CARIBBEAN EXAMINATIONS COUNCIL

REPORT ON CANDIDATES' WORK IN THE CARIBBEAN ADVANCED PROFICIENCY EXAMINATION

MAY/JUNE 2004

MATHEMATICS

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CARIBBEAN ADVANCED PROFICIENCY EXAMINATION MAY/JUNE 2004

INTRODUCTION

This is the sixth year that Mathematics Unit 1 was examined on open syllabus and the fifth year for Unit 2. Fifteen hundred and eighty-two candidates registered for Unit 1 examinations, and six hundred and eighty registered for Unit 2.

Each Unit comprised three papers, Paper 01, Paper 02 and Paper 03. Papers 01 and 02 were assessed externally and Paper 03 was assessed internally by the teachers and moderated by CXC.

Paper 01 in each Unit consisted of 15 compulsory short-answer questions. There were five questions in each of three sections, Sections 1, 2 and 3 corresponding to Modules 1, 2 and 3 respectively. Each question carried marks in the range of 4 – 8. Candidates could earn a maximum of 90 marks for this paper in each Unit, representing 30 per cent of the assessment for the respective Unit.

Paper 02 in each Unit consisted of six compulsory extended response questions. There were two questions in each Section/Module. Each question was worth 25 marks. Candidates could obtain a maximum of 150 marks on Paper 02, representing 50 per cent of the assessment for the Unit. Marks were awarded for Reasoning, Method and Accuracy.

Paper 03 was compulsory. It was assessed internally by the teacher and moderated by CXC. For Unit 1, candidates wrote three tests, one for each Section/Module. For Unit 2, candidates wrote one test for each of Modules 1 and 2, and were required to submit a project based on any aspect of the syllabus. This paper represented 20 per cent of the assessment for the Unit. This is the second year that Paper 03/2 was written by private candidates for Unit I and the first year for Unit 2.

GENERAL COMMENTS

UNIT 1

The performance of candidates showed improvement over previous years and in particular, there were several very good scores in Paper 02. Questions on such topics as Curve-sketching, Remainder Theorem, Sectors and Basic Calculus were very well done. However, there is still room for improvement in dealing with indices, limits, aspects of Coordinate Geometry and Trigonometry, particularly in obtaining solutions to equations involving the basic trigonometric ratios. General algebraic manipulation also requires close attention and improvement in order to complement the progress gained by the candidates in assimilating the formal content of any topic. By and large, the candidates seemed well prepared this year for the experience.

DETAILED COMMENTS

UNIT I

PAPER 01
SECTION A
(Module 1.1: Basic Algebra and Functions)

Question 1

Specific Objective (s): (c) 4, 5; (f) 2 (i)

This question tested the use of the Remainder Theorem in finding the possible values of the constant p.

Most candidates handled the question well but some had difficulties solving the equation $p^2 + p - 2 = 0$ for p. A common error was $(-1)^3 = 1$.

Answers: p = -2 or 1.

Question 2

(a) Specific Objective (s): (a) 3, 4, 5, 7

This part of the question tested basic knowledge of real numbers and the ability of candidates to use simple properties of inequalities as they relate to real numbers. There were very few good solutions to this question. Most candidates substituted specific values of x, y and k to obtain the required answer.

(b) Specific Objective (s): (b) 3; (f) 2 (i)

The question involved the knowledge of the modulus of real numbers and of the quadratic equations.

Most candidates were familiar with both topics, but some answers showed weaknesses in factorization of quadratic equations.

Answers: x = -4, -2, 2, 4.

Question 3

(a) Specific Objective (s): (e)

This part of the question tested knowledge of indices and was very well done.

(b) Specific Objective (s): (e); (f) 2(i); (c) 1, 2

This part of the question required substituting the result of Part (a) above in the given equation and then solving by means of basic knowledge of quadratic equations. Many candidates did not see the relevance of Part (a) to finding a solution and some who did, experienced difficulties in solving the resulting equation. In some instances, candidates showed a lack of understanding of the laws of indices.

Answers: $x = 2$, $x = 0$

Question 4

Specific Objective (s): (d) 1, 3; (f) 1

This question tested the ability of candidates to form the composite of functions and to solve the resulting equations. Several good solutions were submitted, but too many simple algebraic errors were noted, all of which spoiled the general performance on the question.

Answer: $x = -8$

Question 5

Specific Objective (s): (g) (i) 1, 2, 3, 4, 5

There were several attempts on Part (a) of this question. Many candidates obtained the correct answer; quite a few expressed their result in degrees and not radians.

Answer: 1.86 rad

Part (b) was well done by many candidates.

Answer: Approx. 22.1 m

SECTION B
(Module 1.2: Geometry and Trigonometry)

Question 6

Specific Objective (s): (a) 9

This question involved converting the parametric representation of a curve to a Cartesian equation of the curve. There were several attempts at this question with many candidates obtaining about 50 per cent of the marks. The method of substitution presented inordinate difficulties due to weaknesses in the algebraic manipulation of the resulting expressions.

A few candidates spoilt otherwise good responses by simplifying

$\frac{3x^2 - 18x + 27}{4} + 2$ as $\frac{3x^2}{4} - \frac{9x}{2} + \frac{29}{4}$

Some others used the identity $3t^4+2 = A(2t^2+3)^2 + B(2t^2+3) + C$ but were unable to arrive at the correct values of A, B and C thereafter.

Answers: $A = \frac{3}{4}$, $B = \frac{-9}{2}$, $C = \frac{35}{4}$

Question 7

Specific Objective (s): (b)

This question examined linear, fractional and rational inequalities. Most candidates were able to determine critical values associated with the given inequality by employing a variety of methods appropriate for the purpose (for example, graphical, tabular) Some candidates were unable to find the solution after converting the original problem to a more manageable form.

Answers: $x > 2$ or $x < -3$

Question 8

(a) Specific Objective (s): (d) 2, 3

This question involved the use of simple trigonometric identities for cos 2A and sin 2A. Many candidates used inappropriate forms for cos 2A and sin 2A to establish the required identity. This approach made the manipulation of the given expression(s) cumbersome, frequently leading to unnecessary complications. In general, the question was not well done.

(b) Specific Objective (s): (d) 3, 4, 7

The question required the use of the identity $\cos 2\theta = 2\cos^2\theta - 1$ to convert the given equation to a quadratic equation in $\cos\theta$. The solution of quadratic equations is also involved in obtaining the values for θ. Several candidates attempted this question but many displayed weaknesses in obtaining the correct quadratic equation in $\cos\theta$, in solving that equation, and in finding the correct values of θ within the specific range.

