C A R I B B E A N   E X A M I N A T I O N S   C O U N C I L

REPORT ON CANDIDATES' WORK IN THE ADVANCED PROFICIENCY EXAMINATION

MAY/JUNE 2011

APPLIED MATHEMATICS

INTRODUCTION

The revised Applied Mathematics syllabus was examined this year for the third time. This is a two-unit course comprising four papers. Paper 01, consisting of 45 multiple-choice items, Paper 02, consisting of six essay questions and Paper 032 (written by private candidates),made up of three questions, were examined externally while Paper 03 was examined internally by class teachers and moderated by CXC. Contributions from Papers 01, 02 and 032 to each Unit were 30 per cent, 50 per cent and 20 per cent respectively.

Unit 1: Statistical Analysis, tested (i) Collecting and Describing Data; (ii) Managing Uncertainty and (iii) Analysing and Interpreting Data. Unit 2: Mathematical Applications, tested (i) Discrete Mathematics, (ii) Probability and Probability Distributions, and (iii) Particle Mechanics.

GENERAL COMMENTS

In 2011, 580 candidates wrote the Unit 1 examination with and 11 wrote the Alternative to the School-Based paper, Paper 032. For Unit 2, 217 candidates wrote the examination three candidates wrote, Paper 032.

In Unit 1, candidates appeared to be well prepared in Describing and Collecting Data and somewhat in Managing Uncertainty, while Analysing and Interpreting Data seemed to be a challenge for many of them.

In Unit 2, candidates appeared to be well prepared in Discrete Mathematics and Probability and Probability Distributions. Particle Mechanics again seemed to be a challenge for many candidates.

Generally, candidates are still having difficulty with the algebraic manipulations.

DETAILED COMMENTS

UNIT 1

Paper 01 – Multiple Choice

Performance on the 45 multiple-choice items on Paper 01 produced a mean of 55 out of 90 with some scores ranging between 12 and 90.

Paper 02 – Essays

Question 1

This question tested candidates' ability to

(a) distinguish between qualitative and quantitative data, and between discrete and continuous data

(b) distinguish between a parameter and a statistic

(c) distinguish among the sampling methods: simple random, stratified, systematic, quota and cluster

(d) outline the differences between simple random and stratified sampling

(e) use systematic sampling and stratified sampling to obtain a sample.

All candidates attempted the question. Eighty per cent of them performed exceptionally well on this question.

In Part (a) candidates performed exceptionally well.

Answers: (i) A, D, E (ii) B, C (iii) A, E (iv) D (v) B, C

Part (b) was well done by the majority of the candidates.

In Part (b) (i) many candidates were able to recognize different sampling methods. However, other terms were used by candidates that were not specified in the syllabus, for example, snow balling and multi-staging method.

Answers: Simple random, systematic, quota, stratified, cluster

In Part (b) (ii) about two-thirds of the candidates, despite the lack of the necessary parameters needed to answer the question, efficiently, were still able to answer satisfactorily.

Answer: $n=15$

Part (b) (iii) was done exceptionally well by most candidates.

Answer: 8

Part (b) (iv) was poorly answered. Less than one-third of the candidates produced complete answers. The answers were too simplistic and candidates failed to make a proper comparison between the given terms – stratified and simple random sampling.

> *Answers: In simple random, every member of the population has an equal chance of selection while in stratified the members have equal chance of selection only in strata.*
>
> *OR*
>
> *Stratified is more representative of population, while in simple random one stratum may be over or under represented.*

Part (c) was done exceptionally well.

> *Answer: Parameter is a numerical measure calculated from population and statistic is a numerical measure calculated from sample data.*

<u>Question 2</u>

This question tested candidates' ability to

(a) construct and use a stem and leaf diagram to display data

(b) calculate the trimmed mean from given data

(c) determine the quartile

(d) calculate the inter−quartile range

This question was attempted by all candidates and the responses were satisfactory.

Part (a) (i) was done satisfactorily. Candidates understood the concept of median, but some of them used the middle value from the *x*-axis.

