C A R I B B E A N   E X A M I N A T I O N S   C O U N C I L

REPORT ON CANDIDATES' WORK IN THE CARIBBEAN SECONDARY EDUCATION CERTIFICATE EXAMINATION®

MAY/JUNE 2012

MATHEMATICS
GENERAL PROFICIENCY EXAMINATION

GENERAL COMMENTS

The General Proficiency Mathematics examination is offered in January and May/June each year. There was a candidate entry of approximately 95,000 in May/June 2012. Thirty-three per cent of the candidates earned Grades I–III. The mean score for the examination was 66.40 out of 180 marks.

DETAILED COMMENTS

Paper 01 — Multiple Choice

Paper 01 consisted of 60 multiple choice items. This year, 298 candidates each earned the maximum available score of 60 marks in the paper. The mean mark for this paper was 32.70 out of 60 marks.

Paper 02 — Structured Questions

Paper 02 comprised two sections. Section I consisted of eight compulsory questions for a total of 90 marks. Section II consisted of three optional questions: one each from Algebra, Relation, Functions and Graphs; Measurement, Trigonometry and Geometry; and Vectors and Matrices. Candidates were required to answer any two out of three questions from this section. Each question in this section was worth 15 marks.

This year, 13 candidates earned the maximum available score of 120 marks on the paper. Furthermore, based on the data collected, candidates were able to secure full marks in every question on the paper.

The mean mark for this paper was 33.64 out of 120 marks.

Compulsory Section

<u>Question 1</u>

This question tested candidates’ ability to

- subtract and divide fractions involving mixed numbers
- express a fraction in its lowest terms
- determine percentage profit or loss given cost price and selling price
- determine cost price given selling price and percentage profit
- perform currency conversions using given rates
- solve problems involving currency conversions.

The question was attempted by 99.4 per cent of the candidates, 3.1 per cent of whom earned the maximum available mark. The mean mark was 4.60 out of 12.

In general, candidates provided fair responses but there were far too many instances of poor responses in all the areas tested. In Part (a), the majority of candidates knew that they had to calculate the Lowest Common Multiple (LCM) of the denominators but a significant number of candidates made careless errors obtaining the equivalent fractions. The following errors were noted:

- In converting the mixed number $2\frac{4}{5}$ to an improper fraction, some candidates wrote $\frac{24}{5}$ instead of $\frac{14}{5}$
- After simplifying the numerator and the denominator, far too many candidates divided their terms in the reverse order, using the numerator as the divisor instead of the denominator.

In addition, candidates did not state the fraction in lowest terms and some candidates incorrectly wrote $\frac{19}{21} = 1\frac{2}{21}$

For Part (b), the majority of candidates were able to determine the percentage loss as a percentage of the cost price but few were able to reverse this process and determine the cost price given the selling price and percentage profit. Some common errors in Part (i) were:

- Stating the loss instead of the percentage loss, a common error being \$55 – \$44 = 11% instead of \$11
- Computing the percentage loss using the selling price rather than the cost price.

In Part (ii), candidates merely subtracted the 25% from \$100 and obtained \$75 as the cost price. This was an extremely popular incorrect response, demonstrating that candidates did not know how to proceed to solve problems on percentages when the whole was not given.

For Part (c), candidates experienced great difficulty in setting up the required proportions, others made careless errors in simplifying decimal products and obtained unreasonable answers to the various parts.

Solutions

(a) $\frac{19}{21}$ (b) (i) 20% (ii) \$80

(c) (i) TT\$2.50 (ii) EC\$216 (iii) US\$96

Recommendations

When solving fraction problems involving mixed operations, teachers should encourage students to present their work in logical steps, simplifying each step as they proceed towards the solution. The use of the calculator in performing decimal computations should be emphasized. Students must also be taught estimation skills so that they can determine the reasonableness of their answers.

<u>Question 2</u>

This question tested candidates' ability to

- factorize algebraic expressions involving common factors, the difference of two squares and grouping
- solve a linear equation involving fractions
- solve a pair of simultaneous linear equations.

The question was attempted by 97.9 per cent of the candidates, 3.7 per cent of whom earned the maximum available mark. The mean mark was 3.79 out of 12.

Generally, performance on this question was weak, with candidates only demonstrating proficiency in a few of the basic concepts in algebra that were tested. Candidate's inability to perform simple operations with directed numbers was a general weakness throughout.

