ABSTRACT

The object of this work is to extend the theory of scalar or real valued differential forms to forms valued in the universal Clifford algebra $C_{n,s}$. We attack the problem by first developing a general theory of tensors and exterior forms having values in a finite dimensional (associative) algebra over the reals. In Chapter 3, algebraic operations of tensor and exterior (wedge) product are defined and studied for algebra valued forms, and tensor and exterior algebras are constructed.

In Chapter 4, the general theory of algebra valued tensors developed in Chapter 3 is applied to the Clifford algebra, thereby abstracting the particulars of the theory of Clifford valued tensors relevant to this work. This theory is subsequently transferred to the tangent spaces of a smooth (C^∞) finite dimensional manifold, thus providing the base for a theory of Clifford (valued) differential forms.

We also develop a differential calculus of Clifford forms with the aid of the invariant operations of interior product, Lie and exterior differentiations.

Further properties of Clifford forms are presented in Chapter 5. In particular, the Exterior algebra of Clifford forms is constructed, which attracts a differential calculus of its own. It is also in this Chapter that the notions of even and odd Clifford forms are introduced.

An exposition of the physical and mathematical significance of scalar forms and the Clifford algebra is given in Chapter 1. Chapter 2 reviews the properties of scalar forms and the structure of the Universal Clifford algebra.