> *Answer: 25*

In Part (a) (ii), some candidates also used the upper quartile reading and lower quartile reading from the *x*-axis to find the inter-quartile range.

> *Answer: 9 minutes*

In Part (a) (iii), most candidates were able to recognize that 26 persons waited 20 minutes. These candidates were able to follow through to produce the correct answer. However, some candidates read off the value as 23 and were only awarded one mark for subtraction.

> *Answer: 74 persons waited 20 minutes or more at the clinic.*

Candidates generally understood how to construct a stem and leaf diagram as required for Part (b) (i) (a), but had problems with split classes and therefore produced incorrect diagrams.

Answer:

0	*4 4*	
0	*7 8 8 9 9*	
1	*0 0 0 1 2 2 3 3 3 4 4*	
1	*6 6 7 8*	
2	*0*	
2	*5*	
3	*0* *1*	*key: 1|6 means 16*

In Part (b) (i) (b), 98 per cent of the candidates were able to arrive at the correct mode.

Answers: 10 and 13

In Part (b) (i) (c), 65 per cent of the candidates were able to find the position of the median, but were unable to state the median.

Answer: median is 12

Eighty-five per cent of the candidates were able to calculate the position of the upper quartile and lower quartile in Part (b) (i) (d), however, they subtracted the positions instead of the values.

Answer: Interquartile range = 7

In Part (b) (i) (e), most candidates were able to calculate the 8 per cent trimmed mean, which was $\frac{8}{100} \times 25 = 2$, but had difficulty calculating the mean using $\frac{\sum x}{n} = \frac{260}{21}$. Most candidates knew that two values should be subtracted from both sides of the data, however some of them used n as 23 rather than 21.

Answer: 8% trimmed mean $\rightarrow \frac{8}{100} \times 25 = 2$ *(discard top 2 and bottom 2 numbers)*

8% trimmed mean $= \frac{260}{21} = 12.4$

In Part (b) (i) (f), 90 per cent of the candidates were able to describe the shape of the distribution.

Answer: positively skewed

In Part (b) (ii), the majority of the candidates were able to identify that the mean and range were the two measures which were affected by outliers.

Question 3

This question tested candidates' ability to:

(a) calculate the probabilities P(x = a), P(x < a) where $x \sim B\text{in}(\text{n}, \text{p})$

(b) calculate the mean of a binomial distribution

(c) use the normal distribution as an approximation to the binomial distribution where appropriate np > 5 and npq > 5 and apply a continuity correction.

(d) (i) use a cumulative distribution function table

(ii) compute probabilities

(iii) calculate the expected value of E(X) and Var(X) of a discrete random variable X.

Part (a) (i) was attempted by 90 per cent of the candidates, with about 80 per cent giving a satisfactory response. Although many candidates recognized the application of the binomial many of them encountered difficulties using the parameters p and q and the respective indices.

In Part (a) (ii), many candidates were able to recognise that P($X \geq$ 4) = 1- P (X < 4) or

1 - P ($X \leq$ 3). However, a few candidates were able to interpret it as 1- P($X \leq$ 4) and

1- P (X <3)

Part (b) was generally well done.

In Part (c), many candidates were able to use the normal approximation

$$X \sim N(19, 15.2) \text{ and standardized correctly } P\left(\frac{13.5 - 19}{\sqrt{15.2}} < Z < \frac{21.5 - 19}{\sqrt{15.2}}\right)$$

However, some candidates were unable to calculate $\emptyset$ (-1.411) in order to arrive at the correct solution. They encountered difficulties using the standard normal curve and subtracting the required table obtained from 1. This information later assisted them in obtaining $\emptyset$ (-1.411) = (1-0.9208).

In Part (d) (i), many candidates did not attempt to construct the probability distribution table; instead they used the cumulative distribution function for the given table. However, 90 per cent of the candidates who attempted to construct the probability distribution table were successful in their attempt and gave the correct answers.

In Part (d), most candidates were able to recognise that the probability of x>3 is equivalent to P (x = 4) + P(x = 5) + P(x = 6) or 1- P ($x \leq 3$). A few candidates did not interpret the P($x \leq 3$) correctly and therefore obtained the wrong solution. One example of the incorrect solutions was P(x = 0) + P(x =1) +P(x = 2) + P(x = 3).

