C A R I B B E A N   E X A M I N A T I O N S   C O U N C I L

REPORT ON CANDIDATES' WORK IN THE CARIBBEAN ADVANCED PROFICIENCY EXAMINATION MAY/JUNE 2009

PURE MATHEMATICS

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GENERAL COMMENTS

This is the second year that the current syllabus has been examined in the new format of Paper 01 as Multiple Choice (MC) and Papers 02 and 03 in the typical essay-type questions. The syllabus is arranged into two Units, each consisting of three Modules:

Unit 1

- Module 1 - Basic Algebra and Functions
- Module 2 - Trigonometry and Plane Geometry
- Module 3 - Calculus I

Unit 2

- Module 1 - Calculus II
- Module 2 - Sequences, Series and Approximations
- Module 3 - Counting, Matrices and Complex Numbers

There were 5579 candidates who wrote the examinations for Unit 1 in 2009 compared to 4 995 in 2008 and for Unit 2, 2 701 compared to 2 690 in 2008. Performances varied across the entire spectrum of candidates with a significant number obtaining excellent grades. Nevertheless, there continues to be a number of candidates who seem unprepared to write the examinations, particularly for Unit 1. A more effective screening process needs to be instituted to reduce the number of poorly prepared candidates.

DETAILED COMMENTS

UNIT 1

The overall performance in this Unit was satisfactory with a number of candidates excelling in such topics as Trigonometric Identities, Coordinate Geometry, Basic Differential and Integral Calculus and Surds. However, many candidates continue to find Indices, Limits, Continuity/Discontinuity and Algebraic Manipulation challenging. These topics and techniques should be given special attention if improvement in performance is to be achieved. Other areas that need consolidation are general algebraic manipulation of simple terms, expressions and equations, substitution, either as a substantive topic in the syllabus or as a tool for problem solving.

Paper 01 comprised 45 multiple-choice items, with 15 items based on each Module. The candidates performed satisfactorily with a mean score of 21 out of a possible 45. Paper 02 comprised six compulsory questions, two testing each Module. The mean mark on this paper was 51 out of a possible 150.

UNIT 1

PAPER 02

SECTION A

Module 1: Basic Algebra and Functions

Question 1

Specific Objective(s): (a)5; (b)5; (c)1, 3(iii), 5;(g).

This question tested knowledge of surds factors for expressions of the form a^n - b^n, simple skills in equalities and logarithms. Many candidates had difficulty with the algebraic manipulation of $x^4 - y^4$ and the change of base in Part (c).

(a) There were several good and complete answers to this part of the question. The mistakes most frequently encountered related to $\sqrt{4 \text{ x } 7} = 4\sqrt{7}$ and/or $\sqrt{7^2 \text{ x } 7} = 7^2\sqrt{7}$.

(b)(i) The observation that $x^4 - y^4 = (x^2 + y^2) = (x^2 + y^2)(x^2-y^2)$ presented most difficulty for many candidates who tried the method factorization. Many of those who used long division succeeded in gaining full marks for this part of the question.

(ii) The substitution of $x = y + 1$ was poorly done by many of the candidates.
(iii) There were not many good attempts at this part.

The change of base concepts presented enormous difficulties for candidates. More practice in this area is recommended.

Answer(s):

(a) **k = 9,**

(b)(i) $\mathbf{x^3 + x^2y + xy^2 + y^3}$**,**

(c) $\mathbf{x = \frac{1}{16}}$

Question 2

Specific Objective(s): (d) 1, 2, 7; (f) 3, 5(i).

This question examined properties of the roots of quadratic equations of functions and evaluation of a given function defined on intervals of the real numbers.

Part (a) of this question was very well done. However, in Part (b), candidates had difficulty applying the basic definition of a function, while in Part (c) the main weakness arose in recognizing the piece-wise nature of the function *f*. More practice in these topics is recommended.

