C A R I B B E A N   E X A M I N A T I O N S   C O U N C I L

REPORT ON CANDIDATES' WORK IN THE ADVANCED PROFICIENCY EXAMINATION MAY/JUNE 2010

PURE MATHEMATICS

GENERAL COMMENTS

This is the third year that the current syllabus has been examined in the format of Paper 01 as Multiple Choice (MC) and Papers 02 and 03 structured questions. Approximately 5 600 candidates wrote the Unit 1 and 2 800 Unit 2 examinations in 2010. Performances continued in the usual pattern across the total range of candidates with some obtaining excellent grades while some candidates seemed unprepared to write the examinations at this level, particularly in Unit 1.

UNIT 1

The overall performance in this unit was satisfactory, with several candidates displaying a sound grasp of the subject matter. Excellent scores were registered with specific topics such as Trigonometry, Coordinate Geometry and Calculus. Nevertheless, candidates continue to show weaknesses in areas such as Limits and Continuity, Indices and Logarithms. Other aspects that need attention are summation as part of general algebraic manipulation of simple expressions, substitution and pattern recognition as effective tools in problem solving.

UNIT 2

In general, the performance of candidates on Unit 2 was satisfactory. It was heartening to note the increasing number of candidates who reached an outstanding level of proficiency in the topics examined. However, there was evidence of significant unpreparedness by many candidates.

Candidates continue to show marked weakness in algebraic manipulation. Much more emphasis must be placed on improving these skills. In addition, reasoning skills must be sharpened and analytical approaches to problem solving must be emphasized. Too many candidates demonstrate a favour for problem solving by memorized formulae.

DETAILED COMMENTS

Paper 01 – Multiple Choice

Paper 01 comprised 45 multiple-choice items. Candidates performed satisfactorily with a mean score of 46.27 and a standard deviation of 17.02.

Paper 02 – Structured Questions

Section A

Module 1: Basic Algebra and Functions

<u>Question 1</u>

Specific Objectives: (b) 1, 3, 4; (c) 1−5; (f) 3; (g) 3.

The topics examined in this question covered the Remainder and Factor Theorems, simultaneous equations and logarithms, inequalities, quadratic equations and indices.

(a) The majority of candidates correctly applied the Factor Theorem to *f(x)*. Several opted to find the remainder by division but were unable to follow through successfully because of weaknesses in the algebraic manipulation required. Some were unable to factorize the equation for *f(p)* = 0 using the quadratic formula.

(b) Several candidates misinterpreted the basic laws of logarithms, replacing $\log(x - 1) + 2\log y$ with expressions such as $\frac{\log x}{\log 1} + \log 2y$. Such equivalences produced erroneous results and, in some cases, equations which the candidates could not solve. Elimination was tried in some cases without success.

(c) Many candidates multiplied through by $x + 1$ instead of $(x + 1)^2$ and had difficulty afterwards in solving the resulting inequality.

(d) The substitution $y = 2^x$ proved problematic for some candidates, many of whom interpreted 4^x as 2×2^x which led to incorrect answers. Other candidates did not complete the question, giving the answers for y only and not for x as required. More practice is needed to consolidate the areas of weakness identified above.

Solutions:

(a) $p = \frac{3}{2}$ or -1
(b) $x = 2$, $y = 3$
(c) $-\frac{8}{3} < x < -1$
(d) $x = 1$ or 2

Question 2

Specific Objectives: (a) 7; (d) 5−8.

The topics covered in this question related to summation notation and functions, together with the interpretation of the graphs of simple polynomial functions and the existence or non-existence of inverses.

(a) The majority of candidates answered (i) correctly. A common mistake made was to equate S_{2n} with $2 \times S_n$, while some candidates misinterpreted the question as a problem solving mathematical induction. Several candidates encountered simplification problems in (ii). A common error involved writing $-Sn$ as $-n^2/_2 + n/_2$ instead of $-n^2/_2 - n/_2$. In (iii), the correct quadratic equation in n, namely $3n^2 + n - 520 = 0$, escaped many candidates and some who derived it failed to solve it correctly to obtain $n = 13$.

(b) (i) Approximately 80 per cent of the candidates gave $x \geq 3$ as a possible solution. Some obtained the complete solution but failed to write in set notation. Several used the graph and gave the solution as $x \leq 0$, $x \leq 3$.

(ii) Less than 10 per cent of the candidates gave the correct solution. Many assigned values to k and attempted to solve the equation $x^2(3-x) - k = 0$, thereby completely ignoring the graph as a guide to the solution.

(iii) Many candidates showed familiarity with the terms injective, surjective and bijective as they applied to functions but seemed to struggle to separate the terms in respect of the given graphical representations for *f* and *g*. Approximately 80 per cent of the candidates used the horizontal line test (a) and (b).