In Part (a), the inability to recognize the Highest Common Factor (HCF) in an algebraic expression was also a clear weakness. Even though candidates were able to find a common factor they were unable to determine the HCF. Candidates were unable to recognize the difference of two squares and some actually commented that the expression could not be factorized. Candidates also had difficulty in factoring the second group and ignored the negative signs, that is, obtaining $y(x-2y)$ instead of $-y(x+2y)$.

For Part (c), a variety of methods was used to solve the simultaneous equations including the use of Cramer's Rule. However, candidates experienced difficulty in identifying the correct operation (addition or subtraction) to eliminate one of the variables.

Solutions

(a) (i) $2x^2y(x + 3y)$ (ii) $(3x - 2)(3x + 2)$ (iii) $(4x - y(x + 2y)$

(b) $x = 9$ (c) $x = 4$, $y = 1$

Recommendations

Students must be taught to recognize the various types of algebraic expressions so that they can select the correct strategy for factorization. When using common factors they must also be able to recognize when an expression has been completely factorized. A consistent method of clearing fractions in an equation must be taught and these methods must be connected to arithmetical operations with fractions. Errors can also be avoided if students are taught to verify their solutions when solving simultaneous equations.

Question 3

This question tested candidates' ability to

- use a Venn diagram to solve practical problems involving two sets
- determine the number of elements in subsets involving two intersecting sets
- solve problems involving Venn diagrams using algebraic methods
- represent points on a plane given bearings
- solve problems involving bearings using Pythagoras' theorem and trigonometric ratios

The question was attempted by 97.6 per cent of the candidates, 4.3 per cent of whom earned the maximum available mark. The mean mark was 4.63 out of 12.

Performance was generally unsatisfactory; however, a significant number of candidates scored higher in Part (b) than in Part (a).

In Part (a), only a small percentage of candidates was able to obtain $(30 - 9x)$ for the number of students who play only tennis. Many candidates incorrectly obtained $y = 30 - x$ as the answer.

Candidates also had difficulty relating parts of a set to the whole set. In many instances, the expression given did not match the information offered by the candidates in their Venn diagram. In such cases, incorrect equations resulted in unrealistic solutions such as fractions and the candidates were unable to deduce that their answers were incorrect.

In Part (b), the majority of candidates produced correctly labelled diagrams showing the positions of Q and R. Although candidates generally recognized the use of Pythagoras' theorem to solve the triangle, they had problems applying it to the problem.

Candidates were also unable to apply the correct trigonometrical ratio for finding an angle. Many candidates made serious omissions in presenting their work, such as $tan = \frac{15}{20}$ and $\frac{20}{15}$. Some candidates applied the sine and cosine rules but very few arrived at a correct answer using these methods.

A significant number of candidates attempted to find the angle QRP instead of QPR, indicating that basic skills in naming angles are missing.

Solution

(a) (i) $y = 30 - 9x, \quad z = 4$ (ii) a) $x + 34$ b) $x = 2$

(b) (i) (ii) 25 km (iii) 53^0

Recommendations

In interpreting and constructing Venn diagrams, teachers need to ensure that students understand the requisite vocabulary. For example, they need to know how to interpret the word *only* when using Venn diagrams. When responding to questions in solving for algebraic unknowns, they must be guided to differentiate between an expression and an equation.

In solving right–angled triangles, candidates must ensure that their diagrams are labelled correctly with the known values clearly shown. This will facilitate problem solving and identification of the correct ratio to be used.

Question 4

This question tested candidates' ability to calculate

- the length of an arc of a circle
- the perimeter of a sector of a circle
- the area of a sector of a circle
- the volume of a prism using the area of the cross section
- the mass of a prism given its density.

The question was attempted by 81.4 per cent of the candidates, 2.3 per cent of whom earned the maximum available mark. The mean mark was 2.45 out of 10.

Responses to all parts of this question were generally poor, indicating that basic concepts in measurement are generally not understood by many candidates.

In Part (a), candidates had more difficulty calculating the arc length and perimeter of the sector than calculating the area. They were able to choose the appropriate formula for the area and perimeter but the majority used 90^0 instead of 270^0 when substituting for the angle subtended. In particular, candidates displayed poor understanding of the concept of perimeter and many did not include the two radii as part of the total perimeter. In substituting for area, some candidates interpreted πr^2 as $(\pi r)^2$.

