C A R I B B E A N   E X A M I N A T I O N S   C O U N C I L

REPORT ON CANDIDATES' WORK IN THE SECONDARY EDUCATION CERTIFICATE EXAMINATIONS MAY/JUNE 2007

MATHEMATICS

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GENERAL AND BASIC PROFICIENCY EXAMINATIONS

MAY/JUNE 2007

GENERAL COMMENTS

The General Proficiency Mathematics examination is offered in January and May/June each year, while the Basic Proficiency examination is offered in May/June only.

In May/June 2007 approximately 86 835 candidates registered for the General Proficiency examination. Candidate entry for the Basic Proficiency examination was approximately 5 513.

At the General Proficiency level, approximately 34 per cent of the candidates achieved Grades I - III. This represents a 2 per cent decrease over 2006. Thirty-two per cent of the candidates at the Basic Proficiency level achieved Grades I - III compared with 44 per cent in 2006.

DETAILED COMMENTS

General Proficiency

In general, candidates' work revealed a lack of knowledge of basic mathematical concepts, especially the areas of Measurement, Algebra and Transformation Geometry. The optional section of Paper 02 seemed to have posed the greatest challenge to candidates, particularly the areas of Relations, Functions and Graphs; and Geometry and Trigonometry.

One candidate scored the maximum mark on the overall examination compared with six candidates in 2006. Twenty-three per cent of the candidates scored at least half the available marks compared with 26 per cent in 2006.

Paper 01 - Multiple Choice

Paper 01 consisted of 60 multiple-choice items. This year, 120 candidates earned the maximum available mark compared with 84 in 2006. Approximately 60 per cent of the candidates scored at least half the total marks for this paper.

Paper 02 – Essay

Paper 02 consisted of two sections. Section I comprised eight compulsory questions totalling 90 marks. Section II comprised six optional questions: two each in Relations, Functions and Graphs; Trigonometry and Geometry; Vectors and Matrices. Candidates were required to choose any two questions. Each question in this section was worth 15 marks.

This year, one candidate earned the maximum available mark on Paper 02 compared with 17 in 2006. Approximately 15 per cent of the candidates earned at least half the maximum mark on this paper.

Compulsory Section

Question 1

This question tested the candidates' ability to:

– perform basic operations with decimals
– express a common fraction in its lowest terms
– calculate the fraction and the percentage of a given quantity
– express one quantity as a fraction of another
– solve problems involving ratios.

The question was attempted by 98 per cent of the candidates, 13 per cent of whom scored the maximum available mark. The mean mark was 6.9 out of 11.

Responses were generally good with the majority of candidates displaying competence in using the calculator to perform computations and in calculating the fraction or percentage of a quantity.

Those candidates who performed the computation without the use of calculators made errors such as evaluating 3.7^2 as 7.4 or (6.24 ÷ 1.3) as 0.48. A few who used calculators ignored the brackets and performed the operations from left to right.

In part (b) where candidates had to find the missing part of a ratio, many did not understand that 30 parts had to be equated to 1200 and interpreted it as sharing 1200 into 31 parts, hence they divided 1200 by 31 instead of 30.

A large number of candidates correctly calculated the number of students who own personal computers but omitted to subtract from 1200 to calculate the number of students who do not own computers.

A major problem faced by candidates in calculating the fraction of students in the school who own play-stations, was recognizing that the whole consists of 1200 students. Many candidates were able to reduce the fraction to its lowest terms but quite a large number omitted this part of the question.

Solutions:

(a) **8.89**

(b) **(i)** **40** **(ii)** **720** **(iii)** $\mathbf{\frac{3}{25}}$

Recommendations

Teachers need to emphasize the difference between:

- Calculating the missing part of a ratio and dividing a quantity in a given ratio
- A part-part comparison (ratio) and a part-whole comparison (fraction).

Question 2

This question tested the candidates' ability to:

– perform operations involving directed numbers
– use symbols to represent binary operations
– divide algebraic fractions

– substitute numbers for algebraic symbols in simple algebraic expressions
– translate verbal phrases into algebraic symbols
– use simultaneous equations in two variables to solve a real world problem.

The question was attempted by 97 per cent of the candidates, 7 per cent of whom scored the maximum available mark. The mean score was 5.19 out of 11.

Although many candidates were able to evaluate 4*8 correctly, far too many of them interpreted 4*8 as 4×8 and 2*(4*8) as $2 \times$ (4*8). Few candidates made the connection between parts (i) and (ii), failing to substitute the result for 4*8 in part (ii). Computing $4 \times 8 - \frac{8}{4}$ posed a problem for many candidates; a common error was to subtract 8 from 32 and divide the result by 4, hence obtaining 6 instead of 30.

In carrying out the operation of division on fractions, many candidates inverted correctly but only the very able candidates expressed the fraction in its simplest form. In attempting the division, some candidates cross-multiplied displaying poor understanding of operations on algebraic symbols.

