

CARIBBEAN EXAMINATIONS COUNCIL

**REPORT ON CANDIDATES' WORK IN THE
CARIBBEAN ADVANCED PROFICIENCY EXAMINATION**

MAY/JUNE 2006

PURE MATHEMATICS

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CARIBBEAN ADVANCED PROFICIENCY EXAMINATION
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INTRODUCTION

This is the second year that the current syllabus has been examined. There has been a significant increase in the number of candidates writing the examinations, approximately 4430 for Unit 1 compared to 2405 in 2005 and 1500 compared to 885 for Unit 2. Performances varied across the spectrum of candidates with an encouraging number obtaining excellent grades, but there continues to be a large cadre of candidates who seem unprepared for the examination.

GENERAL COMMENTS

UNIT 1

Overall performance in this Unit was satisfactory with a number of candidates excelling in such topics as the Factor/Remainder Theorem, Coordinate Geometry as contained in the Unit, Basic Differential and Integral Calculus, and Curve-sketching. However, many candidates continue to find Indices, Limits, Continuity/Discontinuity and Inequalities challenging. These topics should be given special attention by teachers if improvements in performance are to be achieved. General algebraic manipulation of simple terms, expressions and equations also require attention.

DETAILED COMMENTS

UNIT 1
PAPER 01
SECTION A
(Module 1: Basic Algebra and Functions)

Question 1

This question sought to examine, in Part (a), knowledge relating to substitution and the factor/remainder theorems, and in Part (b), to the use of summation via the Σ notation.

- (a) (i), (ii) Candidates continue to demonstrate weaknesses in making substitutions for values in algebraic expressions. Substituting $x=1$ resulted in the expression $1^4 - (p+1)^2 + p$. However, beyond this result some candidates failed to conclude that $x^4 - (p+1)x^2 + p$ is a factor of $f(x) = x^4 - (p+1)x^2 + p$, for $x=1$.

Many candidates attempted the division

$$x-1 \overline{) x^4 - (p+1)x^2 + p}$$

but could not proceed beyond the point of obtaining

$$\begin{array}{r} x^3 \\ x-1 \overline{) x^4 - (p+1)x^2 + p} \\ \underline{x^4 - x^3} \end{array}$$

A few candidates earned full marks in this question, demonstrating that they fully understood the concepts of substitution and the remainder/factor theorems.

- (b) The majority of candidates who could not show the required result failed to use the fact that

$$\sum_{r=1}^n 1 = n.$$

Substitution for $\sum_{r=1}^n 3r = 3 \frac{n}{2}(n+1)$ posed little difficulties.

Answer: (a) (ii) $p = 4$

Question 2

This question tested the modulus function, sets and simple identities on real numbers:

- (a) Generally, this question was poorly done. Many candidates solved the equation $|x-4| = h$, using the values of $x=2$ and $x=7$, to obtain values of $h=2$ and $h=3$. However, they could not proceed to the largest value of h .

Some candidates attempted to solve

$$(x-4)^2 = h^2$$

and had difficulties in answering the question. Very few candidates gained full marks on this question.

- (b) Expansion of algebraic expressions continues to be an area of weakness, particularly those involving mixed terms such as x, y and $\frac{1}{2}y$. Those candidates who expanded the

left-hand side correctly and simplified the result were able to find the correct value of k .

Answers: (a) Largest $h = 2$

$$(b) k = \frac{3}{4}$$

Question 3

The topics examined in this question were indices, surds and inequalities.

(a) (i), (ii) Rational inequalities that involve solving $\frac{f(x)}{g(x)} \leq (\geq) c$, generally result in

candidates solving $f(x) \leq (\geq) c[g(x)]$.

Partial solutions do not give the full range of values of x for which the inequality is true. A few candidates showed a fair understanding of the methods required for finding the correct solutions. Many candidates used the technique requiring a table with change of signs for $ax + b$ and $x + 1$ to obtain the range of values of x for the solution. No candidate used the technique of the number line and the region of like signs for greater than or equal to zero (≥ 0) or the region of unlike signs for less than or equal to zero (≤ 0).