In Part (d) (iii), many candidates incorrectly used the cumulative distribution function to calculate the value of E(X) and Var(X). However, approximately 75 per cent of them were able to calculate E(X) using the correct method. Additionally a few candidates were unable to recall the correct formula to calculate the Var(X). Other candidates interpreted the question incorrectly by constructing a frequency table to calculate E(X) and Var(X) of the distribution.

Answers

(a) (i) 0.302 (ii) 0.121 (b) 2 (c) 0.660 (d) (i) 0.01, 0.22, 0.41, 0.22, 0.14
(d) (ii) 0.77 (d) (iii) 4.26, 0.972

<u>Question 4</u>

This question tested candidates' ability to:

(a) (i , ii, iv) calculate P(A ∩ B) and P(A∪B)

(iii) calculate the conditioned probability P(A|B) $= \frac{P(A \cap B)}{P(B)}$

(v) state whether A and B are independent events, identify independent events

(b) Construct a tree diagram and use the laws of probability to solve problems involving the diagram.

This question was attempted by more than 95 per cent of the candidates; however, only forty per cent of them attained greater than 50 per cent of the marks in the final score.

In Part (a) (i), candidates assumed that the events A and B were independent events and demonstrated a lack of proficiency of conditional probability. Even though candidates encountered difficulties, some were able to deduce and use

$$P(A|B) = \frac{P(A \cap B)}{P(B)} .$$

In Part (a) (ii), most candidates were able to state the formula for P(A∪ B) and therefore use it to obtain the solution.

In Part (a) (iii), candidates demonstrated understanding of conditional probability since approximately 70 per cent of them were able to arrive at the correct solution.

In Part (a) (iv), candidates demonstrated a lack of understanding of $P(\bar{A} \cap B)$ and therefore were unable to give the solution.

In Part (a) (v), the majority of candidates were able to identify that A and B are not independent. However, some candidates were unable to state that

$$P(A).P(B) = 0.6 \text{ x } 0.4 = 0.24, P(A\cap B) = 0.08$$

In Part (b) (i), most candidates demonstrated a comprehensive understanding of the probability tree and were able to label all the probabilities in their prospective places.

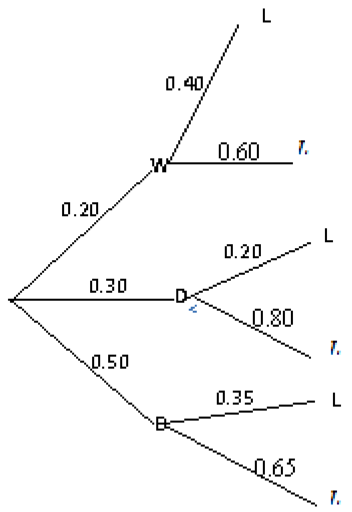
In Part (b) (ii) (a), candidates demonstrated a lack of knowledge in utilizing the tree diagram and deducing a formula. This resulted in candidates' inability to provide a favourable response.

In Part (b) (ii) (b), candidates demonstrated a lack of understanding of $P(B/\bar{L}) = \frac{P(B \cap \bar{L})}{P(\bar{L})}$ and were therefore unable to obtain $\frac{0.5 \ x \ 0.6}{1 - 0.315} = 0.474$ correct to three significant figures.

Answers

(a) (i) 0.08 (ii) 0.92 (iii) 0.133 (iv) 0.32
(v) $P(A \cap B) = P(A).P(B)$, then A and B are not independent.

(b)(i)



(ii) a) 0.315 b) 0.474

Question 5

This question tested candidates' ability to

(a) draw scatter diagrams to represent bivariate data

(b) calculate the mean of *x* and *y* i.e . $\bar{x}$ and $\bar{y}$ and plot on a scatter diagram

(c) calculate equation and draw the regression line y and x passing through $\bar{x}$, $\bar{y}$ on a scatter diagram

(d) calculate and interpret the value of r, the product mean correlation coefficient

(e) and (f) make an estimation using the appropriate regression line.