Responses to part (c) on simultaneous equations were very good. The majority of the candidates correctly derived the two equations, although some of the candidates wrote inequalities instead of equations.

In solving the equations, both elimination and substitution were used although the majority chose the method of elimination. Common errors made included selecting the operation to eliminate a variable and simplifying the algebraic expressions.

When using the substitution method where a fractional expression was involved, some candidates experienced difficulty. A small percentage of candidates used the matrix method, although only a few were able to obtain the correct solution using this method.

Solutions**:**

(a) **(i)** **30** **(ii)** **45**

(b) $\mathbf{\frac{5}{12p}}$

(c) **(i)** **5a + 3b = 105, 4a + b = 63**

(ii) **a = 12, b = 15**

Recommendations

Teachers need to

- emphasize the difference between operations in algebra and operations in arithmetic.
- connect the teaching of operations on fractions in arithmetic with the operations on fractions in algebra.
- encourage students to determine the most efficient strategy to use in solving equations. Both elimination and substitution methods should be taught but students must be able to decide which is better in a given situation.
- emphasize the difference between ***solve*** and ***simplify***, ***equation*** and ***expression***, ***equality*** and ***inequality***.

Question 3

This question tested the candidates' ability to:

- use a phrase to describe a set
- use Venn Diagrams to solve practical problems with not more than three sets
- use instruments to draw and measure angles and line segments
- use instruments to construct a triangle given three sides
- use instruments to construct a perpendicular.

The question was attempted by 93 per cent of the candidates, 1 per cent of whom scored the maximum available mark. The mean score was 5.91 out of 12.

The majority of candidates were able to interpret the Venn Diagram in part (a) by stating correctly the games played by each member. However, describing the region $H' \cap S$ posed problems for many candidates.

Candidates generally succeeded in drawing the triangle with the prescribed dimensions although many omitted to use instruments as evidenced by absence of construction arcs. Weaker candidates experienced problems in measuring the angle and the line segment. Several of them attempted to calculate rather than measure. Generally candidates experienced difficulty constructing the perpendicular from P to meet QR at T.

Some common errors in attempting to locate the point T were constructing the perpendicular bisector of QR and labelling the mid-point as T and bisecting angle PQR and labelling the point at which the bisector cuts QR as T.

Solutions:

(a) **(i)** **a)** **Tennis and Hockey** **b)** **Squash, Tennis and Hockey** **c)** **Hockey**

(ii) **Those members who play Squash (and Tennis) but not Hockey**

(b) **(ii)** **a)** **$59^{o} \pm 1^{o}$** **b)** **5.1 cm + 0.1 cm**

Recommendations

Teachers need to present meaningful contexts in the teaching of set theory giving student opportunities to create their own Venn Diagrams from everyday situations. Oral descriptions of regions should precede written ones so that students grasp the language of sets.

With respect to the teaching of constructions, there is need to emphasize the difference between draw and construct and ensure that students can use instruments properly.

Question 4

This question tested the candidates' ability to:

– make suitable measurements on a map and use them to determine distance and area
– convert centimetres to metres and square centimetres to square metres
– estimate the area of an irregularly shaped figure
– calculate the length of an edge of a square-based prism given its volume and height
– calculate the total surface area of a square-based prism.

The question was attempted by 86 per cent of the candidates, 1 per cent of whom scored the maximum available mark. The mean score was 2.47 out of 11.

Generally this question was poorly done mainly because candidates experienced difficulties with conversion of metric units, and using the linear scale factor to calculate the area represented by 1 cm^2 on the map. Few candidates counted the number of squares to derive the area.

Candidates were able to find the distance between two points on the map and use the scale to convert to the actual distance.

In part (b), the majority of candidates were able to find the area of the cross-section of the prism but they often failed to use the square root to calculate the length of the side AB. Candidates who calculated AB were able to determine the total surface area of the prism.

Solutions:

(a) (i) 120 m (ii) 220 – 232 m (iii) 1 600 m^2 (iv) 416 000 – 46 400 m^2

(b) (i) 8 cm (ii) 608 cm^2

Recommendations

Teachers should delay the introduction of measurement formulae as much as possible and allow students to fully explore basic concepts in measurement. Measuring the area of irregular shapes allows students to grasp the concept of area and how it is measured. The concept of a square unit and the difference between square centimetres and other metric square units also needs to be emphasized.

Question 5

This question tested the candidates' ability to:

– represent inverse variation symbolically
– perform calculations involving inverse variation
– determine the gradient of a line parallel to a given line
– determine the equation of a line given one point and the equation of a line parallel to it.

This question was attempted by approximately 65 per cent of the candidates, 3 per cent of whom scored the maximum available mark.. The mean mark was 2.05 out of 12.