Answer(s): $\theta = 0$ or $\frac{\pi}{3}$

<u>Question 9</u>

Specific Objective(s): (e) 1

The theory of the roots of quadratic equations was examined in this question. In attempting this question, candidates made several simple errors in algebraic manipulation. There were too many mistakes made with signs, simplification of terms, and confusion between a quadratic expression and a quadratic equation. Many marks were lost due to carelessness.

Answer: $x^2 - 5x + 3 = 0$ is required equation

<u>Question 10</u>

(a) Specific Objective(s): (f) 5, 7

This part of the question tested the concept of unit vector. Many candidates were successful in obtaining the correct solution, but too often simple errors were made in calculating the modulus of the position vector of the point P.

Answer: $\frac{1}{\sqrt{10}}(\underline{i} + 3\underline{j})$

(b) Specific Objective(s): (f) 4, 8

The position vector of the point Q on $\overrightarrow{OP}$ produced is required here. Many candidates did not know how to use the information $|\overrightarrow{OQ}| = 5$ and this proved to be a hindrance in obtaining the correct answer.

Answer: $\frac{5}{\sqrt{10}}(\underline{i} + 3\underline{j})$

(c) Specific Objective(s): (f) 10

This question tested knowledge of the condition for two vectors to be perpendicular. Several candidates stated the required condition but some failed to obtain the correct value of t because of faulty manipulation.

Answer(s): $t = -4$

SECTION C
(Module 1.3: Calculus 1)

<u>Question 11</u>

Specific Objective(s): 1 – 5

This question examined some of the basic concepts of limits. Several candidates attempted the question and approximately one-third of them obtained full marks. Some candidates lost credit by omitting reference to limits or by errors in expressing $\frac{5}{4} - 4$ as a fraction.

Answer: $-\frac{11}{4}$ (Intermediate result: $\lim_{x \to -2} f(x) = \frac{5}{4}$)

<u>Question 12</u>

Specific Objective(s): (b) 6

This was a standard question on differentiation from first principles. There were several attempts at this question with a high degree of success. Many efforts suffered from errors in simplification.

Answer: $3x^2$

Question 13

Specific Objective(s): (b) 5, 7, 9, 13,14,17,18,19.

This question tested the concept of derivative of a function and conditions for maxima and minima.

Part (a) was quite well none. Both derivatives were found quite successfully by approximately 95 per cent of the candidates.

Several methods including the completion of the square, were used to solve Parts (b) and (c). Some candidates had difficulty completing the square.

Answer(s): (a) (i) $2rx + s$, (ii) $2r$

(b) $x = -\frac{s}{2r}$, $r < 0$

(c) $t - \frac{s^2}{4r}$ or $\frac{4rt - s^2}{4r}$

Question 14

Specific Objective(s): (b) 2

This question examined the candidates' ability to use calculus methods to obtain the equation of a curve from given conditions. Approximately 60 per cent of the candidates responded favorably to this question. From the attempts, the following observations emerged:

(i) Many candidates substituted x = 1 into the equation of the curve but did not substitute y = 2.

(ii) The gradient of the curve was not correctly calculated.

(iii) Some candidates did not equate the value of the gradient (7) when $x = 1$ was substituted.

(iv) Usually, those candidates who formed two equations from the data were able to gain full marks.

Answer(s): p = 10, q = -13

Question 15

Specific Objective(s): (c) 4, 5, 6, 9

The question combined the modulus function with the concept of volume. Very few candidates responded to this question. Some candidates seemed unfamiliar with the modulus function and with the formula $\int y^2 dx$ for volume.

(a) This part was done fairly well by those who attempted the question although answers were frequently not expressed in coordinate form.

(b) Many candidates lost marks because they used the wrong upper limit for the volume. They used 4 instead of 2. About 5 per cent of the candidates noted that the volume generated was a right circular cone with base radius 2 and height 2 units.

Answer(s): (a) A (0, 2), B (2, 0)
(b) $\frac{8\pi}{3}$ units^3

PAPER 02
SECTION A
(Module 1.1: Basic Algebra and Functions)

Question 1

(a) Specific Objective(s): (c) 1, 3 - 6

This question sought to test the candidates' knowledge of the Factor/Remainder theorem as it applies to polynomials. This question was generally well done. Most candidates obtained full marks.

Some candidates did not specify f(1) = 0 and f(2) = 0 thus improperly setting out the relevant equations to solve m and n. It may be useful to emphasize the importance of proper mathematical statements. One unique method of solving for m and n was demonstrated by a candidate who used the fact that since $(x - 1)$ and $(x - 2)$ are factors of $f(x)$ then the remainder for $f(x)$ when divided by $(x^2 - 3x + 2)$ in terms of m and n must be equal to zero. By deduction, the values of m and n were determined. The understanding of the Factor/Remainder Theorem was clearly established.

Answer(s): $m = -7$, $n = 6$; third factor is $x + 3$.

(b) Specific Objective(s): (c) 1, 6

This question tested knowledge of polynomial identities and the methods involved in determining unknown coefficients. Generally, the performance on this question was unsatisfactory. Many candidates did not get past the expansion of the expression on the right-hand side. The lack of algebraic skills was clearly evident. Grouping of terms and comparing coefficients with the left-hand side were beyond the abilities of a significant number of candidates.

Answers: p = 2, q = -3, r = 7

(c) Specific Objective(s): (a) 4, 5; (b) 1, 2, 3; (c) 1.

This part of the question examined the modulus function and inequalities.

In Part (i), a number of candidates showed that $-5 \leq (2x - 3) \leq 5$. However, as commonly seen in the classroom, solution sets included $x \leq 4$ and $x \leq -1$. Some candidates used the approach of squaring both sides. Obtaining $(x + 1)(x - 4) \leq 0$, candidates solved $x \leq 4$ and $x \leq -1$. Some candidates showed the region for the range of values of x on a quadratic graph and thus were able to state the correct solution set.

Parts (ii) and (iii) were correctly given by those candidates who obtained the correct expressions for x.

Answers: (i) $-1 \leq x \leq 4$, (ii) least value = 0, (iii) greatest value = 5

<u>Question 2</u>

(a) Specific Objective(s): (f) 2 (ii), (iii); (a) 6

This part of the question involved completion of the square for a quadratic function and the identification of the relevant coefficients. The maximum value of the function was also determined.