Answer(s):

(a) $5x^2 + 8x + 8 = 0$

(b)(i) $f = \{(u, 1), (v, 2, (v, 3), (x, 1), (y, 3), (z, 4)\}$

(ii) a) $v \in A$ **has two images in B and** $w \in A$ **has no image in B.**

b) ***For g:*** $A \to B$**, remove from f:** $A \to B$ **one of the ordered pairs (v,2) or (v, 3) and map** $w \in A$ **to some** $b \in B$**.**
eg. g = $\{(u, 1), (v, 2), (x, 1), (w, 1)(y, 3)(z, 4)\}$

c) No. of functions g = $4 \times 2 = 8$

(c)(i) $f(f(20)) = f(5) = \frac{5}{4}$,

(ii) $f(f(8)) = f(2) = -1$,

(iii) $f(f(3)) = f(0) = -3$,

SECTION B

Module 2: Trigonometry and Plane Geometry

<u>Question 3</u>

Specific Objective(s): (b) 1, 2, 3, 5, 7, 8.

This question examined in Part (a), the application of coordinate geometry to the properties of a circle, straight lines, tangents, normal and intersections between straight lines and curves. Additionally, Part (b) of the question examined the concept of finding the angle between two given vectors, position vectors and displacement vectors, as well as finding the area of the triangle.

The majority of the candidates attempted this question, and while a few of them would have attained full marks, a number of candidates had difficulties working out the coordinate geometry, especially the vector questions.

(a) Finding the radius and coordinates of the centre were easily answered. However, many candidates unnecessarily expanded the equation of the circle to find the coordinates of the centre. A number of candidates did not recognize that the gradient of the radius is actually the gradient of the normal at the point and hence did not get the equation of the tangent correct. Many students recognized they had to solve simultaneous equations for Part (iii) but used the equation of the tangent from Part (ii) instead of the equation of the circle, since they did not read the definition of *C* carefully.

(b) This part was attempted by the majority of the candidates who successfully used various methods to calculate the size of the angle between the vectors **p** and **q**. A number of students used $A = \frac{1}{2}bh$ to find the area of the triangle without checking to see if it was a right-angled triangle. However, some candidates used the correct formula $A = \frac{1}{2}\, pq\, sin\theta$ but some used **p.q** rather than $|p||q|$. The majority of candidates found the vector PQ, but had difficulty finding the midpoint of **PQ** since they took OM as ½ PQ instead of OM = OP + PM or $\frac{1}{2}$(OP + OQ). Candidates did not recognize that OR was equal and parallel to PQ. Even those candidates who did well in the majority of the question fell down at this point.

It was evident that aspects of the syllabus needed to be reinforced. More emphasis must be placed on the equation $(x - a)^2 + (y + b)^2 = r^2$, where (a, B) represents the centre of the circle. The use of diagrams in the teaching and answering of exercises on coordinate geometry and vectors should be encouraged in order to strengthen the responses in this area.

Answer(s):

(a) **(i)** **5units; (3, 4)**

(ii) **y =** $-\frac{3}{4}x + \frac{25}{2}$

(iii) **(-1,1); (3,9)**

(b) **(i) a)** **30^o;**

b) **13 square units**

(ii) a) **i + 7j** **b) 4i + 2j**

<u>Question 4</u>

Specific Objectives(s): (a) 4, 5, 9, 12; (b) 1.

This question tested the candidates' ability to use and apply trigonometric functions, identities and equations.

A significant number of students attempted this question. A number of the candidates who attempted Part (a) attempted Part (ii) only. Part (b) and (c) proved to be quite popular with the candidates, with a significant number of candidates scoring the majority of marks in Part (b).

In Part (a) (i), there were few candidates who drew lines parallel to AD and CD respectively, to create the two right-angled triangles. Those who did were then able to use these two triangles to prove the result. Some candidates attempted methods such as sine and cosine rules without success. Most of the candidates who attempted Part (ii) of this question were able to obtain the correct values for r and ∝.

Some of the errors observed included:

- $r = \sqrt{(4 + 9)}$
- $r^2 = \sqrt{(4^2 + 9^2)} \Rightarrow r = 13$
- $\tan \propto = \frac{4}{9}$
- Maximum value is $\theta = \propto$ rather than the x-value

Part (b) was successfully completed by a significant number of candidates.