Solutions:

(a) (i) $S_{2n} = n(2n + 1)$
(ii) $p = \frac{3}{2}$, $q = \frac{1}{2}$
(iii) $n = 13$

(b) (i) $\{0\} \cup \{x \geq 3\}$ (ii) $\{k: 0 \leq k \leq 4\}$

Section B

Module 2: Trigonometry and Plane Geometry

Question 3

Specific Objectives: (b) 4, 5; (c) 3, 9, 10

Overall, candidates performed poorly on this question with approximately 70 per cent scoring less than 12 of the maximum marks (25) and approximately 40 per cent scoring less than 4 marks. Generally, candidates scored higher in Part (a) than in Part (b) of the question. Attempts at Part (a) (i) by the candidates who achieved an acceptable score were usually fruitful. Many of the candidates gained full marks for finding the angle. In addition, many of them used the form $\mathbf{p.q} = |p||q|\cos\theta$. However, a large percentage of candidates did not score maximum marks as they had a problem adding directed numbers accurately or they simply forgot the negative sign in front of the 84. Many candidates used the alternative method which included the graph and trigonometrical ratios. However, a few of them forgot to add the 90° which was required to find the entire angle.

A significant number of candidates experienced difficulty in the relatively easier form of finding the x co-ordinate of the vector in (a) (ii). Many started at $(6\boldsymbol{i} + 4\boldsymbol{j})$ $(x\boldsymbol{i} + v\boldsymbol{j}) = 6x + 4y = 0$ and then stopped. Astoundingly, many candidates declared that $\mathbf{p}.\mathbf{v} = 0$ was either parallel or inverse.

Overall, performance on (b) (i) was also disappointing. Many candidates started at the given equation for C_1 and ignored the given end points completely. Others simply substituted the given points into equation C_1 and then faltered. Some worked backwards and did not use the end points at all.

The responses to (b) (ii) were similarly disappointing. Only a small percentage of candidates attempted to eliminate the x^2 and y^2 hence obtaining $y = x + 5$ which could easily be substituted into C_1 or C_2. The majority who opted to obtain x or y as the subject ended up with an awkward round of algebra, often with limited success. Some of the candidates simply could not appreciate what to do with the equation $y = x + 5$ and proceeded with a completely different and inappropriate strategy.

General recommendations to teachers include:

1. Constant revision of algebraic simplification
2. The provision of opportunities for students to enhance their problem-solving skills

Solutions:

(a) (i) 165°
(ii) (a) $2k\mathbf{i} - 3k\mathbf{j}$, $k \in R$
(iii) **p**, **v** are perpendicular

(b) (ii) Points of intersection are (0, 5) and (-3, 2)

<u>Question 4</u>

Specific Objectives: (a) 4, 5, 13, 14

Most candidates attempted this question but performed poorly overall. Approximately 40 per cent of candidates scored less than 4 marks and approximately 65 per cent scored less than 12 marks.

(a) (i) Many candidates were able to gain marks for stating $3A = \cos^{-1}0.5$ and proceeding from there. However, only a small number of candidates arrived at the three solutions in the given range. The majority ended with the answer $A = \frac{\pi}{9}$ or 20°. In some cases, candidates attempted to find the solutions by using trigonometric identities. Most of them simply stalled thereafter.

(ii) Responses to this part of the question were surprisingly very good. Even some of the relatively lower-scoring candidates completed the exercise with a flourish, gaining the full six marks. Some showed great competence, utilizing standard identities, to prove this more difficult identity.

(iii) This part of the question challenged many candidates including many with high scores. They could not make the link to the earlier parts of the question, even when directed to do so. It was also noted that answers were not given to three significant figures (as indicated at the front of the question paper).

(b) (i) Many candidates were able to make substantial progress on this part of the question. Many of them knew the compound angle formulae and also derived the correct ratios for $\tan \alpha$ and $\tan \beta$, but some lacked the algebraic techniques needed to get the required answer.

(ii) Many candidates were clueless and had no idea how to solve this question. Indeed, there were many 'no responses' seen for this part of the question.

Solutions:

(a) (i) $A = \frac{1}{9}\pi, \ \frac{5}{9}\pi, \ \frac{7}{9}\pi$
(iii) $þ = \cos\frac{\pi}{9}, \ \cos\frac{5\pi}{9}, \ \cos\frac{7\pi}{9} \approx 0.940, -0.174, -0.766$

(b) (ii) $\max(\alpha - \beta) = \tan 0.125 \approx 0.124$ rad.

Section C

Module 3: Calculus 1

Question 5

Specific Objectives: (a) 1, 3−7, 9; (c) 1, 4, 6 (ii)

This question tested knowledge of limits, continuity and discontinuity, and integration.

There were some very good attempts, many of which gained full marks on the question. Nevertheless, some weaknesses in the preparation of the candidates were evident and there were cases of carelessness in pursuing routine procedures.

(a) (i) Many candidates experienced difficulty factorizing the denominator in this part and this curtailed success in obtaining the limit. Some used L'Hôpital's rule with great success.

(ii) Few candidates were successful in obtaining full marks on this part. Those who did used L'Hôpital's to great advantage.