In Part (i), completion of the square continues to be problematic to many candidates. Particularly noticeable was the difficulty experienced when the quadratic expression involved the coefficient of x^2 being less than 0. Some candidates expanded the right-hand side and compared coefficients to determine the values of the constants A, B and C. It was not uncommon to see candidates giving the value of $x = -3$ as the maximum value of the function.

Part (ii) was generally well done. However, it was noted that some candidates plotted a graph from a table of values of *x* as would occur at the CSEC examinations. Many candidates lost marks for not “showing clearly its main features”.

In Part (iii) a significant number of candidates wrote in essay form the reasons for a function not being one-to-one. Some candidates showed f (0) = 0 and f (6) = 0. Very few of them showed that f (*x*) was not one-to-one by using other values of *x*. A considerable number of them used the horizontal line test to show that a common point on the graph corresponded to distinct values of *x*.

Answers: (i) A = 18, B = -2, p = -3; max. pt f(x) = 18
(b) Specific Objective(s): (g) (i) 1, 2, (ii) 1, 2, 3; (a) 6, (d) 5.

This part of the question tested some features of the graphs of the functions sin *x* and | sin *x* | by means of sketches.

Parts (i), (a) and (b) were generally well done.

In Part (ii), very few candidates experienced any difficulty.

In Part (iii), candidates gave solution sets as $x = 0$, $x = \frac{\pi}{2}$, *x* = 2ð. Some candidates gave their answers as 0 ≤ *x* ≤ 2ð. Candidates rarely stated the solution set correctly.

Answers: (iii) Solution set is {0 ≤ *x* ≤ ð} ∪ {2ð}

SECTION B
(Module 1.2: Geometry and Trigonometry)

<u>Question 3</u>

(a) Specific Objective(s): (a) 2; 5 (i), (ii)

This part of the question covered the topic of co-ordinate geometry as it applies to coordinates of points, perpendicularity of lines, gradients and points of intersection of lines. This part of the question was very popular with the candidates and there were many good answers. However, there were a few wrong answers due to faulty manipulation of correct equations.

Answer(s): (i) $2y = 3x – 4$, (ii) $3y + 2x = 7$, (iii) Q (2, 1)

(b) Specific Objective(s): (d) 2, 7

This question tested the ability of the candidates to solve a quadratic equation involving trigonometric functions and to find solutions within a given range. There were many excellent responses to this question. Most candidates were able to do this question completely. Faulty factorization of the quadratic equation was the main cause of errors in the solution.

Answer(s): è = 41.8°, 138.2°

(c) Specific Objective(s): (d) 5, 7

This question focused on the solution of trigonometric functions and equations based on formula for sin A + sin B. In general, candidates seemed unaware of the formula for sin A + sin B in terms of half-angles, $\frac{A+B}{2}$ and $\frac{A-B}{2}$, so that the question was poorly done. Some candidates expanded sin 3*x* in terms of sin *x* as a means of obtaining solutions.

Answer(s): *x* = 0, ð/2, ð

<u>Question 4</u>

(a) Specific Objective(s): (a) 7, (e) 4, 5

This question focused on complex numbers.

In Part (a), the responses were very poor indeed. Insurmountable difficulties arose in separating the real and imaginary parts of the complex number w in terms of x and y.

Answer(s): $w = \frac{(x-1)(x+2)+y^2}{(x+2)^2+y^2} + \frac{3y}{(x+2)^2+y^2}i$

The difficulties encountered in Part (a) adversely affected the progress made in doing Part (b). In some instances, arg w = $\frac{\pi}{4}$ was not properly interpreted.

Answer: $x^2 + y^2 + x - 3y - 2 = 0$

Not many candidates reached Parts (b) (ii) and (iii) due to problems arising in the earlier parts.

Answer(s): (ii) Equation of C is $(x + \frac{1}{2})^2 + (y - \frac{3}{2})^2 = \frac{9}{2}$

(iii) Centre of C = $(-\frac{1}{2}, \frac{3}{2})$, radius of C = $\frac{3}{\sqrt{2}}$

(c) Specific Objective(s): (f) 2, 4, 6, 7, 8

This question concentrated on vectors in the context of a parallelogram. The responses were satisfactory but few in number. Some candidates observed that P is the mid-point of MO or $\overrightarrow{LN}$ and were able to use that insight to calculate $\overrightarrow{MO}$ or $\overrightarrow{LN}$ on the way to obtaining $\overrightarrow{OP}$.

Answer: $\overrightarrow{OP} = -\frac{1}{2}\underline{i} + \frac{9}{2}\underline{j}$

SECTION C (Module 1.3: Calculus 1)

<u>Question 5</u>

(a) Specific Objective(s): (a) 2, 4, 5, 6

This question dealt with limits of a rational function in which both numerator and denominator were quadratic functions of *x* with a common linear factor. This question was attempted by several candidates in a variety of ways. Many tried the straightforward method of factorizing numerator and denominator but often spoiled the analysis with faulty factorization. Use of L'Hôpital's (or L'Hospital's) rule was also evident. Those candidates who substituted $x = 3$ in the original form obtained $\frac{0}{0}$ and then were unable to continue. A few obtained full marks.

Answer: 2

(b) Specific Objective(s): (a) 7, 8

This question was a simple application on continuity. This part was very well done, however, a few of the weaker candidates equated both numerator and denominator to zero, an action that created an unwanted result.

Answer(s): Not continuous at $x = 0$, $x = -1$

(c) Specific Objective(s): (b) 7, 8, 10

This question dealt with the differential calculus, in particular, the formation of a differential equation from a given function.

In Part (i), there were good responses. Basic algebraic errors occurred in the simplifications of expressions involving removal of brackets and collection of similar terms.

A few candidates quoted the quotient rule for $\frac{u}{v}$ incorrectly.

Answer: $\frac{dy}{dx} = \frac{4x}{(x^2 + 1)^2}$

In Part (ii), many candidates did not demonstrate the correct approach to obtaining the result required. They often started with the equation on the question paper rather than by using the result of Part (c)(i) above, forming the expression on the left hand side and proceeding to simplify to the term on the right hand. Some treated it as an equation to be solved for y although y was given at the start, while others expanded $x(x^2 + 1)$ and so lost sight of common terms $x^2 + 1$ in $x(x^2 + 1)$ and $(x^2 + 1)^2$ in $\frac{dy}{dx}$.