Some candidates, however, obtained incorrect solutions mainly due to

- Obtaining incorrect values for cos A and sin B
- Improper use of the relevant identities
- Incorrect substitution

Most candidates attempted Part (c) of the question. Many were able to successfully complete the first two steps of the proof, that is the expansion of tan (A + B), as well as recognizing $\tan \frac{\pi}{4} = 1$.

Many of the candidates failed to realize that some form of rationalization (use of (a + b) (a - b) $=a^2 - b^2$) had to be invoked to successfully complete the process.

Some candidates were successful using the t-approach. It was also observed that those candidates who were successful were adept at manipulating trigonometric identities.

Answer(s):

(a) (ii) $\sqrt{97}$

(b) (i) $\frac{63}{65}$, **(ii)** $\frac{56}{65}$, **(iii)** $\frac{7}{25}$

SECTION C

Module 3: Calculus 1

Question 5

Specific Objectives(s): (a) 1, 3, 4, 5, 8, 10; (b) 4; (c) 3, 4, 5 (ii), 6

This question covered topics on limits, continuity, differentiation from first principles and integration. The question was attempted by most of the candidates. The general performance was below average with only a limited number of candidates scoring more than 20 marks.

(a) In this part, several errors were made in factorizing $x^3 - 8$ which suggests that more practice is required on exercises of this sort.

(b) The graph of the function was done correctly by many candidates. A few recognized that there was a 'break' somewhere in the graph but did not know where it should be placed. Many candidates substituted -1 into $f(x) = 1 + x$ to find $\lim_{x \to 1^-} f(x)$. Very few candidates seemed to know the definition of 'continuity' and as a consequence did not find $f(1)$.

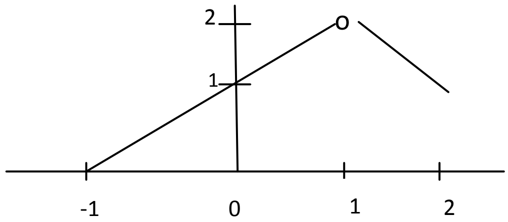
(c) Many candidates did not seem to know what 'differentiation from first principles' meant and some who knew were not able to complete the process successfully.

(d) Many candidates did not include kx and a constant of integration after integrating. Others attempted to find k before integrating.

Answer(s):

(a) -6

(b)(i)



(ii) a) $\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} (3-x) = 3-1 = 2$

b) $\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} (1=x) = 1+1 = 2$

(iii) $\mathbf{f(1) = 3 - 1 = 2 \Rightarrow f\ (x) \text{ is continuous at } x = 1}$

(c) $\frac{dy}{dx} = -\frac{2}{x^3}$

(d) $f(x) = x^3 + 3x^2 - x - 6$

<u>Question 6</u>

Specific Objective(s): (b) 7(ii), 10, 14, 15; (c) 4, 5, 6 (i)

This question tested areas of the differential and integral calculus related to the definite integral and maximum/minimum problems.

(a) This part of the question required finding the first and second derivatives of a trigonometric function and the formation of a differential equation from such derivatives.

The question was very popular with an excellent success rate.

(b) Many candidates obtained parts of the integrals correctly but were unable to complete the question successfully because of errors in the algebraic manipulation of the terms.

(c) A few candidates found difficulty in obtaining the correct expression for *V* in (i). Others lost their way in solving $\frac{dv}{dx}$ = 0 and using $\frac{d^2v}{dx^2}$ correctly.

Despite the weaknesses identified above there were several candidates who obtained full marks for this question.

Answer(s):

(b) ***a* = 4**

(c) (ii) ***x* = 2**

UNIT 1

PAPER 03/B - ALTERNATE TO INTERNAL ASSESSMENT

SECTION A

Module 1: Basic Algebra and Functions

Question 1

Specific Objective(s): (c) 1, 2, 3(ii), (iii), 5; (f) 3; (g) 1, 4

This question tested inequalities, the modulus of real numbers, algebraic expressions involving substitution, logarithms and mathematical modeling.

(a) Although a number of the candidates attempted this part of the question, many of them had difficulty manipulating the modulus sign and this weakness translated into the formation of inappropriate inequalities. There were, however, a few good answers to the problem.

(b) Many candidates did not use the substitution to its full advantage. Some others equated the expression in y to 3 000 and not to 3; nevertheless, there were some encouraging attempts presented by a few candidates.