(b) It is surprising that many candidates are still finding it difficult to use simple manipulation of indices to evaluate results. Evidence of failing to express $\sqrt{2}$ as $2^{1/2}$ and to simplify $8^{-1/3}$ were observed. Many candidates failed to obtain full marks for showing the final expression $2^4(\sqrt{2})$.

Answer(s): (a) (i) $a = 1$, $b = -2$; (ii) $x > 2$ or $x < -1$.

Question 4

This question focused on functions and their properties.

(a) (i) Several candidates were able to get the correct values for p and q . Many candidates were able to simply read the values from the graph by using the boundary values of the domain and the expression for $f(x)$.

(ii) The concept of range was not fully understood by many candidates.

(b) (i),(ii),(iii) Candidates continued to show poor understanding of surjective, injective and bijective functions. Some candidates used the horizontal line test for the injective function. Others used the vertical line test for the surjective function.

Many of the responses were in fact essays explaining when a function is surjective and/or injective. More mathematical examples should be practised by candidates to enhance understanding of these concepts as they relate to functions.

- Answer(s): (a) (i) $p = 2, q = 1$; (ii) $1 \leq f(x) \leq 2$
(b) (i) surjective since $1 \leq f(x) \leq 2$ for each $-1 \leq x \leq 1$
(ii) not injective since $f(-1) = f(1) = 2$
(iii) no inverse since f is not injective

Question 5

This question examined the solution of a system of two simultaneous linear equations with two unknowns.

- (a) Not many candidates used the method of the non-singular matrix to obtain the condition for a unique solution of a system of equations. Candidates used the idea of the coefficients of x and y not being equal in order to find the condition for a unique solution. However, candidates failed to reason that $n \in \mathbb{R}$ hence lost marks for this question.
- (b), (c) It was not difficult for several candidates to deduce the values of m for inconsistent and infinitely many solutions, after finding the correct solution to Part (a). Some incorrect values were given for n .

Answer(s): (a) $m \neq 4$ for any $n \in \mathbb{R}$

(b) $m = 4, n \neq 2$

(c) $n = 2, m = 4$

SECTION B

(Module 2: Plane Geometry)

Question 6

This question tested some of the salient properties of the intersection of perpendicular lines and the perpendicular distance of a given point from a given line.

- (a) Most candidates recognized that the gradient of PQ was 2 and correctly found the equation of the line PQ. Few candidates found it difficult to determine the required gradient but using the gradient obtained, they were able to find an equation.

- (b) Several candidates obtained full marks for this part. Those who followed through from Part (a) also recognized that by solving the pair of equations simultaneously, they would find the coordinates of point Q.
- (c) Most candidates used the correct formula in finding the exact length of the line segment PQ. However, the majority, after having found $\sqrt{5}$ proceeded to approximate to 3 significant figures.

In general, the question was well answered.

Answer(s): (a) $y = 2x + 3$; (b) $Q = (1, 5)$; (c) $PQ = \sqrt{5}$ units

Question 7

The question tested knowledge of the cosine formula.

Many candidates did not know how to find $\cos \frac{2\pi}{3}$. Some seemed unfamiliar with the cosine rule while others did not know the meaning of 'exact length'. For a topic which has been examined so often, too many candidates found the question difficult; however, despite the shortcomings of some candidates, there were a few excellent answers.

Answer(s): (a) $AC = 13$; (b) $AB = 13\sqrt{2}$

Question 8

The focus of this question was quadratic equations in trigonometric functions, and trigonometric identities.

In general, this question was reasonably well-done; however, poor algebraic manipulation hindered several candidates' progress in both parts. Due care is required in writing signs when a substitution is made. Some candidates did not give the answers in Part (a) in radians while a few had difficulties in solving a quadratic equation involving a trigonometric function.

Answer(s): (a) $\theta = \frac{\pi}{6}, \frac{5\pi}{6}$

Question 9

The question examined complex numbers and the roots of quadratic equations as they relate to the coefficients of the equations.