(d) Specific Objective(s): (b) 7, 8, 9, 12

This part dealt with increasing functions using the differential calculus. Generally, candidates were able to find $f'(x)$ and solve for *x* the equation $f'(x) = 0$.

Many stopped at that point and diverted to sketching the curve by investigating the factors of $f'(x)$. In some cases, the correct range emerged with careful reasoning. The factorization of $5x^4 – 5$ presented difficulties for many candidates.

Answer: $-1 < x < 1$

<u>Question 6</u>

Specific Objective(s): 13 – 19, 21

This question dealt with curve-sketching using the methods of calculus. Candidates performed capably on Part (a) of this question.

Answer: Stationary pts. are (1, 0) and (-1, 4)

In Part (b), the majority of candidates used the second derivative effectively, however, some continue to find the conditions for maxima and minima confusing. The change of sign was also used but candidates did not always make the correct inference from the change in sign.

Answer: (1, 0) is a minimum pt; (-1, 4) is a maximum pt.

In Part (c), many candidates did not recognize the emphasis placed on the condition that the curve touched the x– axis at $x = 1$. Most were able to show that $x - 1$ was a factor of $f(x)$ but the repeated nature of the factor $x - 1$ and the fact that (1, 0) was a minimum was not recognised, so that the shape of the curve at $x = 1$ was not always correct.

Part (d) of the question evoked many good responses but was spoiled by some of the considerations identified in Part (c).

Although the majority of candidates were able to identify that integration was required in Part (e), few candidates were able to arrive at the correct value. Many errors occurred at the point of substitution while some had difficulty with the limits of integration. Some candidates attempted to use the trapezium rule but were unable to make the correct conclusion.

Answer: $6\frac{3}{4}$ sq. units

PAPER 03
INTERNAL ASSESSMENT

In general, the internally assessed tests were relevant and appropriate to the objectives stated in both Unit 1 and Unit 2 of the CAPE Mathematics Syllabus. In most cases, questions from past CAPE Mathematics examination papers were used for these examinations. There were instances where one examination paper was set to test all three modules and in most of these cases, the scores of candidates were lower than for those candidates tested on individual modules. The range of difficulty in the tests varied significantly. In some cases, the tests were too detailed in content to be conducted over a 1 to 1½ hour duration. In other instances, the tests did not reflect a sufficiently adequate coverage of the syllabus.

Most of the sample tests were submitted with question papers, solutions and detailed mark scheme indicating clearly the distribution of the marks; nevertheless, there were still too many samples submitted which did not contain all of these components. In one extreme case, the copy of the question paper was not submitted which slowed down the moderation process. Such situations must be avoided in the future.

Assessment was generally excellent with a high level of consistency but there were few occasions, where the allocation of marks was difficult to follow. In the future, fractional (or decimal) scores should not be submitted.

PAPER 03/2

SECTION A
(Module 1.1: Basic Algebra and Functions)

<u>Question 1</u>

(a) Specific Objective(s): (a) 1, 2; (c) 1, 2, 4; (f) 2 (i)

The Remainder/Factor Theorem was tested in this part of the question. There were few candidates for this paper, nevertheless, there was evidence of familiarity with the topic.

Answers: $p = 6$ or p -3

(b) Specific Objective(s): (c) 1; (g) 1, 2, 4

The sector and its properties were the focus of this part of the question. The candidates had difficulties in finding the perimeter of the sector and as a consequence could not obtain the expression for è in Part (i). However assuming that è in Part (i) was correct, some were able to find the expression for the area, A, in Part (ii).

Answer: Perimeter = 2r + rè cms

(c) Specific Objective(s): (f)1, 3

Simultaneous equations, one linear and one quadratic, were examined in this part of the question. There were some good attempts but weaknesses in algebra resulted in incorrect solutions.

Answers: $x = 1$, $y = 2$; $x = -¾$, $y = \frac{-13}{4}$

SECTION B
(Module 1.2: Geometry and Trigonometry)

<u>Question 2</u>

(a) Specific Objective(s): (a) 1, 2, 5 (ii), 6

The coordinate geometry of a given line in the Cartesian plane was explored.

In Part (i), some difficulties were experienced in finding the coordinates of p.

With incorrect coordinates for p, the equation of the line in Part (ii) proved difficult to find.

Answer: (i) p = (1, 1)

(b) Specific Objective(s): (d) 3, 4, 7

The question involved finding solutions to equations for trigonometric functions of multiple angles.

Simple errors were made in Part (i). The connection sin 4è = sin 2(2è) was not noticed by some candidates.

There were few correct solutions to Part (ii).

Answers: (i) sin 4θ = 2 sin 2θ cos 2θ

(ii) $\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{\pi}{12}, \frac{5\pi}{12}$

(c) Specific Objective(s): (e) 1, 3

This part required knowledge of the product of the roots of a quadratic equation in terms of its coefficients. There were a few valiant attempts some of which were spoiled by interpreting i^2 as 1 instead of -1.

Answer: c = 5

(d) Specific Objective(s): (f) 1, 2, 4, 6, 7

This exercise involved vectors. Few candidates completed the major part of the solution.

Answer: Position vector of E is $\underset{\sim}{i} - \underset{\sim}{j}$.

SECTION C
(Module 1.3: Calculus 1)

Question 3

(a) Specific Objective(s): (a) 2, 4, 6

The question examined the limit of a rational function in which the numerator and denominator had a common factor. There were good attempts at the solution.

Answer: limit = 8

(b) Specific Objective(s): (b) 7, 8, 9 ,10

Differential of a rational function was tested here. The candidates failed to simplify the integrand by dividing through by t^2 and then proceed by the quotient formula. There were few completed answers.

Answer: $\frac{dE}{dt} = t - \frac{1}{t^3}$

$= \frac{t^4 - 1}{t^3}$

(c) Specific Objective(s): (c) 3, 4, 5, 6

The evaluation of an integral by using a given result is examined in this part. This was done quite satisfactorily, however, some credit was lost for weakness in properly separating the integrand into two parts.

Answer: $\int_0^4 (4x - f(x))\,\mathrm{d}x) = 26$

(d) Specific Objective(s): (b) 11; (c) 2, 3, 4, 6, 10

This question examined rate of change by means of the differential and integral calculus. There were a few attempts but none was completely successful due to faulty algebraic manipulations.