(b) Some candidates misunderstood the symbols for left hand and right hand limits while others had difficulty with the concept of discontinuity. The word 'deduce' was largely ignored by many.

(c) (i) There were several good returns on this part of the question for the candidates who expanded the expression in the integrand correctly. Many of these candidates were able to follow through with the integration, termwise, of the expanded expression and the evaluation at each of the limit points $x = 1$ and $x = -1$, to obtain full marks for this part.

(ii) Several candidates performed well on this part of the questions. Many candidates who did not, failed to change the integrand completely from x's to u's.

Solutions:

(a) (i) $\frac{2}{9}$ (ii) 2

(b) (i) a) 5, b) 9

(c) (i) at $x = 1$, $-2\frac{2}{3}$; at $x = -1$, $2\frac{2}{3}$

(ii) $\frac{1}{3}(x^2 + 4)^{\frac{3}{2}}$ + k (constant)

Question 6

Specific Objectives: (b) 4, 7, 8, 9 (i), 10

This question covered differentiation, integration and calculus.

Several candidates attempted the question with varying degrees of success. The principles involved seemed to be familiar to most, yet some candidates did not receive maximum reward because of weaknesses in simple algebraic manipulations. More practice is recommended to reduce the incidence of such pitfalls.

(a) (i) Many candidates were strong on differentiating the basic trigonometric functions of sin and tan but some were unable to cope with the composite nature of the functions involved.

(ii) Several candidates had difficulty using the quotient rule often interchanging numerator with denominator in applying it. Nevertheless, there were some excellent answers returned on this question.

(b) (i) The use of the relevant integration theorem seemed not to be recognized by some candidates. In such cases, the candidates performed poorly to the extent that they did not seem to be aware that $\int_1^4 4\,\mathrm{d}x$ had to be obtained.

(ii) It seemed that some candidates had little practice in using substitution to perform integration. However, several of those who knew the technique were able to obtain full marks on this part of the question. Greater attention should be paid to Specific Objectives 6 and 7 of Unit 1 Module 3, in preparing candidates.

(c) (i) This part of the question was well done. The majority of candidates obtained full marks.

(ii) The majority of candidates knew the methods to be used but many encountered problems with the algebra involved. Several methods were used to obtain the correct answer.

Solutions:

(a) (i) $3\cos(3x+2) + 5\sec^2(5x)$ (ii) $-\dfrac{(x^4+3x^2+2x)}{(x^3-1)^2}$

(b) (i) 33 (ii) 14

(c) (i) $P \equiv (-2,4)$, $Q \equiv (1,1)$; (ii) Area = $4\frac{1}{2}$ units^2

Paper 03/B (Alternative Internal Assessment)

Section A

Module 1: Basic Algebra and Functions

<u>Question 1</u>

Specific Objectives: (a) 8, (d) 1, 7, (f) 4, 5 (ii)

This question examined the theory of cubic equations, functions and mathematical induction.

(a) (i) This part of the question was poorly done. There was little evidence of candidates demonstrating the theory of cubic equations. Some candidates merely stated the concepts of the sum of roots, product pair-wise and the product of roots of the cubic equations. However, they were unable to complete the calculations by substituting the correct values.

(ii) Without a correct follow through from (i) candidates could not find the required values of p and q.

(b) (i) Candidates understood the meaning of ‘one-to-one’ but were unable to show the required result mathematically. Instead, they substituted some values for x and concluded that f is one-to-one. No candidate used the horizontal line test.

(ii) A few candidates made correct expansions but failed to make the correct deductions. Generally, this part of the question was satisfactorily done.

(c) The majority of candidates knew that an assumption was necessary, followed by the inductive process. However, the inductive process and the algebra involved proved beyond their abilities. They merely stated conclusions without proof.

Solutions:

(a) (i) $\alpha = -2$

(ii) $p = 12$; $q = 44$

(b) $x = \dfrac{5}{2}$

Section B

Module 2: Trigonometry and Plane Geometry

<u>Question 2</u>

Specific Objectives: (b) 1, 2, 3; Content: (a) (ii), (b) (iii)

This question examined the intersection of lines, equations of straight lines and the area of a triangle using basic concepts of coordinate geometry.

(a) (i) This part of the question was well done.

(ii) This part of the question was well done. Some exceptions included the correct calculations for the gradient of a straight line given two points on the line.

(b) This part of the question was well done.

(c) Candidates had problems calculating the lengths of AC and DC which were required to find the area. The majority of candidates obtained partial marks for some related attempts to find the area.

Solutions:

(a) (i) A (0, 2), B (3, 0), C (6, -2)

(ii) CD: $3x - 2y - 22 = 0$ AD: $5x + y - 2 = 0$

(b) D (2, -8)

(c) Area = 26 square units

Section C

Module 3: Calculus 1

<u>Question 3</u>

Specific Objectives: (b) 6, 15, (c) 2, 3, 4, 5, (ii), (iii), 8 (i)

This question examined indefinite integration of composite trigonometric functions, area under the curve and applications of differentiation to maxima/minima situations. There was also some mathematical modelling included.