In general the performance was very unsatisfactory. It was evident from the responses that candidates were not familiar with the inverse variation. Some incorrect responses were:

$y = kx, y = \frac{k}{x}$ $y = kx^2$, $y = k\sqrt{x}$

In calculating the table values the majority of the candidates were able to make the correct substitution but some experienced difficulties in transposing.

In part (c) many candidates demonstrated knowledge of the formula y = mx + c and attempted to use it but errors in substitution resulted in an incorrect equation.

Solutions:

(a) $y = \frac{k}{x^2}$

(b) **(i)** **K = 18** **(ii)** **r = 5.6** **(iii)** **f = 1.5**

(c) **y = 2x – 1**

Recommendations

The teaching of variation can be used to show the application of mathematics in other disciplines and in real life. The principles of variation are exemplified in many laws in the pure sciences as well as the social sciences and teachers can take the opportunity to design projects using this topic as a foundation.

Question 6

This question tested candidates' ability to:

- determine the centre and scale factor of an enlargement
- locate the image of an object under a refection in the line y = -x
- solve problems involving bearings
- use the cosine rule to calculate an unknown side of a triangle.

The question was attempted by 87 per cent of the candidates, 1 per cent of whom scored the maximum available mark. The mean mark was 1.95 out of 11.

Many candidates attempted either Part (a) or Part (b), but only a few candidates answered both parts.

Most of the candidates could not correctly identify the scale factor or the centre of the enlargement. However, the major difficulty experienced by candidates was the inability to reflect the triangle LMN in the given line y = -x. Many candidates performed a translation instead, and merely shifted the triangle diagonally to a new position in the third quadrant.

Some candidates attempted to use a matrix method to calculate the coordinates of the image but were unsuccessful since an incorrect matrix was used.

In part (b), many candidates correctly chose the Cosine Rule to calculate the length PR. However they could not interpret the given bearings to calculate angle PQR and used cos 70° instead of cos 20° in the formula. In evaluating the result, a common computational error was equating

125 – 100 cos 20° to *25 cos 20°*.

In calculating the bearing, many candidates made the false assumption that the triangle was isosceles and based their calculations on this. Only a few candidates were successful in stating the bearing of R from P.

Solutions:

(a) **(i)** **a)** **2** **b)** **(0,0)**

(ii) **L′′(-4,-1) M′′(-2,-2) N′′(-3, -4)**

(b) **(i)** **5.6 km** **(ii)** **108°**

Recommendations

Teachers should give students practical experience in

- Performing transformations in a variety of situations. For example, reflecting in mirror lines of different orientations should be done so that students can generalize the properties of reflection. Also paper-folding activities involving the use of plain paper should precede the use of graph paper on the coordinate plane.

- Performing out-door activities in which they determine the bearing from one position to another using actual measurements. Stating the bearing using appropriate language and terminology should be explored.

Question 7

This question tested the candidates' ability to:

– complete a frequency table from a set of data

– determine the range from a set of data

– construct a frequency polygon using given scales

– determine the probability of an event using data from a frequency table.

The question was attempted by 92 per cent of the candidates, 1 per cent of whom scored the maximum available mark. The mean mark was 4.17 out of 12.

The majority of candidates were competent in completing the frequency table and in using the correct scales. However, a significant number of candidates did not know that the horizontal axis was the x-axis and the vertical axis was the y-axis. They also failed to use the mid-points on the horizontal scale and used the limits or boundaries instead.

Candidates were generally aware that the frequency polygon was constructed using straight lines. However, some candidates closed the polygon by joining the first and last points, while others did not connect the points in sequence. A few candidates chose to draw a histogram first and then construct the frequency polygon.

A major weakness was the inability to determine the range for the data. Some candidates merely listed the upper-class boundaries, while others interpreted the range as an interval.

The majority of the candidates were able to successfully use the total frequency in calculating the probability, but many could not determine the number of candidates who completed the race in less than 60 seconds.

Solutions:

(a) **3, 7, 4, 5**

(b) **32**

(d) $\mathbf{\frac{7}{32}}$

Recommendations:

- Teachers need to ensure that students can differentiate between a bar chart, a histogram and a frequency polygon.
- Teachers should encourage students to construct a frequency polygon independent of a histogram emphasizing the use of mid-points of the class intervals on the horizontal axis.
- The concept of range as a value needs to be emphasized and students should be taught methods of determining the range using raw data as well as data from frequency tables.

Question 8

This question tested the candidates' ability to:

– express one quantity as a fraction of a given quantity

– calculate perimeter of plane shapes

– calculate the area of a trapezium

– draw a trapezium given its component parts.

The question was attempted by 66 per cent of the candidates, 3 per cent of whom scored the maximum available mark. The mean mark was 2.34 out of 10.

Performance on this question was generally poor. Candidates could not express the parts of the rectangle as a fraction of the whole rectangle. Answers consisted mainly of unit fractions with relatively large denominators.