Answer(s) (i) $\dfrac{dp}{dt} = \dfrac{10}{t^2} + t$

(ii) P = $\dfrac{-10}{t} + \dfrac{t^2}{2}$ + k, k is constant of integration

(iii) Change in P is 8½

GENERAL COMMENTS
UNIT 2

The overall performance of candidates in Unit 2 was very good. Some excellent performances were recorded in both papers, a clear indication that both teachers and the candidates are becoming comfortable with the requirements of the syllabus. However, there were candidates taking the examination who were not yet ready for such an in-depth examination process. There were also weaknesses in the ability of candidates to recall relevant information from Unit 1 or even CSEC and this has, on occasion, reduced their capability in completing standard processes for which the major tenets have been started. The analytical skills of some candidates need to be extended and improved considerably in order to maximize their potential at this level.

By and large, topics such as integration and differentiation in Calculus, partial fractions, Newton-Raphson estimation of roots of equations, and probability theory are familiar to most candidates but there is still uncertainty among many in dealing with mathematical induction, mathematical modeling questions leading to A.P and G.P, and general algebraic manipulation. Limits, indices and logarithms, and solutions to equations involving trigonometric functions still continue to present too many difficulties for candidates and require substantial consolidation.

DETAILED COMMENTS

UNIT 2
PAPER 01
SECTION A
(Module 2.1: Calculus II)

Question 1

Specific Objective(s): (a) 3, 8, 9, 10

This question focused on exponential functions.

Most candidates responded favorably to this question and several good answers were submitted, nevertheless, the following weaknesses were noted:

The solution $x = \frac{1}{2} \ln 3$ was obtained but no explicit mention of this as the value of 'a' was made.

Candidates incurred unnecessary work by substituting $x = \frac{1}{2} \ln 3$ into $y = e^{2x}$ to obtain y = 3 or an approximate value (2.98 or 2.99). Some failed to see that b = 3 from the graph.

A few candidates used $\log_{10} 3$ in place of ln 3.

Answer(s): $a = \frac{1}{2} \ln 3$ or 0.549, b = 3

<u>Question 2</u>

Specific Objective(s): (b) 4, 5, 7, 8

This question examined the product rule, implicit differentiation and the chain rule.

The question was done very well. Candidates should be encouraged to simplify their solutions, for example, the expression $\frac{6x}{3x^2}$ to obtain the simpler form $\frac{2}{x}$.

Some weaknesses were observed, for example,

(i) incorrect use of the rules for logs – some candidates wrote $y = \ln(3x^2) \Rightarrow y = 2\ln 3x$, others interpreted $\ln(3x^2)$ as the product $(\ln)(3x^2)$ and this led to a wrong result for $\frac{dy}{dx}$

(ii) a common error in differentiation was $\frac{d}{dx}(\sin^2 x) = \cos^2 x$.

Answers: (a) $\frac{2}{x}$, (b) $2\sin x\cos^2 x - \sin^3 x$

<u>Question 3</u>

Specific Objectives: (a) 2, 3, 4; (b) 4, 5, 6

This question dealt with exponential functions, quadratic equations, product rule and implicit differentiation.

The majority of the candidates obtained full marks for this question. Some excellent solutions were presented. Some of the weaknesses noted were as follows:

(i) There was some confusion in finding $\frac{d}{dx}(2xy)$.

Typical examples are $\frac{d}{dx}(2xy) = 2 + 2y\frac{dy}{dx}$ or $2x + 2y$.

(ii) There were cases of substituting (1, 1) before differentiation.

(iii) Factorization of the quadratic equation $y^2 - 3y - 4 = 0$ presented difficulty to some candidates.

(iv) Some candidates used log 4 instead of ln 4 while others seemed not to be aware that ln (-1) does not exist.

Answers: (a) $\frac{-1}{2}$, (b) $x = \ln 4$ or 1.39

Question 4

Specific Objective(s): (2, 3)

The topics tested were partial fractions and indefinite integrals.

This question was very well done. Several candidates obtained full marks showing excellent preparation. There were a few blemishes to solutions as follows:

(i) Simplification of $\frac{A}{x} + \frac{B}{x+2}$ to give $\frac{Ax + B(x+2)}{x(x+2)}$

(ii) The omission of dx after the integrand and the integral sign $\int$, and the omission of the constant of integration.

Answers: (a) $\frac{1}{2}\left[\frac{1}{x} + \frac{1}{x+2}\right]$, (b) $\frac{1}{2}\ln x(x+2) + k$ (constant).

Question 5

This question focused on integration by parts.

There were several attempts at this question. Overall, the question was well done, however, subtle weaknesses were evident in some of the solutions presented by the candidates.

In Part (i) a few candidates put $u = x^2$, $\frac{dv}{dx} = \ln x$ and this led to awkwardness in proceeding further; some used $u = x^2$, $v = \ln x$ to calculate $\frac{du}{dx} = 2x$, $\frac{dv}{dx} = \frac{1}{x}$ and then could proceed no further.

In Part (ii), the constant of integration was frequently omitted.

Answers: $\frac{1}{3}x^3\ln x - \frac{1}{9}x^3 + k$ (constant)

SECTION B
(Module 2.2: Sequences, Series and Approximations)

Question 6

Specific Objective(s): (C) 1, 2

This question dealt with the binominal expansion/theorem.

It was not well done despite the several attempts. Many candidates had serious difficulties in solving this problem correctly. It was evident from the responses that the preparation for this topic was limited. Candidates did not apply the binomial theorem to determine the term independent of *x*. A common error noted was that candidates did not pay adequate attention to the term $\left(\frac{-1}{2x^2}\right)^3$ and in fact, got their answer as 7654.5 instead of -7654.5. Also noted was that some candidates expressed the term $\left(\frac{-1}{2x^2}\right)^3$ as $(2x^{-2})^3$. Of course, this resulted in the wrong coefficient, again exhibiting the weaknesses of candidates in working with indices.

Answer: $-\binom{7}{2}(3^7)$ or -7654.5

Question 7

Specific Objective(s): (b) 3

The topic examined in this question was arithmetic progressions.

Most candidates identified the common difference as 3. Algebra continues to pose difficulties to candidates. Consequently, many were not able to find the values for *x* and *y* correctly.

Answer: $x = \frac{7}{4}, y = \frac{-5}{2}$

<u>Question 8</u>

Specific Objective(s): (b) 4, 5, 6

Geometric series and sum to infinity were examined in this question.