(a) This part of the question was poorly done. Approximately three per cent of the candidates obtained maximum marks. A common error among candidates was their inability to use the identity $\tan^2 x \equiv \sec^2 x - 1$ in order to integrate $\tan^2 x$ correctly. A significant number of candidates evaluated $\int (\cos 5x)\,dx$ as $5\sin 5x$ or $-\frac{1}{5}\sin 5x$. Candidates continue to neglect the arbitrary constant of integration for indefinite integrals.

(b) (i) This part of the question was well done.

(ii) The majority of candidates used $\int_0^2 y\,dx$ to find the area of the enclosed region. A few candidates recognized symmetry of the two regions and used $2\int_0^1 y\,dx$ to find the area.

(c) (i) This part of the question was poorly done. Difficulties included being unable to state the perimeter in terms of r and x, and failing to equate the expression for the perimeter to 60.

(ii) Without the correct follow through from Part (i), many candidates could not obtain the area in terms of r only.

(iii) Differentiation of the expression for the area was poorly done. Candidates could not interpret the term $\left(1+\frac{\pi}{4}\right)$ as a constant. In addition, most of the candidates did not follow the instruction to find the **exact** answer.

Solutions:

(a) $\frac{1}{5}\sin 5x + \tan x - x + C$

(b) (i) $p = 1$; $q = 2$ (ii) Area = $\frac{1}{2}$ square units

(c) (i) $x = 30 - r - \frac{\pi r}{2}$

(iii) $r = \frac{60}{4+\pi}$

UNIT 2

Paper 01 – Multiple Choice

Paper 01 comprised 45 multiple-choice items. Candidates performed fairly well with a mean score of 50.39 per cent and a standard deviation of 17.93.

Paper 02 – Structured Questions

Section A

Module 1: Calculus II

<u>Question 1</u>

Specific Objectives: (a) 9, (b) 1, 4, 5, 6, 7, (c) 1 (i), 5, 6, 7

This question examined a real-world situation involving an exponential equation, differentiation of an exponential function, differentiation of an inverse trigonometrical function, implicit differentiation including second derivatives and integration using partial fractions with suitable substitutions.

(a) (i) Approximately 85 per cent of the candidates who responded to this part of the question were able to deduce that the initial temperature occurred when t = 0. Full marks were obtained by all of the candidates who interpreted this question correctly. Some candidates demonstrated a lack of understanding of the term ‘initial temperature’.

(ii) Generally, a significant number of candidates attempting this part of the question found it difficult to interpret the term ‘stabilise’. The general response was below average. Approximately ten per cent of the candidates obtained the correct answer, deducing that the limiting value of the temperature for large increasing t was 65^0 C. It may be useful at the level of instruction to consider terms that may be easily interpreted by candidates who may not be science oriented.

(iii) This part of the question required candidates to solve the equation to find a value for t. Common errors seen were

a) $0 = 65 + 8e^{-0.02t}$

b) $\ln 70 = \ln 65 + \ln 8e^{-0.02t}$

c) $73 = 65 + 8e^{-0.02t}$, using the value 73 obtained in (a) (i)

d) $\ln 5 = 8 \ln e^{-0.02t}$, treating 8 as a power

e) $t = \dfrac{\ln 70 - \ln 65}{8(-0.02)}$

Approximately half of the candidates demonstrated a marked weakness in algebraic manipulation, particularly regarding logarithms. A small percentage of candidates who recognized that logarithms had to be applied used common logarithms instead of natural logarithms. Emphasis must be placed on the properties of exponential and logarithmic equations. This will allow candidates to gain confidence to solve these equations correctly.

(b) (i) For this part of the question approximately 30 per cent of the candidates expressed the equation $y = e^{tan^{-1}\ (2x)}$ as $\ln y = \tan^{-1}(2x)$. However, these candidates were not able to carry out the required implicit differentiation to obtain the correct answer. Common errors made included failing to apply the chain rule when differentiating the composite function $\tan^{-1}(2x)$. Some answers given were

$$\frac{dy}{dx} = \frac{1}{1+4x^2} \text{ x } e^{\tan^{-1}(2x)} \times 2 \quad \text{and} \quad \frac{dy}{dx} = \frac{1}{1+2x^2} \text{ x } e^{\tan^{-1}(2x)} \times 2.$$

Very few candidates obtained full marks on this part of the question.

(ii) Approximately 15 per cent of the candidates attempted to use the result given in Part (b) (i) to show the result required in Part (b) (ii). However, they were unable to carry out the implicit differentiation required when using the form given in Part (b) (i). They failed to show that $\frac{d}{dx}2y = 2\frac{dy}{dx}$, but instead showed $\frac{d}{dx}2y = 2$.

Approximately 80 per cent of the candidates stated the equation $\left(1+4x^2\right)\frac{dy}{dx} = 2y$ as $\frac{dy}{dx} = 2e^{\tan\text{-}1(2x)} \text{x} \left((1+4x^2\right)^{-1}$ and proceeded to use the product rule correctly. These candidates obtained full marks with the correct algebraic manipulation to show the required result.