(c) Some candidates found difficulty in establishing Part (i), while other candidates did not see the relevance of Part (i) to Part (ii). Outside of these instances, there were a few candidates who completed this part of the question successfully.

Answer(s):

(a) $\{\boldsymbol{x} \in \boldsymbol{R}: -\mathbf{2} < x < 0\}$

(b) $x = 1, 3, 2 \pm \sqrt{5}$

(c)(ii) **10**

SECTION B

Module 2: Trigonometry and Plane Geometry

Question 2

Specific Objective(s): (a) 6, 14; (b) 6, 7, 9

This question tested tangents to circles and properties of the locus of a point with coordinates described in parametric form.

(a) Many candidates showed the correct methodology in solving the problem but seemed unprepared to cope with the general point (p, q) on the circle. As a consequence, there were several unfinished solutions to this part of the question.

(b) Several candidates did not appeal to the basic properties of *sin x* and *cos x*, namely, $0 \leq |\ \sin x\ | \ \leq 1,\ 0 \leq |\cos x| \ \leq 1$ and $sin^2\, x + cos^2\, x = 1$ to solve the problems posed in this part of the question, and hence missed the simple approach to the solutions. Attempts at using calculus were made.

Answer(s):

(a)(iii) $p = -5,\ q = 1$ **or** $p = -2\frac{3}{5},\ q = \frac{1}{5}$

(b)(i) **max** $x = 5$, **min** $x = -1$
max $y = 8$, **min** $y = 0$

(ii) $\left(\frac{x-2}{3}\right)^2 + \left(\frac{y-4}{4}\right)^2 = 1$

SECTION C

Module 3: Calculus 1

Question 3

Specific Objective(s): (b) 7 to 10, 13 to 16, (c) 1 to 4

This question covered indefinite integrals and point of inflexion of, and normal to curves, as well as the notion of mathematical modeling.

(a) This part was not well done. The main hindrance to obtaining the correct solution stemmed from the candidates' failure to resolve the integrand into separate terms before attempting to integrate.

(b) The concept of a 'point of inflexion' seemed unfamiliar to many candidates. This resulted in several candidates not being able to find the values of *b* and *c* in (i), without which it was impossible to solve Part (ii) explicitly.

(c) Not many candidates attempted this part of the question, which depended on the notion of small increments, which is knowledge applied to the standard approach to the introduction of differentiation from first principles in calculus.

Answer(s):

(a) $\frac{t^2}{2} - \frac{1}{t^3} + \frac{1}{4t^4} + \textit{constant of integration}$

(b)(i) **b = 3, c = 3**
(ii) **3y = x +16**

(c)(i) **when r = 3,** $\frac{dV}{dt} = 0.72\pi$
(ii) **p=2**

DETAILED COMMENTS

UNIT 2

In general, the performance of candidates on Unit 2 was very satisfactory. Although an increased number of candidates reached an outstanding level of proficiency, some candidates were inadequately prepared for the examinations.

The examination tested some of the newer topics in the revised syllabus and included Calculus of Inverse Trigonometrical Functions and Second Derivative, the use of an Integrating Factor for First-order Differential Equations, Second-order Differential Equations, Maclaurin's Theorem for Series Expansions, Binomial Expansion Series for Rational and Negative Indices, Complex Numbers and the Locus of a Complex Number.

Weaknesses in algebraic manipulation and tasks involving substitution were again evident and candidates found it difficult to solve problems which required these skills. It is imperative that more emphasis be placed on these areas of weakness. Extensive practice in the use of substitution and algebraic manipulation is necessary if candidates are to be well-prepared to show improved performances in these areas.

Paper 01 comprised 45 multiple choice items. The candidates performed fairly well with a mean score of 25 out of a possible 45. Paper 02 comprised six compulsory questions, two testing each Module. The mean mark on this paper was 54 out of a possible 150.

UNIT 2

PAPER 02

SECTION A

Module 1: Calculus II

<u>Question 1</u>

Specific Objective(s): (b) 2, 3, 4, 5, 6, 7

This question examined concepts in differentiation as they apply to trigonometric functions, inverse trigonometric functions, implicit functions and rational functions. Second derivatives emerged in the process leading to the formation of differential equations.

