

Abstract

On M_2 -Polynomials

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The problem of finding a complete invariant for graphs, has been of interest to graph theorists for decades. This thesis investigates the characterizing properties of two F -polynomials, that is the simple M_2 -polynomial, and to a lesser extent, the family-weighted M_2 -polynomial. Several well-known families of graphs, such as paths, cycles, stars, complete graphs and complete bipartite graphs are shown to be characterized by the simple M_2 -polynomial. Establishing characterization of the complete bipartite graph, leads to an interesting extremal result. Furthermore, the problem of characterization is also investigated in a systematic manner. This is done by fixing a particular parameter in the simple M_2 -polynomial. Studies involving certain values of this parameter (close to the extreme values) reveal that many other families of graphs, which include the basic graphs with cyclomatic number 2 (i.e. the theta graph, figure-8 graph and dumbbell), are characterized by the simple M_2 -polynomial. These studies also give some insight into the nature of the problem, and possible methods of handling the general case.

The family-weighted M_2 -polynomial characterizes all the families of graphs, which are characterized by the simple M_2 -polynomial, as well as many others (including wheels). Several methods and reduction formulas, for finding the simple M_2 -polynomial of a graph, are also presented. One such reduction formula is generalized to yield a new reduction formula for the general F -polynomial. Possible extensions of the research are also discussed.

Keywords: Simple Polynomial; Family Polynomial; characterization; identification; cover; component; M_2 -block; minimum configuration; star values; cycle dispensable path.