In ordering the shapes by perimeter, most of the candidates recognized which shapes should be placed first and last and determined the area of the three shapes which had to be rearranged to form the trapezium. Attempts to re-arrange the shapes to form a trapezium proved to be futile and many either omitted this part or failed to form a new arrangement.

Solutions:

(a) **(A, B, D, E, F) =** $\left(\frac{1}{4}, \frac{1}{6}, \frac{5}{24}, \frac{1}{9}, \frac{1}{6}\right)$

(b) **A, D, F, B, E, C, G or reverse order.**

(c) **12 square units**

Recommendations

The use of investigative methods in developing reasoning and problem-solving processes needs to be addressed more aggressively in the classroom. Students must be given opportunities to reason with space as well as with quantity using physical props, where necessary.

Optional Section

Question 9

This question tested the candidates' ability to:

- interpret and make use of the functional notation f(x), $f^{-1}(x)$ and fg(x)
- find the inverse of a simple function
- find an expression for a composite function
- obtain the solution of a quadratic equation by factorization or by formula
- solve a word problem involving a quadratic equation.

The question was attempted by 59 per cent of the candidates, 4 per cent of whom scored the maximum available marks. The mean mark was 4.73 out of 15.

In general this question was not well done. The more able candidates obtained full marks for part (a) of the question. The weaker candidates were able to substitute correctly to evaluate f(-2) but made careless errors in simplifying their result. Candidates also successfully applied the first two steps of the algorithm to find the inverse of a function but in many instances failed to complete it because they lacked skills to manipulate algebraic expressions. Similar problems were encountered in simplifying their expression for gf(x).

In part (b), the majority of candidates recognized that the area of the rectangle was (2x-1)(x+3), but only a small percentage of these were able to expand this expression. Equating their quadratic expression to 294 was not a major problem for many candidates but some attempted to solve without transposing the 294.

Most of the candidates chose to use the formula to solve the quadratic equation and some candidates used the negative **x** value in calculating the dimensions even though it gave a meaningless result.

Solutions:

(a) **(i)** $\frac{-3}{5}$ **(ii)** $\frac{2x+9}{5}$ **(iii)** $\frac{5x-1}{2}$

(b) **(i)** **(2x – 1)(x + 3)** **(ii)** **x = 11** **(iii)** **21 cm × 14 cm**

Recommendations

While this optional question was by far the most popular, teachers must caution students that basic algebraic skills are required in order to attempt these questions successfully. The use of factorization as the first method to try in solving quadratic equations must be emphasized. Students must also be reminded that unlike linear equations, quadratic equations can only be solved when all terms are equated to zero.

Question 10

This question tested the candidates' ability to:

- translate algebraic inequalities into worded statements
- translate verbal statements from and into inequalities
- draw graphs of linear inequalities in one or two variables
- determine whether a set of points satisfy all conditions described by a set of inequalities
- use linear programming techniques to solve problems in two variables.

The question was attempted by 42 per cent of the candidates, 0.2 per cent of whom scored the maximum available mark. Performance was generally fair with a mean mark of 4.82 out of 15.

Candidates were generally able to translate verbal statements into algebraic inequalities. However, most candidates were unable to demonstrate the reverse process. Interpreting the given scale posed no problems for the majority of candidates.

Although quite a significant number of candidates correctly drew all the graphs and obtained the correct region, some made common errors such as:

- writing the inequality $x > 15$ instead of $y > 15$
- writing the inequality $x + y < 60$ instead of $x + y > 60$
- interchanging the variables, gold stars with silver stars
- drawing the line $y = 20$ instead of $x = 20$ and $x = 15$ for $y = 15$
- drawing the line $y = ½ x$ as $x = ½ y$
- using a 'solid' line instead of a 'broken' line for the graphs of inequalities.

The majority of the candidates were able to identify at least one of the two points satisfying the four conditions.

Solutions:

(a) $y \geq 15, \; x + y \leq 60$

(b) The number of gold stars must be less than twice the number of silver stars.

(d) A and C lie in the region which satisfies all conditions.

Recommendations

Teachers need to allow students to construct their own inequalities using a variety of authentic situations so that they can develop the vocabulary relating to inequalities. Terms such as *at least, at most, not greater than, less than or equal to,* need to be emphasized.

Identifying the region defined by an inequality also needs to be more carefully addressed. Students should understand when to use broken lines as boundaries and how to test points to determine the feasible region.

Question 11

This question tested the candidates' ability to:

- recognize and use the trigonometrical ratios of special angles
- calculate the circumference of a circle of latitude
- determine the longitude of a point on the surface of the earth given its distance from another point on the same parallel of latitude.

The question was attempted by 9 per cent of the candidates, 2 per cent of whom scored the maximum available mark. The mean mark was 1.49 out of 15.