The overall performance on the question was poor. Again poor algebraic manipulation was evident and in some instances the roots of the equation were not multiplied to obtain a value for k . In Part (b), multiplying numerator and denominator of the left hand side by $3+4i$ did not occur to many candidates while to some, the rationalisation and equating of real and imaginary parts presented insurmountable challenges. A few candidates did well on this question.

Answer(s): (a) $k = 13$ (b) $u = 11, v = 2$

Question 10

This question tested properties of vectors including perpendicularity.

The response to this question was satisfactory, nevertheless, several candidates lost credit for faulty algebraic manipulation. While the basic concepts seem to be understood, errors due to carelessness spoiled the correctness of answers for many.

Answer(s): $x = -3, y = 1$

SECTION C

Question 11

The question tested basic knowledge of limits and continuity/discontinuity.

Factorization and substitution were the two main areas of weakness in the efforts of the candidates. Most candidates showed familiarity with the concepts but errors in factorization of either numerator or denominator were the main obstacles towards achieving complete and correct solutions.

Answer(s): (a) -3 (b) $x = \pm\sqrt{3}$

Question 12

The focus of this question was differentiation and the critical values of a function.

Most candidates attempted this question but not many obtained maximum marks. Several did not

seem to be familiar with the term 'critical' as it applies to curves. Many errors occurred due to faulty algebra and weakness in simplifications.

Answer(s): (a) critical value at $(4, -\frac{1}{8})$, a minimum

(b) $f'(x) = 4x \sin x^2 \cos x^2$

Question 13

This question tested knowledge of stationary points of a cubic curve and of methods used to obtain the equation of a normal to the curve at a given point.

- (a) Many candidates obtained full marks on this part of the question but others had difficulties in separating out the values of x for the stationary points.
- (b) There were many good answers to this question but again faulty algebraic manipulation was problematic in a few cases.

Answer(s): (a) $A \equiv (-2, 16)$, $B \equiv (2, -16)$

(b) Equation of the normal is $y = \frac{1}{12}x$ or $12y = x$

Question 14

This question focused on area under the curve and the integration of simple functions to obtain such an area.

Several candidates obtained full marks for this question. A few had difficulty expressing the shaded region in Part (a) as a difference of two integrals while others ignored $\int_2^3 dx$ in the calculations in Part (c). A few candidates chose to find the area of the triangle as a means of calculating the required area in Part (c).

Answer(s): (a) $\int_2^3 \frac{16}{x^2} dx - \int_2^3 (\frac{1}{2}x - 1) dx$

(b) $A = 2.42 \text{ units}^2$

Question 15

This question focused on a fundamental principle of definite integrals and the use of substitution towards the evaluation of such integrals.

Not many candidates attempted this question. However, among those who attempted it, a few

earned full marks and most of the others at least 4 marks. The main difficulty seemed to be a misinterpretation of the question which required direct use of

- (i) the given result in Part (a) applied to $f(x) = x \sin x$
- (ii) the identity $\sin(\pi - x) = \sin x$

The only integration involved required candidates to find _____ in Part (c).

UNIT 1
PAPER 02
SECTION A
(Module 1: Basic Algebra and Functions)

Question 1

This question tested the candidates' ability to solve two simultaneous equations in two unknowns, one being quadratic and one being linear, as well as to demonstrate the relationship between the sum and product of roots and coefficients of $ax^2 + bx + c = 0$.

The question was generally well answered with many candidates scoring the maximum 20 marks.

- $\int_0^{\pi} \cos x \, dx$
- (a) In cases where candidates attempted to make y the subject of either formula there were problems in expanding the brackets after the substitution was made.

For example, $x - 3\left(\frac{6 - x^2}{x}\right) + 1$ was **incorrectly** expanded as

$$x - \frac{18 - 3x^2}{3x} + 1$$

- (b) Many candidates
- (i) equated $\alpha + \beta$ incorrectly
 - (ii) represented $\alpha^2 + \beta^2$ incorrectly
 - (iii) failed to put the expression $x^2 + 2x - 2$ equal to zero as required (that is, a quadratic equation).