This question was exceptionally well done.

For Part (a), many candidates were able to plot the scatter diagram accurately. However, candidates *must* make every effort to use materials provided in the exam. Candidates opting to use their own scale inadvertently complicated the question and may have lost marks due to inaccuracies in scale.

In Part (b), calculation of $\bar{x}$ and $\bar{y}$ was very well done. Some candidates failed to realize that the summary statistics had been provided and calculated required statistics from raw data. Most candidates were successful in plotting $(\bar{x}, \bar{y})$.

For Part (c), the majority of candidates were able to identify the equation y on x as being $y = a + bx$ and most candidates were able to substitute correctly into the equation. Inconsistencies in approximated values were the major reason for loss of marks. Candidates must read the instructions of the paper and give answers to *three significant figures unless otherwise specified*. Many candidates failed to draw the regression line on their graph.

Many candidates showed competency in stating the formulae for finding r and substituting accurately into that equation for Part (*d*). The main reason for loss of marks was the incorrect substitution of n = 20. Though many candidates were successful in finding r = 0.965, the majority of candidates were unable to interpret this value as strong positive correlation. Just stating positive correlation was insufficient to obtain the mark assigned.

Part (e) was generally well done as most candidates were able to correctly identify the required equation and substitute x = 45 accurately to get y = 43.5 ≈ 44 years.

Part (f) was well done, however, approximately 40 per cent of candidates were unable to state that the regression line obtained was unreliable as the required value, $x = 45$, was outside the range of values given in the table.

Answers:

(b) $\bar{x}$ = 28.6 and $\bar{y}$ = 28.8
(c) $y = 3.15 + 0.897x$
(d) r = 0.965 which implies *strong positive* correlation
(e) y = 43.5 ≈ 44 *years*
(f) This value is not reliable as x = 45 lies outside the range of values given for x.

<u>Question 6</u>

This question tested candidates' ability to

(a) apply the central limit theorem where n ≥30 and use the fact that Var $\bar{X}$ = $\frac{\sigma^2}{n}$ where $\bar{X}$ is the sample mean, μ the population mean and n the sample size.

(b) determine the degree of freedom for a given test and level of significance

(c) (i) formulate a null hypothesis H_0 and an alternate H_1
(ii) calculate the expected frequencies, E, when the null hypothesis is true
(iii) (a) determine the appropriate number of degrees
(b) determine the critical region for a given test
(c) determine the test statistic
(iv) give a valid conclusion of the test

Part (a) was well done by the majority of the candidates who recognized the need to use the central limit theorem.

For Part (b) most candidates correctly obtained the degrees of freedom. There were a few candidates who were unable to determine the level of significance. They mistook the *p* value = 0.95 to be the level of significance.

In Part (c)(i), about 50 per cent of the candidates were not sure how to state the null and alternate hypotheses. Most of these candidates then had inaccurate conclusions.

Part (c)(ii) was generally well done. Most candidates calculated the expected values correctly. There were a few candidates who did not state the formula and gave answers to more than one decimal place or rounded off their answers to the nearest whole number. This resulted in the total sample size being more than 300.

Candidates need to be reminded that final answers should give the same result as the observed values.

More that 90 per cent of the candidates correctly calculated the degrees of freedom correctly.

In Part (c)(iii)(b), about 40 per cent of the candidates did not read the critical value correctly which resulted in their critical region being incorrect. The value of the test statistic, Part (c)(iii) c) was also incorrectly, calculated mainly because candidates' expected values were incorrect.

Most candidates were able to state the conclusion clearly, for Part (c)(iii)(iv). However, their reasons were unclear for the most part. This is an area that needs attention.

Answers

(a) $\bar{X} \approx N\left(100, \frac{16}{50}\right)$ (b) 4 d.f. ; 5% s.f.