It was evident that candidates were not familiar with the surd and fractional form of the trigonometric ratios. Some attempted to use the calculator to find $\sin^{-1} \frac{\sqrt{3}}{2}$ but encountered problems because of incorrect use of the order of operations when inputting the inverse function. A few candidates were able to successfully use the trigonometric identity $\sin^2 x + \cos^2 x = 1$. A number of candidates did not use Pythagoras' Theorem to find the unknown side of the triangle.

In part (b), a number of candidates substituted R instead of $R\cos\theta$ in calculating the radius of the circle latitude 37^0 N.

Using the given arc length to find the difference in latitude posed a problem for many candidates; many were unable to rearrange their expression to evaluate the angle. Some who were successful failed to subtract 50^0 to obtain the value of x.

Solutions:

(a) (i) a) $\frac{1}{2}$ **b)** $\sqrt{3}$

(ii) $\frac{1}{2}$

(b) (i) 31 948 km

(ii) 10.7°

Recommendations

Teachers should allow students to derive the trigonometric ratios for special angles using drawings and measurements prior to using the calculators.

The use of three-dimensional models in illustrating various concepts in relation to earth geometry must also be explored so that students can have mental props in visualizing the angles and distances in solving these type of problems.

Question 12

This question tested the candidates' ability to:

- solve problems using the properties of regular polygons
- calculate the area of a triangle given two sides and an included angle
- calculate the area of regular polygon (octagon) from the area of one triangle
- use theorems in circle geometry to calculate the measure of angles.

The question was attempted by 14.5 per cent of the candidates 1 per cent of whom scored the maximum available mark. The mean mark was 3.2 out of 15.

Responses to this question were poor. Candidates were generally able to find the area of the octagon by multiplying the area of one triangle by 8.

Many candidates did not understand the properties of a regular polygon and failed to recognize that the angle at the centre was $\frac{1}{8}$ of 360^0. It was assumed that triangle XYZ was equilateral and hence many gave 60^0 as their response.

Some candidates calculated the interior angles of the octagon and used a lengthy route to obtain the required angle.

In calculating the area of the triangle, those who did not recognize the use of $\frac{1}{2}$ ab sinθ attempted to calculate YZ but these attempts generally did not produce correct results, because of computational errors.

Solutions:

(a) **(i)** **45^0** **(ii)** **12.7 cm^2** **(iii)** **101.6 cm^2**

(b) **a)** **90^0, angle in a semi-circle**

b) **90^0, angle between a tangent and a radius**

c) **23^0, angle between tangent and chord is equal to the angle subtended by the same chord in the alternate segment**

d) **113^0, opposite angles of a cyclic quadrilateral are supplementary.**

Recommendations

The topic of circle theorems continues to pose challenges for many students and proficiency in this area is necessary if students are to develop logical thinking and problem-solving skills. Teachers need to give students opportunities to make oral presentations in order to clarify their reasons, and to be concise in their explanations.

Question 13

This question tested the candidates’ ability to:

- show the relative position of points on a diagram
- add vectors using the triangle law
- use a vector method to prove two lines are parallel
- use vectors to represent and solve problems in geometry.

The question was attempted by 15 per cent of the candidates 1 per cent of whom scored the maximum available mark. The mean mark was 4.91 out of 15.

Most candidates were able to locate the position of R, the midpoint of a line but the fraction ⅓ gave some difficulty and the point ‘S’ was not always placed nearer to O along OM. Routes for resultant vectors were arrived at in a variety of ways and although these were mostly correct, incorrect substitution of vectors resulted in the loss of marks. In many cases candidates did not know the direction of the vector.

Candidates knew the condition for vectors to be parallel but could not simplify their expressions to express one vector as a scalar product of the other.

Solution:

(b) **(i)** $\underline{\mathbf{k}} - \underline{\mathbf{m}}$ **(ii)** $\underline{\mathbf{m}} - \frac{1}{2}\underline{\mathbf{k}}$ **(iii)** $\frac{1}{3}\underline{\mathbf{m}} - \underline{\mathbf{k}}$ **(iv)** $\frac{1}{3}\underline{\mathbf{m}} - \frac{1}{2}\underline{\mathbf{k}}$

Recommendations

The abstract nature of this topic demands that teachers place greater emphasis on practical situations. Particular attention to the use of proper notation in writing vectors is also critical to developing proficiency in this topic.

Question 14

This question tested the candidates' ability to:

– evaluate the determinant of a 2 × 2 matrix
– obtain the inverse of a non-singular 2 × 2 matrix
– perform addition, subtraction and multiplication of matrices
– perform multiplication of matrices by a scalar
– describe a transformation geometrically given an object and an image
– determine the matrices associated with translation and rotation
– use matrices to solve simple problems in geometry.

The question was attempted by 27 per cent of the candidates, 1 per cent of whom scored the maximum available mark. The mean mark was 4.01 out of 15.