(a)(i) This part of the question was well done although many candidates did not use the identity $sin^2 3x + cos^2 3x = 1$ to simplify the given expression for *y*. As a consequence, many answers were not given in the simplest form. No penalty was applied for non-simplification.

(ii) Many candidates found difficulty in differentiating $cos\ x^2$. Several interpreted $cosx^2$ as $(cos\ x)^2$ or $(cos\ x)x$.

(iii) This part of the question was not well done. Too many candidates did not know how to cope with the implicit nature of the expression for y.

(b)(i) This part of the question was generally well done. However, amongst the candidates who did not perform satisfactorily, many equated $cos^{-1}x$ with $\frac{1}{cosx}$.

(ii) Candidates found this exercise manageable and readily recognized the relevance of the chain rule to the results. Mistakes were made in a), in differentiating $\sqrt{1-t}$ while in b) the second derivative $\frac{d^2y}{dx^2}$ proved to be a major challenge for many. A common mistake made in this case was $\frac{d^2y}{dx^2} = \frac{d^2y}{dt^2} \times \frac{d^2t}{dx^2}$.

Answer(s):

(a) (i) $\frac{dy}{dx}$ = ***10 sin5x cos5x*** **(in its simplest form)**

(ii) $\frac{dy}{dx} = -\frac{x\, sinx^2}{\sqrt{cosx^2}}$

(iii) $\frac{dy}{dx} = x^x(1 + lnx)$

(b)(ii) $\frac{d^2y}{dx^2} = -\frac{\sqrt{1-t}}{4}$

Question 2

Specific Objective(s) (c) 7, 8, 13

This question examined knowledge of the trapezium rule and integration by parts in the internal calculus. Inverse trigonometric functions and approximations were also involved.

(a) This part of the question asked for the sketch of the particular function $\sqrt{1-x^2}$ on the interval $0 \leq x \leq 1$. Some candidates were unable to determine the correct quadrant or the circular property of the function.

(b) Several candidates could not find the width of the strips, but nevertheless, showed competent knowledge of the trapezium rule.

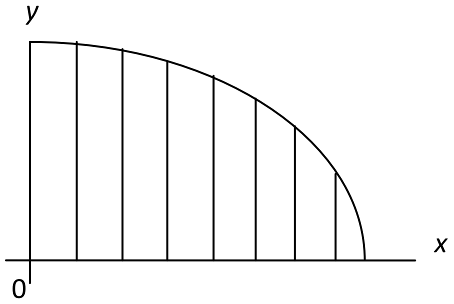
(c) (i) The majority of candidates were able to obtain the first 5 of the 9 marks allocated to this integration but could not proceed to completion of this part.

(ii) Several candidates did not see the link between (c)(i) above and failed to obtain the result for *I*.

(iii) Many answers for this part were stated in terms of degrees and not radians as expected.

(iv) The majority of candidates were unable to combine (c)(i) to (iii) to obtain the approximation to π. However, there were a few good answers to this question.

Answer(s):
(a)



The width of each strip = 0.2

(c)(iii) $\int_0^1 \sqrt{1 - x^2}\, dx = \frac{\pi}{4}$

(iv) $\pi \cong 0.759 \times 4 = 3.036$

SECTION B

Module 2: Sequences, Series and Approximations

<u>Question 3</u>

Specific Objective(s): (a) 1, 2; (b) 1, 7, 11, 12

This question tested candidates' abilities to use the recurrence relation of a sequence to obtain values of the common ratio of a convergent geometric series, the method of differences for the summation of series and the sum to infinity.

(a) The majority of candidates answered Part (i) correctly but had serious difficulties in obtaining t_n in Part (ii).

(b) Few candidates coped well with this part of the question and among those attempting the question, some had severe challenges resolving the inequality produced.

(c) Part (i) was easily obtained by many of the candidates several of whom fell down as they proceeded through to Part (ii) in order to find S_n. Many resorted to partial fractions in order to cope with Part (ii)

It is recommended that extended practice in questions of this nature be undertaken to consolidate the fundamental concepts portrayed in this question.