(c) H_0: no association between gender and response of school teachers
$\mathbf{H}_1$: there is an association between gender and response

Observed values:

	In favour	against	No opinion	total
Male	102	62	11	175
female	73	45	7	125
total	175	107	18	300

(iii) a) 2; b) c.r. $\chi^2 > 5.991$ c) $\chi^2_{calc} = 10.995$

(iv) reject H_0

Paper 03/A -Internal Assessment

The internal assessment for Applied Mathematics tested candidates' ability to apply the theories learnt in the syllabus to real–world situations.

After evaluating the internal assessment projects, it was found that:

1. Innovation and creativity were lacking in over 50 per cent of the samples as candidates often chose topics that were used by fellow classmates.

2. Project titles were often vague.

3. Some candidates did not list the variables in the purpose section.

4. Some candidates misused the sampling technique in the method of data collection section. For example, in comparisons of gender, simple random sampling was used when stratified random sampling was the more appropriate technique to use.

5. In many instances, charts, graphs and diagrams were given which were not relevant to the purpose of the project.

6. Questionnaires and long tables of data were unnecessarily included in the presentation of data in about 20 per cent of the samples. These should be appended at the end of the projects.

Areas of Strengths and Weaknesses Identified

Strengths

1. Recalling basic definitions, for example, a sample, a population
2. Drawing a stem and leaf diagram
3. Reading χ^2 and t−tables
4. Standardizing values using the normal distribution
5. Recognizing when the binomial distribution is to be used

Weaknesses

1. Candidates were unable to obtain a sample mean given the end points of a confidence interval
2. Candidates were unable to define a critical region given the critical value
3. Candidates incorrectly interchanged the H_0 and the H_1 for the χ^2 test

UNIT 2

Paper 01 – Multiple Choice

Performance on the forty-five multiple-choice items on Paper 01 produced a mean of 61 out of 90 with scores ranging between 18 and 90.

Paper 02 – Essays

<u>Question 1</u>

This question tested the candidates' ability to:

(a) formulate conditional propositions

(b) derive a Boolean expression from a given logic circuit,

(c) represent a Boolean expression by a switching circuit.

(d) establish the truth value of

- negation of simple propositions,
- compound propositions that involve conjunctions, disjunctions and negations,
- conditional and bi conditional propositions,
- use truth tables to determine if propositions are equivalent

(e) use the laws of Boolean Algebra (de Morgan's Law) to simplify Boolean expressions

All candidates attempted this question but only about 40 per cent attained a score greater than 20.

Part (a) was fairly well done. All the candidates who attempted this section successfully filled out the truth table. However, a few candidates did not comment on the equivalence of p and $(\sim p \vee \sim q) \Rightarrow (p \wedge \sim q)$.

Part (b) was fairly well done. Most candidates were able to write the Boolean expression for the logic circuit whilst some used the incorrect logic notation (+ and .) for (**or** and **and**). Candidates should ensure that the correct notations are used.

Part (c) was satisfactorily done. Candidates were unable to distinguish between a switching and a logic circuit.

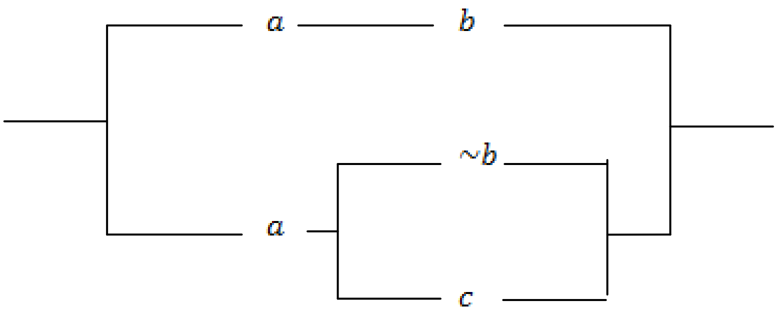
Part (d) was very well done. Candidates were able to convert statements to a Boolean expression.

Part (e) was poorly done. Candidates were unable to apply the laws of Boolean algebra to prove the identity.