Responses to this question were barely satisfactory although some parts were well done. In part (a), many candidates were able to perform the scalar multiplication and to solve their resulting matrix equation. Although candidates were familiar with the method of finding the inverse, careless errors were common in stating the adjunct or in calculating the determinant.

In solving the matrix equation, candidates were able to equate corresponding terms in their 2 × 2 matrix, but poor algebraic skills prevented them from arriving at the correct solution. For example 3a + 2 was often expressed as 5a.

Describing the geometric transformations which represented the transformations posed difficulties for the majority of candidates. A large number of candidates omitted this part of the question while others gave an incomplete description by simply naming the transformation without stating the specific characteristics.

A number of candidates successfully obtained the matrix for the rotation. However, there was a tendency to use a 2 × 2 matrix instead of a column matrix to describe the translation.

Solutions:

(a) **(i)** $\begin{pmatrix} 3a & 3b \\ 3c & 3d \end{pmatrix}$ **(ii)** $\begin{pmatrix} 2 & -3 \\ -3 & 5 \end{pmatrix}$ **(iii)** $\begin{pmatrix} 3a+2 & 3b-3 \\ 3c-3 & 3d+5 \end{pmatrix}$

(iv) **a = 4, b = 1, c = -2, d = 0**

(b) **(i)** a) **A translation of -4 units parallel to the y-axis**

b) **A rotation of 180^0, about the origin OR an enlargement, scale factor -1 about the origin.**

(ii) a) $\begin{pmatrix} 0 \\ -4 \end{pmatrix}$ b) $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$

(iii) **P′ (6, -2)**

(iv) **Q′ (-5, 4)**

Recommendations

Teachers must seek to strengthen basic algebraic skills prior to teaching topics in matrices. The principles involved in solving simple algebraic equations still apply when solving matrix equations and students must be allowed to make this connection.

Similarly, the understanding of basic geometric transformations is a necessary prerequisite for the understanding of matrix transformations. Hence teachers should take the necessary steps to ensure that basic conceptual knowledge is in place before introducing students to these abstract topics.

DETAILED COMMENTS

Basic Proficiency

The Basic Proficiency examination is designed to provide the average citizen with a working knowledge of the subject area. The range of topics tested at the basic level is narrower than that tested at the General Proficiency level.

This year approximately 32 per cent of the candidates achieved Grades I - III.

Paper 01 - Multiple Choice

Paper 01 consisted of 60 multiple-choice items. The maximum mark was not attained by any of the candidates. Approximately 41 per cent of the candidates scored at least half the maximum mark on this paper.

Paper 02 – Essay

Paper 02 consisted of 10 compulsory questions. Each question was worth 10 marks. The highest score earned was 95 out of 100. This was earned by two candidates. Thirteen per cent of the candidates earned at least half the total marks on this paper.

Question 1

This question tested candidates' ability to:

– perform any of the basic operations with rational numbers

– approximate a value to given number of significant figures

– solve problems involving fractions and decimals.

The question was attempted by 98 per cent of the candidates, 0.5 per cent of whom scored the maximum available mark. The mean mark was 3.12 out of 10.

The candidates were able to use calculators to evaluate 1.05^2 but they generally lacked knowledge of significant figures. Examples of incorrect responses were 690.000 or 69. Some candidates wrote their answer in standard form.

In part (b), candidates were able to recognize that the total number of shares was 7 and that 3 shares were equivalent to \$45. However, some candidates assumed that one share was the \$45 while others equated \$45 to 7 shares.

In part (c), most candidates knew that they had to add the fractions but the weaker candidates associated the 'and' with multiplication and proceed to multiply $\frac{1}{4}$ by $\frac{3}{8}$ instead of $\frac{1}{4} + \frac{3}{8}$. Candidates were generally aware that they had to find the fraction that played basketball by subtracting before they found the required percentage.

Solutions:

(a) **(i)** **687.96** **(ii)** **690**

(b) **\$105**

(c) **(i)** $\mathbf{\frac{5}{8}}$ **(ii)** **37.5%**

Recommendations

- The concept of significant figures should be reinforced and more practice exercises given.
- More non-routine problems based on ratio must be given to students, and more word problems involving fractions.

Question 2

This question tested candidates' ability to:

– perform operations involving directed numbers

– use symbols to represent binary operations and perform simple computations

– use the laws of indices to manipulate expressions with integral indices

– apply the distributive law to insert or remove brackets in algebraic expressions

– solve a simple linear inequality in one unknown.

The question was attempted by 95 per cent of the candidates, 1 per cent of whom scored the maximum available mark. The mean mark was 2.3 out of 10.

Generally, candidates performed unsatisfactorily. They experienced difficulty in all parts of the question.

Candidates were generally unable to apply the laws of indices. For example, they incorrectly equated $2p^2 \times 3p^3$ with $6p^6$ or 5p6.

