ABSTRACT

Stochastic Models for Structured Financial Products:
Theoretical Background and Multilevel Monte-Carlo Simulation

Stephen Christopher Barnes

In this thesis, we examine the theoretical and practical basis for estimating the mean percentage return of structured financial products. This requires us to explore the fundamentals of stochastic differential equations (SDEs). We give a detailed construction of the Itô Integral and apply Itô's Lemma. We then proceed by examining Girsanov's theorem in the context of finance, specifically in option pricing where it is used to change the real world probability measure of the discounted stock price process to a risk neutral probability measure. In doing so, we illustrate the change of measure concept in a discrete setting and develop a theorem that specifies an interval for which the mean of a random variable with three outcomes can be changed to in such a way that keeps its original variance intact. We then examine a relatively new simulation technique, [8], called the Multilevel Monte Carlo (MLMC) method, that we use to simulate both the price estimates of a call option and the mean percentage return of selected structured financial products. Simulations are executed using Euler-Maruyama and Euler-Milstein numerical schemes for approximating solutions of the SDE models. Comparisons of the computational cost are made with the standard Monte Carlo (MC) method for different tolerances ϵ of the simulation estimates' root mean square error. We show in all instances that the MLMC outperforms the MC method. Lastly, we give insights into the convergence results when using the more complicated payoffs seen in our examples of structured financial products.

Keywords: Stephen Christopher Barnes; Financial Engineering; Stochastic Numerical Scheme; Computational Complexity; Structured Financial Products; Stochastic Models; Multilevel Monte-Carlo Simulation; Girsanov's Theorem.