(c) (i) Approximately 50 per cent of the candidates used the substitution given, $u = e^x$, to express $\int \frac{4}{e^x+1}dx$ as $\int \frac{4}{u+1}dx$ in this part of the question and obtained the result $4 \ln (u + 1) + C$. They failed to replace dx with $\frac{1}{u}\ du$. A number of candidates obtained the form $\int \frac{4}{u(u+1)}$ correctly. However, they failed to carry out the correct integration since they did not recognize that partial fractions were necessary at this point. A few candidates who carried out the correct procedure for the integration left their answer as $4 \ln u - 4 \ln\left(u+1\right) + C$.

(ii) Approximately 70 per cent of the candidates were able to obtain the expression $\int \frac{4e^{-x}}{1+e^{-x}}dx$. However, these candidates could not proceed with the correct integration.

They failed to recognise the systematic integration of the form $\int \frac{\mathrm{f}'(x)}{\mathrm{f}(x)}\mathrm{d}x$. A significant number of those candidates who carried out the integration incorrectly obtained $4\ln\left(1+\mathrm{e}^{-x}\right)+C$.

Solutions:

(a) (i) 73℃

(ii) 65℃

(iii) 23.5 hours

(c) (i) $4\ln(\mathrm{e}^{x}) - 4\ln(\mathrm{e}^{x}+1) + \mathrm{C}$ or $4\ln\left(\frac{\mathrm{e}^{x}}{\mathrm{e}^{x}+1}\right) + C$

(ii) $-4\ln(1+\mathrm{e}^{-x}) + C$ or $4\ln\left(\frac{\mathrm{e}^{x}}{1+\mathrm{e}^{x}}\right) + C$

<u>Question 2</u>

Specific Objectives: (b) 2, 5, (c) 8, 10, 11

This question required differentiation of ln x using product and chain rules, reduction formula and the solution of a differential equation using an integrating factor.

(a) (i) Most candidates recognized that this part of the question required the use of the product and chain rules. However, application of the chain rule to differentiate the logarithmic function $(\ln x)^{n}$ proved to be difficult in many cases. Approximately 50 per cent of the candidates obtained full marks.

(ii) Generally, most of the candidates used integration by parts using the approach $\int(1)(\ln x)^{n}\,\mathrm{d}x = x\ln - \int x\frac{\mathrm{d}}{\mathrm{d}x}(\ln x)^{n}\,\mathrm{d}x$. No candidate used the result from (a) (i) to derive the reduction formula. This part of the question was well done by the majority of candidates.

(iii) This part of the question was poorly done. Many candidates demonstrated a lack of simple use of the derived reduction formula. No candidate successfully managed $I_1 = \int(\ln x)\mathrm{d}x$. In addition, the weakness in algebraic manipulation was again evident in substituting successive values of n in the reduction formula, making correct use of brackets. Approximately ten per cent of the candidates obtained full marks.

(b) (i) A small percentage of candidates demonstrated the ability to find a suitable integrating factor and proceeded to show the required general solution of the differential equation.

Approximately ten per cent of candidates were able to state $I = \mathrm{e}^{\int \frac{2}{t+10}\mathrm{d}t}$ but were unable to complete this integration correctly. The remaining candidates did not demonstrate any knowledgeable attempt to find the integrating factor.

(ii) Using the general solution to the differential equation in (b) (i), some candidates were able to obtain the correct answer to this part of the question. However, a significant number of candidates did not understand the term 'initially' and substituted random values.

Solutions:

(a) (i) $n(\ln x)^{n-1} + (\ln x)^n$

(iii) $x(\ln x)^3 - 3x(\ln x)^2 + 6x(\ln x) - 6x + C$

(b) (ii) 24.2 kg

Section B

Module 2: Sequences, Series and Approximation

Question 3

Specific Objectives: (b) 2, 4, (c) 3

This question examined candidates' ability to define the r^{th} term of a sequence, obtain the n^{th} partial sum of a finite series, find the first term and common difference of an arithmetic progression and their application of the binomial theorem.

(a) (i) Approximately 70 per cent of the candidates had difficulty obtaining the r^{th} term of the sequence.

(ii) In this part of the question most candidates attempted to sum the given series as an A. P. or a G. P. Those candidates who were able to express the r^{th} term correctly used the standard formulae for $\sum_{r=1}^{n} r^2, \sum_{r=1}^{n} r$ and $\sum_{r=1}^{n} c$ to find the required sum. Evidence of a weakness in algebraic manipulation resulted in some candidates not being able to simplify the answer.

(b) This part of the question was well done by approximately 75 per cent of the candidates. Some candidates incorrectly defined the equation required to express the 9^{th} term as 3 times the 3^{rd} term. As a result, incorrect values were obtained for the first term *a*, and the common difference *d*.