In other cases, $2p^2 \times 3p^3$ was interpreted as $2^2 \times 3^3 \times p = 4 \times 27 \times p = 108p$.

In part (iii), candidates multiplied the entire expression by 3 instead of the terms in the bracket. Thus $3(2x + 1) - 4x$ was equated to $6x + 3 - 12x$, instead of $6x + 3 - 4x$. They also experienced difficulty in collecting the like terms and simplifying.

In part (b) $4 \times (-5^2)$ was interpreted to mean $(4 \times - 5)^2$.

Solutions:

(a) **(i)** $\mathbf{6p^5}$ **(ii)** $\mathbf{p^2}$ **(iii)** $\mathbf{2x + 3}$

(b) **100**

(c) **(i)** $\mathbf{x < 2}$ **(ii)** $\mathbf{x = 1}$

Recommendations

- Emphasis should be placed on the laws of indices and operations with directed numbers.

Question 3

This question tested candidates' ability to:

– solve simple problems involving payments in installments as in the case of hire purchase

– solve problems involving insurances

– solve problems involving compound interest.

The question was attempted by 95 per cent of the candidates, 5 per cent of whom scored the maximum available mark. The mean mark was 3.31 out of 10.

Candidates showed a fairly good understanding of the topic hire purchase. However, many did not complete the question by finding the difference between the cash price and the hire purchase price.

Candidates showed very limited knowledge of insurance calculations. Most added the fixed land charge to the price of the house and then tried to find 0.5% of the total. Some found 5% instead of 0.5%.

Compound interest was very often calculated as simple interest.

Solutions :

(a) **$35**

(b) **$1 350**

(c) **$6 615**

Recommendations

- Students should be taught to distinguish between simple and compound interest.
- More emphasis should be placed on all areas of consumer arithmetic.

Question 4

This question tested candidates' ability to:

– calculate the area enclosed by a trapezium

– use measurements on a map and a scale to calculate actual distance and vice versa

– solve problems involving measurements .

The question was attempted by 93 percent of the candidates, 7 per cent of whom scored the maximum available mark. The mean mark was 3.83 out of 10.

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In part (a) most candidates were able to calculate the actual distance in km. However many could not do the calculation to find the distance on the map.

Solutions:

(a) (i) 11.2 km (ii) 7.5 cm

(b) (i) 6 cm (ii) 88 cm²

Recommendations

- Students need to do more examples in measurement involving irregular shapes.
- More emphasis on the use of scale drawings and converting from maps to actual measures and vice versa.

Question 5

This question tested candidates' ability to:

– solve problems involving cost price, percentages and discount

– solve problems involving salaries and wages.

The question was attempted by 95 per cent of the candidates, 13 per cent of whom scored the maximum available mark. The mean mark was 5.06 out of 10.

The majority of the candidates were able to calculate the total cost of books and magazines. They also understood the concept of doubling to obtain the overtime hourly rate. Although many of the candidates were able to calculate the discount, they were unable to follow through to determine the actual discount price. Many of the candidates were unable to calculate the overtime wage of $80 for the week.

Solutions:

(a) (i) $8 (ii) $16 (iii) $440

(b) (i) a) $150

b) $45

(iii) $179.40

Recommendations

- Students should be given more practice in solving word problems in consumer arithmetic.

Question 6

This question tested candidates' ability to:

– translate verbal phrases into algebraic symbols and vice versa
– solve simultaneous linear equations in two unknowns algebraically
– use a linear equation to solve word problems.

The question was attempted by 89 per cent of the candidates, 3 per cent of whom scored the maximum available mark. The mean mark was 1.97 out of 10.

Candidates generally had knowledge of methods used in solving simultaneous equations. However, some candidates failed to subtract correctly after equating the coefficients. A number of candidates obtained the result below.

$$\begin{aligned} x + 2y &= 7 \\ 6x + 2y &= 12 \\ \hline -5x &= 5 \\ &\text{or} \\ 5x &= -5 \end{aligned}$$

After the candidates found the value for one variable, they incorrectly substituted it for the other variable, for example, after obtaining $x = 1$, some mistakenly used this value to substitute for y instead of x in the equation.

In part (b), candidates interpreted the statement "more than" as the inequality symbol and wrote $p > 36$ instead of $p + 36$. In simplifying algebraic expressions, some common errors were: Equating $p + 36$ with $36p$ or with $36p^2$. Forming an equation from the given information also posed problems for candidates.

Solutions:

(a) **$x = 1, y = 3$**

(b) **(i)** **a)** **$p + 36$** **b)** **$2p + 36$**

(ii) **$p = 9$**

Recommendations

- The simplification of algebraic expressions needs to be practised by students, as well as the solution of simultaneous equations.

Question 7

This question tested candidates' ability to:

– use instruments to draw and measure angles and line segments
– use instruments to construct triangles
– use Pythagoras' Theorem to solve simple problems
– use trigonometric ratios in the solution of right-angled triangles.