Answer(s):

(a) (i) $t_2 = 16$, $t_3 = 21$, $t_4 = 26$
(ii) $t_n = 5n + 6$

(b) $-\frac{1}{3} < x < 7$

(c)(i) $f(r) - f(r+1) = \frac{1}{(r+1)(r+2)}$ **(ii)** $S_n = 2 - \frac{4}{n+2}$ **(iii)** $S_\infty = 2$

Question 4

Specific Objective(s): (b) 13; (c) 1, 3

The topics examined in this question were the binomial theorem, Maclaurin's theorem and power series expansions.

Overall, the candidates' performances in this question were below the expected level. Only approximately one-third of the candidates obtained more than 13 marks out of a possible 25. Areas of good performances involved Parts (a) (ii) and (b) (ii).

(a)(i) Candidates experienced difficulty in using the general binomial coefficient ${}^{n}C_{r}$ in problems of this kind.

(ii) Some good performances were registered in this section. Some of the candidates who answered poorly ignored the fact that terms in the separate expansions of $(1+ 2x)^5$ and $(1 + px)^4$ should have been multiplied instead of added to obtain the correct results.

(b)(i) The expansion of ln (1 + *x*) appeared to be unfamiliar to many candidates. Without this basic expansion, ln (1 + 2*x*) became much harder to obtain.

(ii) Several errors were made in deriving the various derivatives of *sin2x.*

(iii) Many candidates did not link this part to the earlier results obtained in the question and hence lost direction in trying to proceed. More practice is recommended.

Answer(s):

(a) (i) $n = 4,$

(ii) $p = -3 \text{ or } \frac{-11}{3}$

(b)(i) $\ln(1 + 2x) = 2x - 2x^2 + \frac{8}{3x^3} - 4x^4 + \ldots..$

(ii) $\sin 2x = 2x - \frac{4}{3}x^3 + \frac{4}{15}x^5 - \ldots.$

(iii) $\ln(1+\sin 2x) = 2x - 2x^2 + \frac{4}{3}x^3 \ldots..$

SECTION C

Module 3: Counting, Matrices and Complex Numbers

Question 5

Specific Objective(s): (a) 1, 2, 3, 7, 10; (c)1, 3, 4, 5

This question tested simple counting techniques, elements of probability and properties of complex numbers.

(a) There were several attempts at this part of the question with a high degree of success. The majority of candidates who attempted the question obtained full marks.

(b) A large number of the candidates who did this part of the question obtained full credit for their efforts. Some candidates, however, had difficulty in writing down the correct combinations. Many candidates tried to capitalize on the result in (b)(i) above but experienced challenges.

(c) Most candidates who attempted this part of the question were able to substitute and expand correctly. Some failed to achieve this end because of faulty algebraic manipulation of the expressions. Many candidates did not appreciate that the theory of quadratic equations applied and hence did not obtain the discriminant. Others who obtained the discriminant did not observe that the result of (c) (i) was relevant.

Answer(s):

(a) 50

(b)(i) $\frac{5}{22}$,

(ii) $\frac{6}{11}$

(c)(i) u = 1 + 4 i **or** $-1-4i$

(ii) $z = 2 + 3i$ ***or*** **1 –** i

Question 6

Specific Objective(s): (b) 1, 2, 6, 7

This question examined properties of determinants and matrices and solutions of simultaneous linear equations in three variables.

(a) There were many attempts at this part of the question with a high degree of success. However, poor algebraic manipulation was the cause of many errors in the solutions.

(b)(i) Several candidates who attempted this part of the question gained full marks.

(ii) Most candidates were able to express the system of equations in the required form. Many candidates saw the relevance of Part (ii) a) to the solutions of the system.
Several others used the 'otherwise' path and employed different approaches to solving the system of equations.

Answer(s):

(a) x = 2, 3 or 6

(b)(i) a) AB = 20I, b) $A^{-1} = \frac{1}{20}B$

(ii) a) $\begin{pmatrix} 1 & -1 & 1 \\ 1 & -2 & 4 \\ 1 & 3 & 9 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix}=\begin{pmatrix} 1 \\ 5 \\ 25 \end{pmatrix}$

b) x = 1, y = 2, z = 12

UNIT 2

PAPER 03/B (ALTERNATIVE TO INTERNAL ASSESSMENT)

SECTION A

Module 1: Calculus II

Question 1

Specific Objective(s): (c) 11, 12, and MM

One part of the question posed a mathematical problem against the background of a differential equation. Both parts examined the candidates' skills in solving such equations.