(c) (i) This part of the question was generally well done although some errors were made in simplifying the coefficients. Too many candidates did not state the correct range of values of x for which the expansion is valid.

(ii) A significant number of candidates did not attempt to rationalize the denominator by recognizing the resulting difference of squares. Many of those candidates who attempted to rationalize the denominator used $(1 - x) - \sqrt{(1+2x)}$ as the 'conjugate' of $(1 + x) + \sqrt{(1+2x)}$. Other candidates used the result from Part (i) to expand the denominator and could not show the required expression. Only a few candidates obtained full marks.

(iii) Using the result from Part (ii), the majority of candidates expanded the expression $\frac{1}{x}(1 + x - \sqrt{(1+2x)}\ )$ up to and including the term in x^2. Consequently, the resulting expression could not be $\frac{1}{2}x(1-x)$ as required. It was clear that candidates did not take into account that the expansion was divided by x, thus requiring the expansion up to and including the term in x^3.

Solutions:

(a) (i) $(3r - 1)(2r + 1)$

(ii) $S_n = \frac{n}{2}(4n^2 + 7n + 1)$

(b) $a = d = 2$

(c) (i) $1 + x - \frac{x^2}{2} + \frac{x^3}{2} - \ldots \quad -\frac{1}{2} < x < \frac{1}{2}$; $|x| < 1$

Question 4

Specific Objectives: (b) 1, 11, 12, (c) 1, (e) 1, 2, 4

This question examined candidates' ability to manipulate and prove expressions involving ${}^nC_r$, the sum of finite series using the method of differences and application of the Newton-Raphson procedure.

(a) (i) A small percentage of candidates obtained full marks for this part of the question. A significant number of candidates expressed ${}^nC_{r-1}$ as $\frac{n!}{(r-1)!(n-r-1)!}$. Many candidates stated the correct expression for ${}^nC_r + {}^nC_{r-1}$ but were unable to complete the algebraic manipulation to obtain the required result. Some candidates substituted numbers as a means of proof.

(ii) (a) Many candidates merely stated $f(r) - f(r+1) = \frac{1}{r!} - \frac{1}{(r+1)!}$. The algebraic manipulation and understanding of factorials required for showing the result were beyond the ability of these candidates.

(b) Most candidates recognized that the method of differences was needed to find the sum required. This part of the question was generally well done.

(c) A few candidates demonstrated an understanding of deducing the limiting value of this type of sum. Many of the candidates attempted to use the formula for the sum to infinity of a geometric progression but found it impossible to apply. A significant number of candidates did not respond to this part of the question.

(b) (i) Most of the candidates used the Intermediate Value Theorem but did not state the continuous property of the polynomial. Very few candidates obtained full marks.

(ii) Generally, this part of the question was well done. Some errors were made in substitution and there were incorrect calculations.

Solutions:

(a) (ii) b) $1 - \frac{1}{(n+1)!}$

c) $\lim_{n\to\infty} S = 1$

(b) (ii) 0.725

Section C

Module 3: Counting, Matrices and Complex Numbers

<u>Question 5</u>

Specific Objectives: (a) 1, 3, 11, 12, 13 (c) 1, 2, 4, 5

This question examined the concepts of counting, using permutations and combinations, basic probability theory and complex numbers.

(a) (i) Approximately half of the candidates wrongly applied the concept of all the letters of the word SYLLABUS as being unique, thus giving the solution as 8! Some candidates subtracted the repeated letters from the total number of letters and gave the solution as $\frac{6!}{2!2!}$.

(ii) This part of the question was poorly done and was not attempted by the majority of the candidates. Among the responses seen were ${}^6C_5$, ${}^8C_5$ and $\frac{5!}{2!2!}$. Some candidates were awarded partial marks for applying the concept but not accounting for all of the combinations.

(b) (i) This part of the question was well done. Approximately 75 per cent of the candidates obtained full marks.

(ii) This part of the question was fairly well done although some candidates did not demonstrate an understanding of independence and mutual exclusivity of events. Many of the candidates attempted to define the terms 'independence' and 'mutually exclusive' and related the concepts to arbitrary sets of events rather than to the given set of events.

(c) (i) Generally, this question was fairly well done. Many candidates were unable to rationalize the denominator correctly, failing with the algebra involved.

(ii) A significant number of candidates deduced the root 1 + i but were unable to find by division, or otherwise, the third root. Some candidates used the theory of quadratic equations to deduce the third root by inspection.

Solutions:

(a) (i) $\frac{8!}{2!2!} = 10\,080$

(ii) 30

(b) (i) 0.17

(ii) a) not mutually exclusive

b) not independent

(c) (i) 2 + 4i

(ii) 1 + i, – 3

Question 6

Specific Objectives: (b) 1, 3, 4, 5, 6, 7, 8

This question examined the augmented matrix, row-echelon reduction, consistency of a system of linear equations and inverting a matrix.

(a) (i) This part of the question was well done although some candidates merely wrote down the matrix for the system of equations.

(ii) In this part of the question, a common error seen was obtaining a row of zeroes in rows other than the last row. However, the question was generally well done.