Candidates failed to make a connection between Part (a) and Part (b) and did not recognize that the area of the sector was in fact the cross–sectional area of the tin. Hence, some candidates recalculated the area. They also experienced problems substituting in the formula $V = Ah$ while weaker candidates attempted to use $l \times w \times h$.

Candidates generally disregarded the units that should have been used in the various parts of the questions.

Solutions

(a) (i) 16.5 cm (ii) 23.5 cm (iii) 28.875 cm^2

(b) (i) 577.5 cm^3 (ii) 4216 kg

Recommendations

The poor performance on this topic suggests that teachers should revisit the teaching of basic concepts in measurement rather than focus on the use of formulae. Measurement language and vocabulary need to be addressed, in particular the meaning of cross section, height and base as they relate to solids. Candidates need to understand how to calculate the perimeter, area and volume of regular as well as irregular shapes. Close attention must be also paid to the units for measuring these attributes.

<u>Question 5</u>

This question tested candidates' ability to

- use mathematical instruments to construct a triangle given two angles and a corresponding side
- measure the length of a line segment
- given two points, determine
    - the gradient of the line
    - the equation of the line
    - the midpoint of the line
    - the length of the line.

The question was attempted by 84.1 per cent of the candidates, 2.1 per cent of whom earned the maximum available mark. The mean mark was 3.33 out of 12.

The performance of candidates on this question was generally unsatisfactory. In Part (a), many candidates did not use the required instruments as no construction lines were shown. Constructing the 45^0 angle posed greater difficulty for candidates than constructing the 60^0 angle. A significant number of candidates labelled their diagram incorrectly and thus failed to measure the required line.

In Part (b), many candidates were successful in calculating the gradient of the line. Candidates wrote the correct formulae for the equation and length of a line but made careless errors in substituting and simplifying the values. Determining the equation of a line proved to be most challenging for the majority of candidates. After calculating the gradient and the y-intercept, many candidates could not write down the equation of the line. A large number of candidates plotted the given points on a graph and drew a straight line but they were unable to use the diagram to answer any of the questions.

Solutions

(a) (ii) $RQ = 5.9 \pm 0.1$ cm

(b) (i) $\frac{4}{3}$ (ii) $y = \frac{4}{3}x - 2$ (iii) (3, 2) (iv) 10

Recommendations

Teachers must encourage students to make a sketch of the diagram, prior to constructing. This will allow them to review the labelling and the given information before creating an accurate diagram.

Concepts in coordinate geometry should be well grounded through the use of drawings prior to developing the formulae. The teaching of algebraic techniques such as simplification and substitution should precede topics in coordinate geometry.

Question 6

This question tested candidates' ability to

- locate the centre of enlargement given an object and its image
- state the scale factor and coordinates of the centre of enlargement
- determine the ratio of the area of the image to the area of the object
- draw a triangle given its coordinates
- describe fully the transformation which maps a given triangle onto its image.

The question was attempted by 80.8 per cent of the candidates, 1.1 per cent of whom earned the maximum available mark. The mean mark was 2.56 out of 11.

Overall, performance on this question was poor, with many candidates displaying limited knowledge of geometric transformations.

The areas of bad performance were in locating the centre of enlargement and in describing the single transformation that mapped triangle LMN onto triangle ABC.

Candidates showed little knowledge of the method to find the centre of enlargement. A large number connected the corresponding vertices of LMN and PQR but failed to extend the lines to a point of convergence. In many cases, they connected all vertices to the origin and stated that as the centre of enlargement.

Most candidates were able to correctly recognize the scale factor as 2 but it was often written as a ratio of 1:2 or as a column vector. Weaker candidates interpreted scale factor as the scale used on the axes on the graph. Also, the centre of enlargement was incorrectly written as (5,1) instead of (1,5) and, in a few cases, as a column vector.

Candidates showed limited knowledge of the relationship between the scale factor and the ratio of the areas, that is, k^2. Many took pains to calculate the areas using lengthy methods and wasted valuable time. Some candidates were able to accurately plot the points A, B and C but there were several cases where the points were incorrectly labelled.

Part (e) of the question was very poorly done. Candidates frequently stated two consecutive transformations that were often incorrect. Many of those who recognized the rotation had difficulties in describing it. They seemed unable to distinguish clockwise from anticlockwise and in many cases if the correct angle of 90^0 was written, the centre of rotation was omitted. A common incorrect answer was that the single transformation was an enlargement as candidates confused Part (a) with Part (e).