The success rate in this question was not very high, although many candidates attempted it. More exposure to such problems is required at the instructional level.

Answer(s):

(a) $xy = (x + 1)(c - \frac{1}{2}e^{-x^2})$, **c is constant**
(b) $y = e^{-x} - e^{4x} - sinx$

SECTION B

Module 2: Sequences, Series and Approximations

Question 2

Specific Objective(s) 6, 8, 9 and MM

The question tested the principle of mathematical induction as well as arithmetic and geometric progressions.

The majority of candidates who attempted this question performed satisfactorily.

(a) Candidates knew how to verify the initial step in the proof for $n=1$, but some had difficulty with the induction step in proceeding from $n = k$ to $n = k + 1$.

(b) Most candidates used the formula for S_n to find the sum of all the terms, however, a few went the route of trying to calculate the value of the common difference and were unsuccessful. Despite the incidence of such cases, this part of the question was very well done.

(c) Many candidates failed to realize that the problem posed involved a geometric progression, nevertheless, some used the simple interest formula for each year and were able to reach the correct result. Most candidates did not approximate the answers to the nearest dollar.

Answer(s):

(b) Sum = -300;

(c)(i) $29 877, (ii) $ 273 743

SECTION C

Module 3: Counting, Matrices and Complex Numbers

Question 3

Specific Objective(s): (a) 1, 2, 4, 7, 9, 11; (c) 5, 7, 10

This question examined basic principles in counting methods, probability and the locus of complex numbers.

In general, candidates exhibited familiarity with the topics examined. At the level of detail, however, weaknesses were evident.

(a) Some candidates demonstrated an understanding of combinations. However, interpretation of exclusiveness was weak and led to incorrect solutions.

(b) Many candidates used a Venn diagram to reason out what was required, while some tried to use formulae but failed to obtain the correct answers.

(c) This part of the question presented an enormous amount of difficulty. Many were unable to obtain the circle in (i) and hence (ii) was not manageable in such circumstances.

Answer(s):

(a) **251**

(b)(i) $\boldsymbol{P(A \cup B) = 0.7}$, **(ii)** $\boldsymbol{P(A \cap B') = 0.5}$, **(iii) 0.6**

(c)(i) **locus is the circle** $\boldsymbol{(x-1)^2 + (y+1)^2 = 5^2}$

(ii) **radius = 5, centre** $\boldsymbol{\equiv (1, -1)}$

PAPER 03 – INTERNAL ASSESSMENT

This year 170 Unit 1 and 158 Unit 2 Internal Assessments were moderated. Far too many teachers continue to submit solutions without unitary mark schemes. In some cases neither question papers with solutions nor mark schemes, were submitted. The majority of the samples submitted were not of the required standard. Teachers MUST pay particular attention to the following guidelines and comments to ensure effective and reliable submission of the Internal Assessments.

The Internal Assessment is comprised of three Module tests. The main features assessed are:

- Mapping of the items tested to the specific objectives of the syllabus for the relevant Unit
- Content coverage of each Module test
- Appropriateness of the items tested for the CAPE level
- Presentation of the sample (Module test and students' scripts)
- Quality of the teachers' solutions and mark schemes
- Quality of the teachers' assessments-consistency of marking using the mark schemes
- Inclusion of mathematical modeling in at least one Module test for each Unit

GENERAL COMMENTS

1. Too many of the Module tests comprised of items from CAPE past examination papers.

2. Untidy “cut and paste” presentations with varying font sizes were common place.

3. Teachers are reminded that the CAPE past examination papers should be used ONLY as a guide.

4. The stipulated time for Module tests (1 to $1\frac{1}{2}$ hours) must be strictly adhered to as students may be at an undue disadvantage when Module test are too extensive or are inadequate.

5. The following guide can be used: 1minute per mark. About 75% of the syllabus should be tested and mathematical modeling MUST be included.