Answers:

(b) $[p \wedge (q \vee \sim r)] \vee r$

(c)



(d) (i) $p \Rightarrow q$ (ii) $\sim q \Rightarrow \sim p$

<u>Question 2</u>

This question tested candidates’ ability to:

(a) solve a minimization assignment problem (4×4) by the Hungarian Algorithm

(b)(i) use the activity network algorithm to draw a network diagram to model a real-world problem

(ii) calculate the earliest start time, latest start time and float time

(iii) identify the critical path in an activity network

This question was well done. Candidates attempting Part (a) were knowledgeable of the Hungarian algorithm, however, some candidates did not apply the concept of stopping the algorithm when the number of lines of shading is equal to the dimension of the matrix.

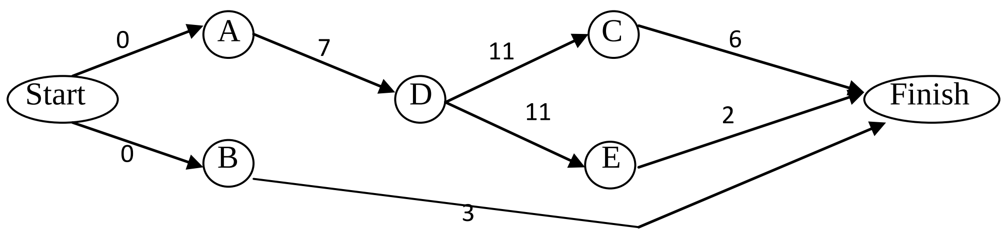
In Part (b) (i), some candidates drew two unnecessary edges (AE and AC) since D was preceded by A. The labelling of the duration and activity in the network were not done correctly by some candidates. Candidates need to be reminded that the START and FINISH nodes should be included in the network diagram.

In Part (b) (ii), candidates encountered difficulty in calculating the latest start time and identifying the order of the critical path.

Answers:

(a) A – 1, B – 4, C – 3, D – 2.

(b) (i)



(ii)

(iii)

Activity	Earliest Start Time	Latest Start Time	Float
A	0	0	0
B	0	21	21
C	18	18	0
D	7	7	0
E	18	22	4

(iii)(a) Start – A – D – C – Finish (b) 24 hours

Question 3

This question tested candidates' ability to

(a) calculate the number of selections of n distinct objects taken r at a time

(b) calculate the number of ordered arrangements on n objects taken r at a time

(c) identify probability models, the geometric distribution

(d) calculate the E(X) for the geometric distribution

Part (a) (i) was well done except for a few candidates who did not divide by the 3. Some candidates treated all "M's" individually. Some of them were not sure and did both.

Part (a) (ii) was partially done by the majority of candidates. The most common error was either not multiplying by ‘3’ or not dividing by ‘3!’. Several candidates again treated each “M” individually. A few candidates, instead of multiplying by ‘3’, multiplied by ‘3!’.

Part (a) (iii) was well done. The most common error was doing ‘5! x 4!’.

Part (b) was well done except for some candidates who did “${}^{n}C_{r} + {}^{m}C_{s}$” instead of “${}^{n}C_{r} \times {}^{m}C_{s}$.

Part (c) (i) was poorly done since candidates did not recognize the distribution as ‘Geometric’ and/or did not use the appropriate parameter.

Part (c) (ii) (a) was fairly well done.

In Part (b), candidates interpreted the question wrongly and solved for ‘greater than 3’ and not ‘greater than 2’.

In Part (c), many candidates interpreted the question wrongly and solved for ‘less than or equal to 4’ instead of ‘less than or equal to 3’.

Part (d) was well done.

Answers:

(a) (i) 840 (ii) 360 (iii) 120

(b) 209

(c) (i) X ~ Geo(1/3) (ii) a) 2/9 (b) 4/9 (c) 19/27 (d) 3

Question 4

The question tested candidates’ ability to:

(a) calculate probabilities using the Poisson distribution

(b) calculate probabilities using the binomial distribution

(c) calculate probabilities using appropriate counting techniques

Part (a) (i) was fairly well done. Most errors were due to candidates using an incorrect or incomplete formula. ‘γ^{x}’ many times treated as ‘γx’.

In Part (a) (ii), several candidates were unable to correctly interpret the inequality.