Answer(s): (a) $x = -\frac{9}{4}, y = -\frac{5}{12}$ and $x = 2, y = 1$

(b) (i) $\alpha + \beta = -4, \alpha\beta = 1$

(ii) $\alpha^2 + \beta^2 = 14$

(iii) Equation:
 $x^2 + 2x - 2 = 0$

Question 2

This question tested the principle of mathematical induction and the use of the sigma (Σ) notation.

Many candidates performed below average in this question, especially in (b) and (c). Only a very small percentage of candidates earned the maximum 20 marks.

- (a) From the responses it was evident that candidates showed some improvement over previous years. However, few candidates earned full marks.

Some weaknesses observed were:

- The use of the right hand (RHS) only (instead of both LHS and RHS) to establish that $P(1)$ is true.
- The inductive step was incorrectly obtained by some candidates who replaced k with $k + 1$, thus obtaining $P(k+1) = \frac{1}{2}(k+1)(k+2)$ instead of $P(k+1) = \frac{1}{2}k(k+1) + k + 1$.
- Incorrect conclusions involving $\forall n \in Z, \forall n \in R$ instead of $\forall n \in Z^+, \forall n \in N$ or equivalent.

- (b) (i) There were poor responses to this part. Candidates demonstrated a lack of understanding of the concept tested in this part of the question. Some candidates multiplied the expression by 2 thereby **incorrectly** obtaining

$$2\left(\frac{1}{2}n(n+1)\right) \text{ for } \sum_{r=1}^{2n} r.$$

- (ii) This part of the question was poorly done by the majority of candidates who failed to

recognize that $\sum_{r=n+1}^{2n} r = \sum_{r=1}^{2n} r - \sum_{r=1}^n r$.

- (c) Many candidates failed to see that this part was a continuation from (b) (i) and (ii), so that very few correct answers were obtained.

For example, $\sum_{r=n+1}^{2n} r = 100$ was incorrectly interpreted by the weaker candidates as

$n+1=100$, that is, $n=99$.

Answer(s): (b) (i) $\sum_{r=1}^{2n} r = n(2n+1)$

$$(ii) \sum_{r=n+1}^{2n} r = \frac{1}{2}n(3n+1)$$

(c) $n=8$

SECTION B (Module 2: Plane Geometry)

Question 3

This question dealt with the geometry of the circle and tested the candidates' ability to find the centre and radius, given the equation of a circle in the Cartesian form, to obtain parametric equations from the Cartesian form and to find the points of intersection of a curve with a straight line.

- (i) This part was generally well done, however, some candidates encountered problems converting the given equation into the form $(x-a)^2 + (y-b)^2 = r^2$. Problems arose when candidates had to complete the square. Candidates who expanded to find 'f' and 'g' usually forgot to change signs for the coordinates of the centre. Others factorised **incorrectly** to find the centre usually by grouping like terms, for example, $x(x+2) + y(y-4) = 4$ to obtain **incorrectly** that radius = 4 and centre = (2,-4).
- (ii) This part was poorly done. Candidates generally did not show any understanding of the concepts involved. A preferred method was substitution of the parametric equations using $\sin^2 \theta + \cos^2 \theta = 1$ to obtain the original equation given in (i).
- (iii) This part of the question was generally well done; however, substitution of $y = 1-x$ into $x^2 + 2x + y^2 - 4y = 4$ was the preferred method leading to the quadratic equation $2x^2 + 4x - 7 = 0$ from which was obtained the required solution.
- (b) This question was well done. Most candidates, however, did not exhibit a full understanding of 'General Solution' and stopped after finding the principal values of θ .

Answer(s): (a) (i) Centre $(-1, 2)$; radius = 3 units

$$(b) \quad \theta = 2n\pi \pm \frac{\pi}{3}, \quad 2n\pi \pm \pi$$

Question 4

This question covered topics related to trigonometric functions of the form $a \cos x + b \sin x$ and complex numbers.

Most candidates showed familiarity with the concepts involved in both parts of this question, however, in Part (a) the notion of stationary point confused some candidates.

In Part (b), many candidates did not see the connection between (iii), (i) a) and b), and the roots of quadratic equations.