This question was generally well done. However, the simplification of $\frac{4\left[1-\left(\frac{1}{2}\right)^n\right]}{1-\frac{1}{2}}$ presented difficulties to candidates and in some cases was not pursued beyond this point. A significant number of candidates went on to answer Part (b) using the formula for the sum to infinity, $S_\infty = \frac{a}{1-r}$. Candidates did not demonstrate the use of the deduction $\lim_{n\to\infty}\left(\frac{1}{2}\right)^n \to 0$.

Answer: (a) $8(1 - (½)^n)$, (b) 8

<u>Question 9</u>

Specific Objective(s): (c) 1, 2

The topics examined were binomial expansion and equating of coefficients.

This question was done poorly. The majority of candidates scored no more than three marks. Candidates stated the formula as given on the formula sheet but could not expand in terms of a and n. Consistent with the observations in Question 6, it seems that there was a problem in the preparation of this topic on the binomial theorem. Very few candidates were able to do this question properly.

Answers: $a = \frac{2}{3}$, n = 9

<u>Question 10</u>

Specific Objective(s): (d) 1

The topics tested were errors and approximations.

Generally this question was done satisfactorily. Some candidates experienced difficulty in determining the denominator to calculate the percentage error. Many of them used the estimated measurement instead of the true value.

Answer: 1.73%

SECTION C
(Module 2.3: Probability and Mathematical Modelling)

<u>Question 11</u>

Specific Objective(s): (a) 1, 2, 3

The topics examined were sample space and probability

There were several candidates who attempted this question and obtained some credit for what was presented. Nevertheless, there appear to be gaps in their understanding of some of the fundamental concepts associated with the topic(s). Some of these gaps resulted in the following weaknesses in the respective parts of the question:

(a) Many candidates could not describe the sample space.

(b) (i) Some candidates failed to recognize that the required probability that both balls are of the same color is based on the event set $\{R_1R_2, B_1B_2\}$.

(ii) Many candidates experienced difficulty in setting out the probability that at least one ball is black. Some candidates attempted to use the complementary event giving probability 1 – P(no black) without success while others did not recognize the listing in the set $\{R_1 B_1, R_1 B_2, R_2B_1, R_2B_2, GB_1, GB_2, B_1B_2\}$ as appropriate for this part of the question.

Answer(s): (a) $\{R_1R_2, R_1G, R_1B_1, R_1B_2, R_2G, R_2B_1, R_2B_2, GB_1, GB_2, B_1B_2\}$

(b) $\frac{2}{10}$ or $\frac{1}{5}$

(c) $\frac{7}{10}$

Question 12

Specific Objective(s): (a) 3, 4, 5

In this question, the topic examined was probability.

Most candidates attempted this question. There were few good responses. Some candidates were unable to formulate an algebraic expression linking the probabilities of A, B and C. Others demonstrated very little understanding of probability theory often producing solutions with probabilities greater than 1. Some candidates used a ratio method to determine the solution and those familiar with the law of total probability were able to solve the question successfully.

Answers: P(A) = $\frac{4}{7}$, P(B) = $\frac{2}{7}$, P(C) = $\frac{1}{7}$

Question 13

Specific Objective(s): (a) 5, 6, 7

The topics tested were probability and events.

This question was very well done with several candidates obtaining excellent scores. For the majority of candidates, Part (a) was straightforward. In Part (b), the basic rule of probability was known to a large number of candidates but algebraic errors occurred in many calculations giving rise to incorrect answers.

Part (c) was the most popular part of this question and a variety of methods was employed, including Venn diagrams, to produce correct results. However, a few candidates multiplied the probabilities $\frac{1}{3}$ and $\frac{1}{4}$ to obtain $\frac{1}{12}$ instead of $\frac{1}{3} - \frac{1}{4}$ which was the correct route.

Answers: (a) P (A) = $\frac{1}{3}$, (b) P(B) = $\frac{2}{3}$, (c) P $(A \cap B')$ = $\frac{1}{12}$

Question 14

Specific Objective(s): (a) 3,4, 8, 10

The topics tested were probability – mutually exclusive and independent events.
In Part (a), most candidates were able to identify accurately a mutually exclusive event. As a consequence, this part was very well done.

Several candidates attempted Part (b). Most of them were able to arrive at a correct solution. However, some candidates still had difficulty identifying that for independent events A and B, $P(A \cap B) = P(A) \times P(B)$. Generally, candidates were very familiar with the content of the question.

Answers: (a) (ii), (b) $P(A \cap B) = \frac{1}{10}$

Question 15

Specific Objective(s): (a) 3 – 6, 10, 12, 13; (b) 3

The topics tested were sample space and probability

Part (a) was poorly done. Most candidates did not understand the question and instead of writing down the sample space for the event as {HHM, HMH, MHH}, they tried to calculate the probability for the event occurring. Those candidates who attempted to write down the sample space were successful but a few wrote the outcomes descriptively rather than by symbols, H and M, a weakness which should be corrected for the future.

There were many successful attempts at Part (b). The majority of candidates correctly used the Binomial Distribution to calculate the probability of hitting the target at least once. Few candidates properly defined their random variable x as either a hit or a miss. A few candidates had problems in understanding that $P(x = 1)$ was equivalent to $1 - P(x = 0)$ with x defined as a hit. Some candidates used the tree diagram to calculate the probability although in the process some of the outcomes were omitted.

Answer(s): (a) {HHM, HMH, MHH}, (b) Prob. $= 1 - (0.6)^3 = 0.784$

PAPER 02
SECTION A
(Module 2.1: Calculus II)

Question 1

Specific Objective(s): (a) 9, 10

The topics examined were logarithms, exponential function, equations and graphs. Parts (a) and (b) of this question were well done by most candidates, however, there were a few errors in algebraic manipulation which led to spurious results.

Some of these errors are as follows:

$3 \log_a 3 = \log_a 9$

$\log_3 (x + 6) = \log_3 x + \log_3 6$

$-3 \log_a 3 - \log_a 5 = -\log_a \frac{27}{5}$

Parts (c) and (d) were also well done. A few candidates did not use the given scales to draw the graphs but otherwise there were some very good responses.

Part (e) (i) was well done.

Many candidates tried to do Part (e) (ii) without reference to the graphs despite the instruction "use your graphs….".

Answer(s): (a) a = $\frac{2}{3}$; (b) $x = 3$; (e) (i) $x = 0$, (iii) $x = 2$

Question 2

Specific Objective(s): (b) 3, 4, 5, 7, 8; (c) 4.

The topics tested were differentiation, integration by substitution, and parametric equations.