6. Multiple choice questions will NOT be accepted in the Module tests.

7. Cases were noted where teachers were unfamiliar with recent syllabus changes i.e.

    - Complex numbers and the Intermediate Value Theorem are now tested in Unit 2.
    - Three dimensional vectors, dividing a line segment internally and externally, systems of linear equations have been REMOVED for the CAPE syllabus (2008).

8. The moderation process relies on the validity of the teachers’ assessment. There were few cases where students’ solutions were replicas of the teachers’ solutions – some contained identical errors and full marks were awarded for incorrect solutions. There were also instances where the marks on the students’ scripts did not correspond to the marks on the Moderation sheet.

9. Teachers MUST present evidence of having marked each individual question on the students’ script before a total is calculated at the top of the script. The corresponding whole number score out of 20 should be placed at the front of the students’ scripts. To enhance the quality of the design of the Module tests, the validity of the teacher assessment and validity of the moderation process, the Internal Assessment guidelines are listed below for emphasis.

Module Tests

I. Design a separate test for each Module. The Module test MUST focus on objectives from that Module.

II. In cases where several groups in a school are registered, the assessments should be coordinated, common tests should be administered and a common marking scheme used.

III. One sample of FIVE students will form the sample for the centre. If there are less than five students ALL scripts will form the sample for the centre.

IV. In 2009, the format of the Internal Assessment remains unchanged.

MULTIPLE CHOICE EXAMINATIONS WILL NOT BE ACCEPTED AS INTERNAL ASSESSMENTS.

GUIDELINES FOR MODULE TESTS AND PRESENTATION OF SAMPLES

1. COVER PAGE TO ACCOMPANY EACH MODULE TEST

The following information is required on the cover of each Module test.

- Name of the school and territory, Name of teacher and the Centre number.
- Unit Number and Module Number
- Date and Duration (1-$1\frac{1}{2}$ hours) of Module test.
- Clear instructions to candidates
- Total marks allotted for Module test.
- Sub-marks and total marks for each question MUST be clearly indicated.

2. COVERAGE OF THE SYLLABUS CONTENT

- The number of questions in each Module test must be appropriate for the stipulated time of (1-$1\frac{1}{2}$hours).
- **CAPE past examination papers should be used as a guide ONLY.**
- Duplication of specific objectives and questions must be avoided.
- Specific objectives tested must be from the relevant Unit of the syllabus.

3. MARK SCHEME

- Detailed mark schemes **MUST** be submitted, that is, **one mark should be allocated per skill.** (not 2, 3, 4, etc marks per skill)
- **FRACTIONAL/DECIMAL MARKS MUST NOT BE AWARDED (i.e. DO NOT ALLOCATE $\left(\frac{1}{2}\right)$MARKS).**
- The total marks for Module test MUST be clearly stated on the teachers' solutions sheets.
- A student's marks **MUST** be entered on the front page of the student's scripts.
- Handwritten marks schemes **MUST** be NEAT and LEGIBLE. The unitary marks **MUST** be written on the right side of the page.
- **Diagrams MUST be neatly drawn with geometrical/mathematical instruments.**

4. PRESENTATION OF SAMPLE

- Student's responses MUST be written on letter sized paper $\left(8\frac{1}{2}\ x\ 11\right)$.
- Question numbers MUST be written clearly in the left hand margin.
- The total marks for EACH QUESTION on student's scripts MUST be clearly written in the left or right margin.
- ONLY ORIGINAL students' scripts MUST be sent for moderation.
- Photocopied scripts WILL NOT BE ACCEPTED.
- Typed Module tests MUST be NEAT and LEGIBLE.
- The following are required for each Module test:

  - ❖ A question paper.
  - ❖ Detailed solutions with detailed Mark Scheme.
  - ❖ The question paper, detailed solutions, Marks Schemes and 5 students' samples should be batched together for each Module.

- Marks are recorded on PMath1 – 3 and PMath2 – 3 forms and must be rounded off to the nearest whole number. If a student scored zero, then zero must be recorded. If a student was absent, then absent must be recorded.

- The guidelines at the bottom of these forms should be observed. (See Page 57 of the syllabus, No. 6).