Some good answers were received:

Answer(s): (a) (i) $R = 4.1$, $\alpha = 14^\circ$;

(ii) $x = 104^\circ$

$$(b) (i) \text{ a) } 5+i, \text{ b) } 18-i, \text{ c) } -\frac{6}{25} - \frac{17}{25}i$$

$$(ii) \text{ Equation: } z^2 - (5+i)z + (18-i) = 0$$

SECTION C **(Module 3: Calculus 1)**

Question 5

This question covered the topics of differentiation from first principles of the function $y = \sin 2x$ and the application of differentiation in obtaining the gradient and equation of a tangent to a given curve.

This question was generally well done by several candidates. Part (a)(iii) proved to be challenging for some candidates, particularly those who experienced difficulties in obtaining A and B correctly in (a) (ii).

In Part (b), some candidates had minor difficulties in differentiating $y = hx^2 + \frac{k}{x}$ but apart from those, many candidates found this part easy.

$$\text{Answer(s): (a) (i) } \lim_{\delta x \rightarrow 0} \frac{\sin \delta x}{\delta x} = 1; \text{ (ii) } A = 2x + \delta x, B = \delta x$$

$$(b) (i) \quad h = 2, k = -1; \quad (iii) \text{ Equation: } 2y = 12x - 9$$

Question 6

The topics tested in this question involved integration by means of the rectangular rule, and differentiation and integration of rational functions.

- (a) (i) Most candidates used the trapezium rule instead of the required rectangular rule.
- (ii) Many candidates were unable to show the required equation for the approximate area S using the given sum $\sum_{r=1}^{n-1} r = \frac{1}{2}n(n-1)$. The factorization of the expression for S in (a) (i) was clearly not recognized by the candidates.
- (b) (i) This part was well done. Students correctly identified that the quotient rule was needed. There were some instances where candidates rearranged the expression for $f(x)$ and used the product rule as an alternative.
- (ii) Candidates' performance on this part of the question was satisfactory. Most students took notice of the "Hence" part of the question and realized that the integral of the expression given involved using a scalar multiple of $f(x)$.
- (c) There were some good results for this question. Some candidates, however, did not notice that the integration process involved a negative index and proceeded to treat the index as a positive number. For most candidates, the 'solving process' was well done.

Answer(s): (b) (ii) $\frac{6}{5} = 1.2$; (c) $u = 2$.

UNIT 1
PAPER 03/B (ALTERNATE TO INTERNAL ASSESSMENT)
SECTION A
(Module 1: Basic Algebra and Functions)

Question 1

This question focused on the modulus function, indices, quadratic equations and properties of functions in general.

- (a) (i) There were a few good responses to this part among the small number of candidates, nevertheless, some candidates obtained only one value of x because one or other of the two possible equations $x + 4 = 2x - 1$, $x + 4 = -(2x - 1)$ was ignored.

(ii) Indices continue to present difficulties to many candidates. Too often candidates

incorrectly obtained $\frac{x^2}{4}$ from the expression $\frac{3^{x^2}}{81}$.

(b) Candidates seemed not to understand the basic definition of a function and so had difficulty in doing Parts (i) and (ii), and in recognising a function as a set of ordered pairs.

Answer(s): (a) (i) $x = 5$ or $x = -1$ (ii) $x = 4$ or $x = -2$

(b) (i) v maps to both 1, 3 or w is not mapped to any $b \in B$.

(ii) Delete one of the arrows from v to 1 or 3 and map w to any $b \in B$.

(iii) $g = \{(u,1), (v,1), (x,2), (y,4), (w,b), \text{ any } b \in B\}$ or
 $g = \{(u,1), (v,3), (x,2), (y,4), (w,b), \text{ any } b \in B\}$

SECTION B (Module 2: Plane Geometry)

Question 2

This question tested a linear function model of an experiment, trigonometrical identities and some basic properties of complex numbers.