Part (a) (i) was very well done by many candidates.

In Part (ii), a number of candidates experienced difficulties with the form of the function $\tan^2(x^3)$ and did not seek to use a substitution to simplify this. A common response to this question was $6x^2 \sec^2(x^3)$. Another variant was $6x^2 \tan x \sec^2 x$.

Part (b) was not well done. Steps were often omitted, thus making the logic incomprehensible in many cases. There were also instances where there was not complete replacement of *x* by *u* in the integrand so that the simplicity of the integral in *u* was missed. The constant of integration was very frequently omitted.

The elementary calculus in Part (c) was very well done, however, some candidates seemed to have forgotten the connection between tangent and normal, and so could not obtain the equation of the normal.

Answer(s): (a) (i) $\frac{xe^{x}}{(1+x)^2}$ (ii) $6x^2 \tan(x^3) \sec^2(x^3)$

(b) $e^{\sin x} + k$ (constant)

(c) $\frac{dy}{dx} = \frac{1}{2}$ (2t – 1), equation of normal: $4y + 4x + 3 = 0$

SECTION B
(Module 2.2: Sequences Series and Approximations)

Question 3

Specific Objective(s): (a) 1, 2, 3, 5; (b) 3, 5

The topics tested were sequences, mathematical induction and A.P.

Responses in Part (a) of this question were generally poor and indicated extensive unfamiliarity with many of the basic concepts involved. Weaknesses which could be identified were as follows:

Unfamiliarity with the suffix notation u_{n+1}.

The concepts of convergent, divergent and periodic seemed not to be clearly understood by many candidates.

Many solutions were expressed in terms of u_n and not explicitly as required.

Answer(s): (ii) $\{u_n\} = \{1, 2, 1, 2, ..\}$; $\{a_n\} = \{3, 3, 3, 3, \ldots\}$;
$\{b_n\} = \{1, -1, 1, -1, \ldots\}$

(ii) $\{u_n\}$ is periodic; $\{a_n\}$ is convergent; $\{b_n\}$ is periodic

In Part (b), the induction step was not always set out clearly in the responses, nevertheless, there were some reasonable attempts.

In Part (c), the value of d = -3 (some, d = 3) was obtained by most candidates who attempted this question but then they experienced difficulty in finding the value of *n*. Whatever legitimate value was obtained for *n* was used to compute the sum of the AP.

Answers: d = -3, *n* = 34, sum of AP = 765

<u>Question 4</u>

Specific Objective(s): (e) 1, 2, 3

The topics examined were functions, approximation of roots and binomial expansion.

Generally, Part (a) was well done with candidates securing most of the marks allocated. Most candidates overlooked the concept of continuity in Parts (i), b) and c). A few candidates established Part (i) b) by considering the point of intersection of the two graphs y = x^3 and y = 2 – 2x.

Very few candidates explicitly mentioned the Intermediate Value Theorem but used the concept, nevertheless.

The Newton-Raphson formula for the second approximation in (ii) was sometimes written incorrectly as a $-\left[\frac{f^1(x)}{f(x)}\right]$, a$\left[\frac{f^1(x)}{f(x)}\right]$ or a$\left[\frac{f(x)}{f'(x)}\right]$. A few candidates proceeded beyond the 2^{nd} approximation, quite unnecessarily.

Part (b) was generally well done with full marks being awarded in many cases. Weaknesses were observed in the following instances:

(i) Incorrect expansion of the Binomial Theorem although this was given on the formula sheet

(ii) Inability to collect the like terms together in order to obtain the coefficients of x^2 and x^3

In general, candidates seemed to have had practice in handling problems of this type.

Answers: (a) (ii) 0.818; (b) a = 4, coeff. of x^3 is - 80

SECTION C
(Module 2.3 Probability and Mathematical Modelling)

Question 5

Specific Objective(s): (a) 1 – 6, 8, 12; (b) 3

This question focused on probability.

This question was well done. Most candidates obtained full marks. Those who did not, lost marks for incorrect labeling of the branches of the tree diagram and for stating the relevant probabilities wrongly. Many candidates incorporated Parts (a) and (b) by completing a single tree diagram with the branches correctly labeled and the correct probabilities stated.

Answers to Part (c) (i) and (ii) were generally well done. Candidates used the tree diagram to deduce the correct probabilities rather than to re-calculate the probabilities at each stage.

Answers: (c) (i) $\frac{37}{240}$ (ii) $\frac{53}{144}$

Question 6

Specific Objective(s): (b) 3

This question focused on mathematical modelling

Overall, this question was poorly done.

In Part (a), a significant number of candidates could not obtain the expression for the area of the container despite drawing a diagram of the figure. Also evident was the fact that those candidates who did not obtain the expression for the area of the container did not go on to use the given expression to solve the problem.

A common error in the differentiation was $\frac{d}{dx}\left(\frac{216}{x}\right) = \frac{d}{dx}\left(216x^{-1}\right)$

$$= 216\,x^{-2}$$

$$= \frac{216}{x^2}.$$

This resulted in candidates getting a negative value for *x*. Realizing that this was not valid as a solution, they simply changed the value found to a positive value.

Candidates were completely lost with Part (b) of this question. In fact, most of them were awarded one mark for stating that profit is selling price less cost price. Those candidates who proceeded beyond that point incorrectly subtracted

$\$\left(\frac{1}{2}x^2 + 50x + 50\right)$ from $\$(80 - \frac{1}{4}x)$.

No attention was paid to the cost of making *x* articles per day and the relationship of the selling price of each article. Some students attempted to solve the equation

$\$\left(\frac{1}{2}x^2 + 50 + 50\right) = \$(80 - \frac{1}{4}x$. Much work is still needed in the area of modelling.

Most candidates gained full marks on Part (c) of this question. However, as commonly seen in the classroom, expressing answers in specific terms, for example, significant figures, decimal places or exactly, is either ignored by candidates or perhaps not clearly understood. Despite the instruction, 'to the nearest dollar', in bold print, candidates gave their answers to one and two decimal places.

Few candidates recognized the geometric progression and used this method to calculate the total amount of rent paid.

Answers: (a) (ii) x = 3

(b) (i) $\$(30x - \frac{3x^2}{4} - 50)$

(ii) $x = 20$

(iii) \$250

(c) \$443

PROJECTS

In general, the standard of the project reports was very high and the topics selected were both interesting and relevant to the respective syllabus content.