Solutions

(b) Scale factor = 2, centre (1, 5)

(c) 4

(e) A rotation of 90^0 in an anticlockwise direction with centre (0,0).

Recommendations

The topic of transformations appears not to have been extensively taught; students were very unfamiliar with the concept. Teachers need to emphasize the characteristics that define each transformation as well as the properties of each one. The specific vocabulary associated with each transformation needs to be emphasized.

<u>Question 7</u>

This question tested candidates' ability to

- calculate cumulative frequencies from a frequency table
- draw a cumulative frequency curve (ogive) for a set of data
- use a cumulative frequency curve to estimate the median
- use a cumulative frequency curve to determine the probability of an event.

The question was attempted by 89.6 per cent of the candidates, 3.2 per cent of whom earned the maximum available mark. The mean mark was 4.19 out of 11.

The majority of candidates were competent in completing the cumulative frequency table and using the correct scales. However, a significant number of candidates did not draw the cumulative frequency curve using the cumulative frequencies but plotted their frequencies instead. Also, some candidates plotted their cumulative curve using the lower limits and midpoints instead of the upper–class boundaries. Some even interchanged the horizontal and vertical axes.

A significant number of candidates were unable to use their cumulative frequency curve to estimate the median or the number of persons 75 years or younger who visited the clinic. Often, candidates who drew the lines on the graph failed to state the determined values. Candidates were unable to calculate the probability correctly. While some read their graph correctly, they simply left their answers as whole numbers, displaying no knowledge of the concept of probability. Some candidates incorrectly used the sum of the cumulative frequencies as the total number of persons in the sample.

Solutions

(a) 35 and 47 (c) (i) 64 years (ii) $\frac{43}{50}$

Recommendations

Teachers must ensure that students construct and interpret cumulative frequency curves from real-world data so that they can develop a sound understanding and appreciation for the statistical concepts they will encounter. In representing statistical graphs, care must be taken to explain what variables are to be plotted when setting up the axes for different types of graphs.

<u>Question 8</u>

This question tested candidates' ability to

- recognize a spatial pattern in a given sequence of drawings
- continue a pattern by drawing a given shape in the sequence
- use a pattern to generate subsequent terms in a number sequence
- use a pattern to derive a rule for the n^{th} term in a number sequence.

The question was attempted by 93.2 per cent of the candidates, 3.2 per cent of whom earned the maximum available mark. The mean mark was 6.30 out of 10.

In general, candidates' responses ranged from satisfactory to quite good. Those who had difficulty could not interpret the table but ignored the breaks in the columns and assumed that it was continuous. Hence, they treated Part b (ii), as though it were Figure 5, continuing from Figure 4.

In Part (a), almost all of the candidates were able to complete a fourth figure in the sequence. However, in Part (b), some candidates had difficulty recognizing the pattern of square numbers and multiplied by two instead. Generating the n^{th} term for the sequence posed most challenges for candidates and many used numbers instead.

Solutions

(b) (i) 16, 2(4) + 1 = 9 (ii) 10, 2(10) + 1 = 21

(iii) 400, 2(20) + 1 = 41 (iv) n^2, $2n + 1$

Recommendations

Teachers should continue to engage students in activities involving pattern recognition and continuation. Students should be encouraged to generate their own patterns using concrete objects or abstractly. The use of algebra as a tool to describe and generalize a rule should be emphasized.

Question 9

This question tested candidates' ability to

- solve a pair of simultaneous equations in which one is linear and the other is a quadratic
- deduce whether or not a given line is a tangent to a curve
- construct inequalities for given conditions
- determine the region satisfied by a system of three inequalities
- state the coordinates of the vertices of the region satisfied by inequalities
- determine the maximum profit.

The question was attempted by 45.7 per cent of the candidates, 1.0 per cent of whom earned the maximum available mark. The mean mark was 2.72 out of 15.

Generally, candidates' responses to this question were unsatisfactory. While many candidates knew that they had to eliminate one variable in solving the simultaneous equations and proceeded to do so, weak algebraic techniques prevented them from arriving at correct solutions. Simplifying the quadratic equation posed problems for the weaker candidates and quite a large number of them were unable to solve the equation. The majority of candidates were able to state that the line $y = 8 - x$ is a tangent to the curve but only a small minority was able to justify why this was so.