In Part (a) (iii), candidates failed to use the appropriate parameter for fortnight and continued using the parameter given for weekly average.

In Part (b), most candidates identified the correct distribution and parameters. Few candidates failed to find 2/3 of 12 and proceeded to solve using the value of X as 2/3 instead of using a value of 8. Again, a few candidates were unable to interpret the inequality correctly. Most candidates were aware of how to use the Binomial formula.

In Part (c) (i), most candidates obtained full marks. The few candidates who did not, treated the problem as though the marbles were being replaced.

Part (c) (ii) was poorly done since candidates struggled to obtain the correct amount of arrangements that there were thus resulting in loss of marks.

Part (c) (iii) posed a challenge to candidates since many of them failed to obtain the correct amount of arrangements and their corresponding probabilities.

Answers

(a) (i) 0.0498 (ii) 0.353 (iii) 0.161

(b) 0.8885

(c) (i) 1/21 (ii) 2/7 (iii) 4/9

Question 5

This question tested candidates' ability to

(a) apply Newton's laws of motion to
- a particle moving along an incline plane with constant acceleration,
- a system of two connected particles
- resolve forces on an inclined plane.

(b) solve problems involving power

This question was not well done since only 20 per cent of the candidates attained a mark greater than 20. Most candidates attempted Part (a). Some candidates did not recognize that $\sin^{-1}\left(\frac{1}{4.4}\right)$ was the angle.

In Part (a) (i) (a), some candidates did not include the normal reaction on the inclined plane.

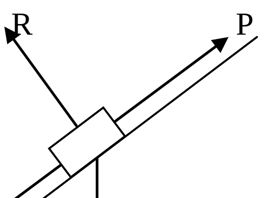
For Part (a) (ii) (b), some candidates did not convert the speed to ms^{-1}.

In Part (a) (ii) (c), candidates were familiar with the equation to compute Power but used the initial velocity instead of the final velocity.

Part (b) was fairly well done.

Answers:

(a) (i)



200N

θ 1100g N

(ii) a) $0.5\ ms^{-2}$ b) $3250\ N$ c) 65kW

(b) (i) $\frac{10}{9} ms^{-2}$ (ii) $\frac{400}{9} N$

<u>Question 6</u>

This question tested candidates' ability to:

(a) draw and use velocity – time graphs

(b) calculate the work done by a constant force.

This question was attempted by most candidates with only ten per cent attaining marks greater than 20.

In Part (a) (i), most candidates were able to interpret the given information to produce the graph.

For Part (a) (ii), three methods were used to answer this question but the majority of the candidates were unsure of what was required. Candidates should be consistent with the units $(minutes \rightarrow hrs/s)$.

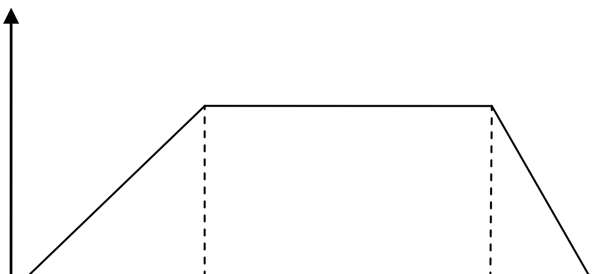
In Part (b) (i), candidates were familiar with the formula to compute work done but some used the incorrect component of the force.

For Part (b) (ii), candidates were unable to resolve the forces on the inclined plane. In addition, candidates were unable to apply the formula:

Work done = Work done against gravity + work done against frictional force

Answers:

(a) (i) v/kmh^{-1}



v

0.5km 1.5 km 0.3 km

O $\frac{1}{10}$ t/h

(ii) $961\ kmh^{-2}$

(b) (i) $276\ J$ (ii) $239N$

UNIT 2

Paper 03/A – Internal Assessment

1. Statement of Task

   - Needs to be more specific
   - Must include a definition of the variables

2. Method of Data Collection

   - Most candidates did not understand what it means to collect data

3. Evaluation

   - Approximately ten per cent of candidates did not state a conclusion.
   - Conclusions need to be more precise and related to the analysis and statement of task