Candidates exercised good initiative and judgement in reporting their findings. The variety of tasks executed was very encouraging, although, as has been the case in the past, there was a definite bias towards statistical analysis. A few projects also included applications of differential equations.

The assessment criteria appeared to be well understood by the teachers and candidates, and achievements levels were rewarded in a consistent manner, except for a few cases where there was evidence of overgenerous compensation. In such instances, the mathematical model was developed but the data generated were insufficient to ensure the reliability of the model. In a few cases, the time constraint seemed to have militated against the execution of what would otherwise have been interesting conclusions to well-designed project proposals.

PAPER 03/2
SECTION A
(Module 2.1: Calculus II)

Question 1

Specific Objective(s): (a) 2, 5; (b) 3, 4, 8; (c) 6

The topics tested in this question were exponential function, differentiation and integration.

Part (a) of the question was well done. The few candidates who took the examination performed well here.

Good responses were received for Part (b).

Part (c) was not well done. In too many instances, the question was treated as an equation to be solved. Candidates experienced tremendous difficulty in using

$\frac{dy}{dx}$ from Part (a) to obtain the exact result to this question.

The success rate was very low on Part (d). Very often $\sqrt{1+\left(\frac{dy^2}{dx}\right)}$ was equated incorrectly to $1 + \frac{dy}{dx}$.

Answers: (b) $\frac{dy}{dx} = \frac{1}{2}(e^x - e^{-x})$

SECTION B
(Module 2.2: Sequences, Series and Approximations)

<u>Question 2</u>

Specific Objective(s): (a) 1, 2

In this question, the topics tested were sequences, series and mathematical modelling.

In Part (a), almost all the candidates were able to find p_1 and p_2.

In Part (b), few of a very small number of candidates were able to express p_{n+1} in terms of p_n.

Generally, Part (c) was not well done but there was one good response to this question.

Part (d) was done satisfactorily.

Answers: (a) $p_1 = 1100$, $p_2 = 1220$

(b) $p_{n+1} = (1.2)\, p_n - 100$

(d) $n = 17$

Question 3

Specific Objective(s): (a) 1, 2, 3, 4, 7, 10, 12

This question tested probability.

There were some good responses for Parts (a) and (b).

In Part (c), candidates experienced great difficulty in determining the number of favorable cases in this problem. It seemed not to be known that for any number formed from the digits 1, 2, 3, 4, 5 to be divisible by 5 then its last digit must be 5.

Answers: (a) (i) 0.35 (ii) 0.43 (iii) 0.2

(b) (i) $P(B) = \frac{3}{4}$, $P(A \cup B) = \frac{5}{6}$

(c) Prob. $= \frac{1}{5}$

MODEL ANSWERS
UNIT I, PAPER 01

Question 2

(a) Given that $x > y$ and $k < 0$ for the real numbers x, y and k, show that $kx < ky$.

Solution: $x, y \in R$ and $x > y \Rightarrow x - y > 0$

$\Rightarrow k(x - y) < 0$ for $k < 0$

$\Rightarrow kx - ky < 0$

$\Rightarrow kx < ky$

OR $x, y \in R$ and $x > y \Rightarrow y - x < 0$

$\Rightarrow k(y - x) > 0$ for $k < 0$

$\Rightarrow ky - kx > 0$

$\Rightarrow ky > kx$

$\Rightarrow kx < ky$

<u>Question 15 (b)</u>

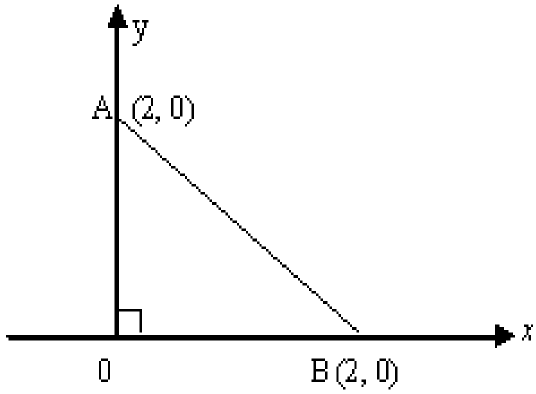
Solution: Reqd. volume = $\int_0^2 \pi y^2 \, dx$

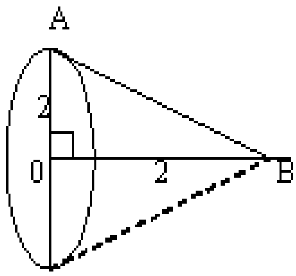
$= \int_0^2 \pi \ |x - 2|^2 \, dx$

$= \int_0^2 \pi \ (x - 2)^2 \, dx$

$= \pi \left. \frac{(x-2)}{3} \right|_0^2$

$= \frac{8\pi}{3}$ units^3





OR Base radius OA = 2 units

Height = 2 units

Reqd volume = volume of cone

$= \frac{1}{3} \pi (2^2) (2)$

$= \frac{8\pi}{3}$ units^3

UNIT II, PAPER 01

Question 9

Solution $(1 + ax)^n = 1 + n a x + \frac{n(n-1)}{2} a^2 x^2 + \ldots\ldots$

$\Rightarrow na = 6, \ \frac{n(n-1)}{2} a^2 = 16$

Now, $\frac{n(n-1)}{2} a^2 = 16$ $\quad \Rightarrow na \left[\left(\frac{n-1}{2}\right) a\right] = 16$

$\Rightarrow 6 \left(\frac{n-1}{2}\right) a = 16$

$\Rightarrow 3(n - 1) a = 16$

$\Rightarrow 3na - 3a = 16$

$\Rightarrow 18 - 3a = 16$

$\Rightarrow a = \frac{2}{3}$

$\Rightarrow n = 6 \times \frac{3}{2} = 9$

UNIT II, PAPER 02

Question 6 (c)

Solution

The rent paid after 15 years is

$$\$\left[64 + 64 \text{ x} \left(\frac{7}{8}\right) + 64 \text{ x} \left(\frac{7}{8}\right)^2 + ...+ 64 \text{ x} \left(\frac{7}{8}\right)^{14}\right]$$

$$= \$\ 64 \left(1 + \frac{7}{8} + \left(\frac{7}{8}\right)^2 + ...+ \left(\frac{7}{8}\right)^{14}\right)$$

$$= \$\ 64 \left(\frac{1 - \frac{7}{8}^{15}}{1 - \frac{7}{8}}\right)$$

$= \$64 \times 8 \times 0.865063$

$= \$443$ to nearest \$.