\boldsymbol{x = -6.16 \text{ or } x = 0.16}$

Recommendations

Candidates should be apprised of the purpose for writing the quadratic expression in the form suggested. Attention should be paid to the maximum or minimum value of such a function and the value of the variable for which this occurs.

Mastery of algebra is an essential skill for all students of mathematics at this level. Candidates therefore need to develop competence in a wide range of knowledge and skills in algebra.

Question 10

This question tested candidates' ability to

- use circle theorems and properties of a circle to determine the measure of an angle
- illustrate information on bearings
- apply the sine and cosine rules to calculate angles and distances

The question was attempted by 48.7 per cent of the candidates, 1.09 per cent of whom earned the maximum available mark. The mean mark was 3.09 out of 15.

Part (a) was poorly done. Although most candidates deduced that triangle OFG is isosceles, and as a result were able to arrive at the correct value for angle OGF, they did not recognize that GFED was a cyclic quadrilateral and as a consequence could not apply the cyclic quadrilateral theorem.

In Part (b), very few candidates were able to generate an accurate diagram from the information given. Those who drew correct diagrams recognized when to use the cosine and the sine formulae for finding the length of JL and the measure of angle JLK. Many of them, however, proceeded to incorrectly apply both formulae. Most candidates did not know which angle represented the bearing of J from L.

Solutions:

(a) (i) 31° (ii) 56° (iii) 124°
(b) (ii) a) 144° b) 174.15 km c) 245.7°

Recommendations

Teachers should engage students in more practice on problems associated with circle theorems. Candidates need to approach the topic of bearings from a practical standpoint. Applications of the sine and cosine formulae need to be reinforced.

Question 11

This question tested candidates' ability to

- obtain the 2×2 matrix which transforms two points in the plane to their images
- use a matrix to obtain the image of a given point in the plane
- describe the transformation defined by a given matrix
- derive a displacement vector
- prove that three given points are collinear

The question was attempted by 39.3 per cent of the candidates, 0.13 per cent of whom earned the maximum available marks. The mean mark was 1.55 out of 15.

In Part (a), candidates demonstrated a high level of proficiency in writing the coordinates of a point as a position vector, but thereafter, they experienced considerable difficulty. They were unable to set up a system of equations which could be solved to produce the components of the matrix. Further, many candidates could not obtain the image of a point under the transformation described by the matrix. Describing the transformation, M, proved difficult.

In Part (b), candidates failed to obtain the vectors OR and RS. In addition, proving that P, R and S are collinear was outside of their level of competence.

Solutions:

(a) (i) $\begin{pmatrix} \mathbf{0} & \mathbf{1} \\ \mathbf{-1} & \mathbf{0} \end{pmatrix}$ **(ii)** $\boldsymbol{Z} = (\mathbf{-1}, \mathbf{5})$ **(iii) Clockwise rotation of** $\mathbf{90}^{\circ}$ **about the origin, O.**

$(\boldsymbol{b})$ **(i) a)** $\overrightarrow{\boldsymbol{OP}} = \begin{pmatrix} \mathbf{2} \\ \mathbf{7} \end{pmatrix}$ **b)** $\overrightarrow{\boldsymbol{OR}} = \begin{pmatrix} \mathbf{6} \\ \mathbf{4} \end{pmatrix}$
$(\mathbf{ii})$ **a)** $\overrightarrow{\boldsymbol{RS}} = \begin{pmatrix} \mathbf{8} \\ \mathbf{-6} \end{pmatrix}$

Recommendations

Teachers should engage students in problems associated with transformations using matrices, position and displacement vectors and proving that three points are collinear.