(iii) A number of candidates did not understand the term 'consistent'.

(iv) This part of the question proved to be the most challenging to the majority of candidates. Most of them attempted to find a unique solution. Others expressed the variables x and y in terms of z but did not choose an arbitrary constant.

(b) (i) a) This part of the question was well done although some arithmetic errors were made. A number of candidates squared the elements of the matrix A instead of finding A × A.

b) This part of the question was well done.

(ii) This part of the question was well done.

(iii) A significant number of candidates used the cofactor method to find the inverse of A. However, they did not recognize the product AB was equal to 3I.

Solutions:

(a) (i) $$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 2 & 1 & -1 & -1 \\ 1 & 2 & 4 & k \end{array}\right)$$

(iii) $k = 1$

(ii) $$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & -1 & -3 & -1 \\ 0 & 0 & 4 & k-1 \end{array}\right)$$

(iv) $$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2\lambda - 1 \\ 1 - 3\lambda \\ \lambda \end{pmatrix}$$

(b) (i) a) $$\begin{pmatrix} 2 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$$

(iii) $$\frac{1}{3}\begin{pmatrix} 1 & -2 & 1 \\ -2 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

b) $$\begin{pmatrix} 1 & -2 & 1 \\ -2 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

(ii) $\begin{pmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{pmatrix}$

Paper 03/B – Alternative to Internal Assessment

Section A

Module 1: Calculus Ii

Question 1

Specific Objectives: (c) 1 (iii), 5, 11

This question examined partial fractions, solution of a logarithmic differential equation and proportional increase.

(a) This part of the question was poorly done. Candidates showed a marked weakness in manipulating fractions of the form $\frac{A}{x}+\frac{Bx+C}{x^2+1}$. Invariably, candidates expressed $\frac{1-x^2}{x(x^2+1)}$ as $\frac{A}{x}+\frac{Bx}{x^2+1}+C$ and $\frac{A}{x}+\frac{Bx}{x^2+1}+\frac{C}{x^2+1}$.

(b) (i) (a) This part of the question was poorly done in its entirety. A small number of candidates separated the variable successfully but could not complete the integration.

(b) Approximately 40 per cent of the candidates obtained full marks on this part of the question, using the follow through from (i) and making the correct substitution.

(ii) Candidates merely substituted t = 2 in the equation. This part of the question was poorly done.

Solutions:

(a) $\frac{1}{x}-\frac{2x}{x^2+1}$

(b) (ii) $2^{2/3} - 1$

Section B

Module 2: Sequences, Series and Approximations

Question 2

Specific Objectives: (b) 1, 10, 11, (c) 1, MM

This question examined the geometric progression, the limiting sum of an infinite series using Maclaurin's expansion and the applications of an exponential series.

(a) This part of the question was well done.

(b) This part of the question was hardly attempted. In fact, candidates seemed unfamiliar with the form of such series.

(c) (i) This part of the question was well done.

(ii) Only a small percentage of the candidates were able to state the correct value in terms of *t*.

(iii) A few candidates solved this part of the question using the index approach. A significant number of candidates used an arithmetic approach, calculating the depreciated value for successive years.

Solutions:

(a) $r = \dfrac{5}{6}$

(b) $3e - 1$

(c) (i) \$13 125

(ii) $\$15\,000 \left(\dfrac{7}{8}\right)^t$

(iii) \$ 4 510

Section C

Module 3: Counting, Matrices And Complex Numbers

<u>Question 3</u>

Specific Objectives: (a) 1, 2, 4, 7 (b) 1, 2, 6, 8, MM

This question examined selections with and without restrictions, classic probability and the solution of a system of linear equations using a matrix approach.

(a) (i) This part of the question was poorly done. Candidates applied combinations for calculations instead of reasoning.

(ii) The approach in Part (i) to finding a solution was repeated in this part of the question.

(b) This part of the question was poorly done since incorrect methods in Part (ii) resulted in incorrect answers being obtained.

(c) (i) This part of the question was well done.

(ii) This part of the question was well done.

(iii) Responses to this part of the question were poor. Candidates appeared to have no knowledge of the row-reduction method, the inverse method, or the solution of 3 linear equations with 3 unknowns.

Solutions:

(a) (i) 48

(ii) 100

(b) $\frac{1}{5}$

(c) (i)
$$\begin{aligned} 20x + 40y + 60z &= 1120 \\ 40x + 60y + 80z &= 1720 \\ 60x + 80y + 120z &= 2480 \end{aligned}$$

(ii)
$$\begin{pmatrix} 20 & 40 & 60 \\ 40 & 60 & 80 \\ 60 & 80 & 120 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1120 \\ 1720 \\ 2480 \end{pmatrix}$$

(iii) $x = 12$; $y = 10$; $z = 8$

Paper 03/1 – Internal Assessment

This year, 171 Unit 1 and 141 Unit 2 internal assessments were moderated. Far too many teachers continue to submit solutions without unitary mark schemes. In some cases, neither question papers with solutions nor mark schemes were submitted. In an increasing number of cases, marks awarded were either too few or far too many. For example: an entire IA module test was worth 20 marks and in another case a simple polynomial division was awarded 78 marks.