- (a) The initial value $d = 0$ corresponding to $w = 500$ in the table in (i) was routinely missed by the candidates and this adversely affected the outcomes to the entire Part (a). Few correct answers were received.
- (b) Both identities presented difficulties due mainly to faulty algebra, however, there were one or two correct derivations in both cases.
- (c) Candidates showed some familiarity with complex numbers but a few seemed not to know how to find $\arg(z)$ in Part (ii). None used the fact that $z \bar{z} = |z|^2$ which connected Part (i) with Part (iii).

Answer(s): (a) (i)

d (day)	0	25
w (gm)	500	1500

(ii) (a) $w = f(d) = 40d + 500$

(b) $w = 900$

(iii) $d = 42$

(c) (i) $|z| = 1$

(ii) $\arg(z) = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{6}$

(iii) $z\bar{z} = 1$

SECTION C
(Module 3: Calculus 1)

Question 3

The topics covered in this question were limits, integration and volume of rotation.

- (a) There were some good answers to the limits posed in this part.
- (b) This part was not well done. The separation of $\int_2^3 [f(x) + 4] dx$ did not come readily to all but a very few of the small number of candidates.
- (c) That rotation was around the y-axis was ignored by almost all of the candidates.

Answer(s):

(a) (i) $\lim_{x \rightarrow 4} \frac{\sqrt{x-2}}{x-4} = \lim_{x \rightarrow 4} \frac{1}{\sqrt{x}+2} = \frac{1}{4}$

(ii) $\lim_{x \rightarrow 4} \frac{\sqrt{x}-2}{x^2-5x+4} = \lim_{x \rightarrow 4} \frac{\sqrt{x}-2}{x-4} \lim_{x \rightarrow 4} \frac{1}{x-1}$

$$= \frac{1}{4} \times \frac{1}{3}$$

$$= \frac{1}{12}$$

(b) $\int_2^3 [f(x) + 4] dx + \int_3^5 f(x) dx = \int_2^5 f(x) dx + \int_2^3 4 dx$

(c) (i) $\frac{\pi}{2}$ (ii) π

GENERAL COMMENTS

UNIT 2

In general, the performance of candidates in Unit 2 was of a high standard with a small number of candidates reaching an outstanding level of proficiency. However, there were some candidates who were inadequately prepared for the examination.

Topics in Calculus, simple probability, approximation to roots of equations and series seemed well covered but weaknesses continue to manifest themselves at the level of algebraic manipulation, including substitution, which frustrate the processes required to complete the problem-solving exercises posed in the questions. As recommended in previous years, extended practice on respective themes needs to be undertaken in order to eradicate such deficiencies and raise the level of performance in the identified areas of weakness.

The results on the whole were very encouraging.

DETAILED COMMENTS

UNIT 2 PAPER 01 SECTION A (Module 1: Calculus II)

Question 1

This question examined the use of logarithms in solving equations. The majority of the candidates performed well on this question.

- (a) Candidates performed better in this part of the question than in Part (b). Despite this, some errors were evident. For instance, $(\log x)^2$ was incorrectly interpreted as $\log x^2$ or $2 \log x$.
- (b) Many candidates were aware of the principle/procedure involved, but premature rounding off of values affected the accuracy of the answer. In some cases, $\log 5 - \log 3$ was incorrectly represented as $\frac{\log 5}{\log 3}$ instead of $\log (5/3)$.

Answer(s): (a) $x = 4$ or $x = 2$

(b) $x = 3.15$

Question 2

This question tested the candidates' ability to differentiate (using the chain rule or otherwise), combinations of trigonometric and logarithmic functions as well as to find the derivative of $e^{f(x)}$ and $\ln f(x)$, where $f(x)$ is a differentiable function of x .

The question was generally well done by the many candidates who attempted it, with approximately half of them scoring the maximum mark.