The question was attempted by 79 per cent of the candidates, 1 per cent of whom scored the maximum available mark. The mean mark was 2.19 out of 10.

The responses to this question were fair. However, a number of candidates had difficulty drawing line segments and measuring angles accurately. Some candidates summed all three sides of the triangle to determine the angle BAC.

The majority of candidates had some knowledge of Pythagoras' Theorem and a number of candidates wrote XY = 13cm without showing any working. This may be as a result of previous knowledge of Pythagorean Triples.

The trigonometric ratios were not well understood by many candidates. A significant number did not attempt this part of the question.

Solutions:

(a) (ii) 57°

(b) (i) 13 cm (ii) 9.96 cm

Recommendations

- Students must develop the ability to construct polygons using geometrical tools and to measure angles.
- Students need more practice in the applications of Pythagoras' Theorem and the trigonometric ratios.

Question 8

This question tested candidates' ability to:

– interpret graphical data
– recognize the gradient of a line given its equation
– determine the equation of a line given the gradient and one point on the line.

The question was attempted by 93 per cent of the candidates, 0.25 per cent of whom scored the maximum available mark. The mean mark was 2.19 out of 10.

In (a) (i) quite a few of the candidates were able to obtain the 35 seconds.

In b (ii) many candidates were familiar with the equation of a straight line y = mx + c. However, they were unable to make the appropriate substitution.

Most candidates were unable to interpret and calculate the distance in a(ii) and a few candidates stated the time rather than the distance

Most candidates were unable to obtain 12.5 seconds for (a) (iii) but instead stated 10, 10.5 and 15 seconds. Some candidates incorrectly calculated speed as $\frac{time}{distance}$ while others stated their speed as 200 metres in 8 seconds.

In part (b), many of the candidates were unable to identify the gradient as the coefficient of x in the equation y = 2x – 1. They also failed to substitute the given point in the equation to determine the value of c, thus stating the equation of the line incorrectly.

Solutions:

(a) **(i)** **35 secs** **(iv)** **5 secs (Chris fell)**

(ii) **75 m** **(v)** **8 m s $^{-1}$**

(iii) **12.5 secs**

(b) **(i)** **2** **(ii)** **y = 2x + 3**

Recommendations

- Students need to be exposed to more problems involving graphs.
- Teachers need to emphasize that the coefficient of x is always the gradient of a straight line in the equation y = mx + c.
- Students need more practice in determining the equation of parallel and perpendicular lines.

Question 9

This question tested candidates' ability to:

– describe a translation using column vectors

– perform an enlargement given the centre and scale factor

– state the relationship between the area of an object and an image under an enlargement.

The question was attempted by 76 per cent of the candidates, 1 per cent of whom scored the maximum available mark. The mean mark was 1.54 out of 10.

The performance in this question was generally poor. Most of the candidates were able to identify the coordinate of the point K (2, 2) but were unable to determine the column vector $\begin{pmatrix} 2 \\ -6 \end{pmatrix}$ for the translation.

The majority of the candidates were unable to draw the correct image of the triangle KLM after the enlargement.

Although many candidates correctly stated the formula for the area of a triangle, they were unable to calculate the area of the triangle drawn.

Solutions:

(a) **K (2, 2), K′(4, -4)**

(b) $\begin{pmatrix} \mathbf{2} \\ \mathbf{-6} \end{pmatrix}$

(c) **K′′ (4,4), L′′(4,10), M′′(8,4)**

(d) **12 square units**

Recommendations

- Teachers must ensure that students know the difference between column vectors and coordinates.

- Enlargements and the other transformations should be taught using both graph paper and paper with no grid lines.

Question 10

This question tested candidates' ability to:

– construct a simple frequency table for a given set of data

– interpret data from a pie-chart

– determine the median of a set of data presented in a frequency table

– determine the probability of an event from data presented in frequency table.

The question was attempted by 88 per cent of the candidates, 1 per cent of whom scored the maximum available mark. The mean mark was 3.39 out of 10.

The majority of candidates were able to recognize that 90 out of 360 was $\frac{1}{4}$ to determine the number of candidates whose favourite music was j …………………………………..azz. Completion of the frequency table was also well done. However, many candidates could not determine the median value from the table or calculate the probability.
The majority of the candidates also had difficulty converting degrees from the pie chart into percentages.

Solution:

(a) **(i)** **15 students** **(ii)** **30%**

(b) **(i)**

Height	**Freq**
6	**4**
7	**3**
8	**1**

(ii) **a)** **6 cm** **b)** $\frac{1}{4}$

Recommendations

- Measures of central tendency (mean, mode, median) need to be clearly defined and sufficient examples provided so that students can differentiate between them.

- Students should be exposed to more experimental probability by conducting simple experiments in the classroom.