The majority of samples submitted were not of the required standard. Teachers must pay particular attention to the following guidelines and comments to ensure effective and reliable submission of the internal assessments.

The internal assessment is comprised of three module tests.

The main features assessed are:

- Mapping of the items tested to the specific objectives of the syllabus for the relevant Unit
- Content coverage of each module test
- Appropriateness of the items tested for the CAPE level
- Presentation of the sample (module test and students' scripts)
- Quality of teachers' solutions and mark schemes
- Quality of teachers' assessments, that is, consistency of marking using the mark schemes
- Inclusion of mathematical modelling in at least one module test for each unit

GENERAL COMMENTS

1. Too many of the module tests comprised items from CAPE past examination papers.
2. Untidy 'cut and paste' presentations with varying font sizes were common place.
3. Teachers are reminded that the CAPE past examination papers should be used **only** as a guide.
4. The stipulated time for module tests must be strictly adhered to as students may be at an undue disadvantage when Module tests are too extensive or insufficient.
5. The following guide can be used: one minute per mark. About 75 per cent of the syllabus should be tested and mathematical modelling **must** be included.
6. Multiple-choice questions will **not** be accepted as the entire module test but the test may include some multiple-choice items.
7. Cases were noted where teachers were unfamiliar with recent syllabus changes. For example,

    - Complex numbers and the Intermediate Value Theorem are now tested in Unit 2.
    - Three dimensional vectors, dividing a line segment internally and externally, systems of linear equations have been removed from the CAPE syllabus (2008).

8. The moderation process relies on the validity of teachers' assessment. There were a few cases where students' solutions were replicas of the teachers' solutions – some contained identical errors and full marks were awarded for incorrect solutions. There were also instances where the marks on students' scripts did not correspond to the marks on the moderation sheet.
9. Teachers must present evidence of having marked each individual question on students' scripts before a total is calculated at the top of the script. The corresponding whole number score out of 20 should be placed at the front of students' scripts. To enhance the quality of the design of the module tests, the validity of the teacher assessment and the validity of the moderation process, the internal assessment guidelines are listed below for emphasis.

Module Tests

(i) Design a separate test for each module. The module test **must** focus on objectives from that module.

(ii) In cases where several groups in a school are registered, the assessments should be coordinated, common tests should be administered and a common marking scheme used.

(iii) One sample of **five** students will form the sample for the centre. If there are less than five students **all** scripts will form the sample for the centre.

(iv) In 2010, the format of the internal assessment remains unchanged.

GUIDELINES FOR MODULE TESTS AND PRESENTATION OF SAMPLES

1. **COVER PAGE TO ACCOMPANY EACH MODULE TEST**

The following information is required on the cover of each module test.

- Name of school and territory, name of teacher, centre number
- Unit number and module number
- Date and duration of module test
- Clear instructions to candidates
- Total marks allocated for module test
- Sub-marks and total marks for each question **must** be clearly indicated

2. **COVERAGE OF THE SYLLABUS CONTENT**

- The number of questions in each module test must be appropriate for the stipulated time.
- CAPE past examination papers should be used as a guide ONLY.
- Duplication of specific objectives and questions must be avoided.
- Specific objectives tested must be from the relevant unit of the syllabus.

3. **MARK SCHEME**

- Detailed mark schemes MUST be submitted, that is, one mark should be allocated per skill (not 2, 3, 4 marks per skill)
- Fractional or decimal marks MUST NOT be awarded. (that is, Do not allocate $\frac{1}{2}$ marks).
- A student’s marks MUST be entered on the front page of the student’s script.
- Hand written mark schemes MUST be NEAT and LEGIBLE. The **unitary** marks MUST be written on the right side of the page.
- **Diagrams MUST be neatly drawn with geometrical/mathematical instruments**.

PRESENATION OF SAMPLE

- Students' responses MUST be written on letter sized paper ($8\frac{1}{2}$ x 11).
- Question numbers MUST be written clearly in the left hand margin.
- The total marks for EACH QUESTION on students' scripts MUST be clearly written in the left or right margin.
- ONLY ORIGINAL students' scripts MUST be sent for moderation.
- Photocopied scripts WILL NOT BE ACCEPTED.
- Typed module tests MUST be NEAT and LEGIBLE.
- The following are required for each Module test:

    - ❖ A question paper
    - ❖ Detailed solutions with detailed unitary mark schemes.
    - ❖ The question paper, detailed solutions, mark schemes and five students' samples should be batched together for each module.

- Marks recorded on PMath–3 and PMath2–3 forms must be rounded off to the nearest whole number. If a student scored zero, then zero must be recorded. If a student was absent, then absent must be recorded. The guidelines at the bottom of these forms should be observed. (See page 57 of the syllabus, no. 6.)