Part (b) had slightly better responses than Part (a). Writing the inequalities was the most difficult part of the question for candidates but they were successful in identifying the common region. Naming the vertices of the region and using the profit equation to determine the maximum profit was attempted by the more able candidates. However, there were far too many errors in reading the coordinates of the vertices of the region and some candidates stated the maximum without showing evidence of substitution of the other vertices in the profit equation.

Solutions

(a) (i) $x = -4, -4 \quad y = 12$

(ii) The line is a tangent to the curve because it touches the curve at one point only.

(b) (i) $y \geq \frac{1}{2}x,\ x \geq 2,$ (iii) (2, 1) (2, 10) (8, 4) (iv) $46

Recommendations

Translating worded phrases into inequalities needs to receive more attention. Students must recognize the difference between a worded expression that represents an equation and one that represents an inequality. They should be exposed to performing translations in both directions, words to symbols and symbols to words. In addition, they should also be taught how to verify their algebraic expressions by substituting numbers, for example, if *x* is at least 8, then *x* must be 8, 9, 10, and so on.

Question 10

This question tested candidates' ability to

- solve for unknown values in a triangle using the sine and cosine rule
- solve geometric problems using properties of
    - lines and angles
    - circles and circle theorems
    - congruent triangles
    - isosceles triangles.

The question was attempted by 47.1 per cent of the candidates, 1.3 per cent of whom earned the maximum available mark. The mean mark was 2.72 out of 15.

Performance on this question was generally unsatisfactory.

In Part (a), a, large number of candidates recognized the use of the sine and cosine rules but experienced difficulty applying them. For example, candidates made incorrect substitutions of values in these formulae and could not apply basic techniques to solve for the unknown. A common error in using the cosine rule is illustrated below:

$49 = 64 + 100 - 160 cosQ$

$49 = 164 - 160 cosQ$

After this step, candidates wrote

$49 = 4\ cosQ$ instead of $160 cosQ = 164 - 49$

For Part (b), many candidates experienced difficulty obtaining OUZ, failing to recognize the properties of isosceles triangles. Circle theorems were also hardly recognized and some candidates used rather lengthy procedures to determine the measure of the unknown angles. Many candidates wrote answers only, omitting working or explanations to support their answers.

Recommendations

In preparing students to answer optional questions, teachers must ensure that they have the necessary foundation skills. In these questions the students must have mastered basic algebraic skills and basic concepts in geometry. In addition, students must be encouraged to give explanations, making reference to geometric theorems, when solving problems involving angle calculations.

Solutions

(a) (i) 6 cm (ii) 44^0

(b) (i) a) 35^0 b) 55^0 c) 20^0 (ii) a) ΔYOX b) $\Delta\ ZUX$

Question 11

This question tested candidates' ability to

- combine vectors to determine resultants
- state the geometrical relationship between two vectors
- sketch the relative position of given points
- solve for an unknown variable in a matrix equation
- determine the inverse of a 2 x 2 matrix
- use a matrix method to solve simultaneous equations.

The question was attempted by 53.1 per cent of the candidates, 1.7 per cent of whom earned the maximum available mark. The mean mark was 3.28 out of 15.

Although this question was the most popular optional question, the responses were not encouraging.

In Part (a), candidates were able to add the vectors but experienced great difficulty describing the geometrical relationship and representing the points to show their relative positions.

In Part (b) (i), candidates experienced difficulty with matrix multiplication and solving matrix equations. They were more successful in finding the determinant of the matrix in Part (ii) but this was done separately as they failed to see the connection between Parts (i) and (ii). Only the more able candidates were able to use matrix methods to solve the simultaneous equations.

Solutions

(a) (i) a) $\begin{pmatrix} 3 \\ 2 \end{pmatrix}$ b) $\begin{pmatrix} 9 \\ -6 \end{pmatrix}$ (ii) BC = 3 BA

(b) (i) $a = 3,\ b = -2$ (ii) $-\frac{1}{2}\begin{pmatrix} -3 & 4 \\ -1 & 2 \end{pmatrix}$ (iii) $x = 4,\ y = -1$

Recommendations

Teachers must allow students to explore the geometry of vectors before introducing the matrix algebra. They must be able to identify routes for vectors using visual props and then proceed to writing equations. In solving equations using matrix methods, students must be encouraged to present their solution in stages so that full working is shown at each stage. In this way they can review their solutions and avoid carrying over computational errors.