- (a) Many candidates omitted the brackets and wrote

$$e^{2x+\sin x} 2+\cos x \text{ or } 2+\cos x e^{2x+\sin x}$$

instead of $(2+\cos x) e^{2x+\sin x}$

- (b) The most common errors were:

(i) $\frac{d}{dx}(\tan 3x) = \sec^2 x \text{ or } \sec^2 3x$

(ii) $\frac{d}{dx}(\ln(x^2 + 4)) = \ln\left(\frac{2x}{x^2 + 4}\right) \text{ or } \frac{1}{x^2 + 4}$

Answer(s): (a) $\frac{dy}{dx} = e^{2x+\sin x} (2 + \cos x)$

(b) $\frac{dy}{dx} = 3\sec^2 3x + \frac{2x}{x^2 + 4}$

Question 3

This question tested the ability of candidates to use the concept of implicit differentiation to obtain the gradient of the curve at a point P and to use it to find the equation of the normal at a point P on the curve.

- (a) There were many good solutions to this part of the question. However, some candidates experienced problems with the implicit differentiation and the product rule. The transposing of terms in the equation posed a challenge in a few cases as well.
- (b) This part of the question was generally very well done. About 95% of the candidates were able to obtain the correct gradient of the normal from the gradient of the curve in Part (a), and the subsequent equation.

Answer(s): (a) $\frac{dy}{dx} = \frac{-1}{2}$

(b) Equation: $y = 2x + 5 \text{ or } y - 2x = 5$

Question 4

This question examined the candidates' knowledge about applying the chain rule to find the first and second derivatives of trigonometric functions involving $\sin 2A$ and $\cos 2A$.

The response to this question was very good with many candidates earning the maximum mark.

- (a) Many candidates differentiated $\sin 2A + \cos 2A$ as composite functions. This was efficient and full marks were obtained. However, few candidates transformed $\sin 2A + \cos 2A$ using the trigonometric identities $\sin 2A = 2 \sin A \cos A$, and $\cos 2A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A$. This was longwinded and often included errors where candidates were unable to complete the solution successfully. Some candidates used the chain rule effectively in this regard.
- (b) Candidates had a good level of success in obtaining $\frac{d^2 y}{dx^2}$ using composite functions. Some candidates used other methods which were complex/complicated, without success. The proof of $\frac{d^2 y}{dx^2} + 4y = 0$ was well done using the correct substitution for $\frac{d^2 y}{dx^2}$ and y .

Answer(s): (a) $\frac{dy}{dx} = 2 \cos 2x - 2 \sin 2x$

(b) Since $\frac{d^2 y}{dx^2} = -4 \sin 2x - 4 \cos 2x$

$$= -4y$$

$$\therefore \frac{d^2 y}{dx^2} + 4y = 0$$

Question 5

Candidates were required to use given substitutions to integrate functions.

- (a) This part of the question was generally well answered with the majority of candidates earning the maximum of 4 marks for this part. There were some attempts at integration by parts although the question specifically stated "use the substitution given".
- (b) This part proved to be a bit more problematic, as it involved three substitutions. There were difficulties in obtaining $dx = u \, du$, with some candidates incorrectly obtaining $dx = \frac{du}{u}$ instead.

The omission of the constant of integration was not frequently seen. The manipulation of indices, and transposition continue to be challenging to some candidates.

Answer(s): (a) $\frac{1}{9}\sin^9 x + k$ (constant of integration)

(b) $\frac{1}{10}(2x+1)^{5/2} - \frac{1}{6}(2x+1)^{3/2} + k$ (constant of integration).

or $\frac{1}{10}\sqrt{(2x+1)^5} - \frac{1}{6}\sqrt{(2x+1)^3} + k$ (constant of integration).

SECTION B (Module 2: Sequences, Series and Approximations)

Question 6

- (a) This part of the question tested candidates' knowledge and ability to manipulate recurrence relations. Most candidates attempted this question, however, there was a high percentage of candidates who responded incorrectly. The main source of error encountered was that candidates substituted values for n and showed by example that $u_{n+2} = -u_n$ and $u_{n+4} = u_n$. Most candidates showed $u_{n+4} = u_n$ by first finding u_{n+3} and substituting (a long method). They generally did not note the relation

$$u_{n+4} = -u_{n+2} \rightarrow u_{n+4} = -(-u_n).$$

- (b) This part of the question required candidates to write specific terms from the recurrence relations. Most candidates were able to identify correctly the required terms. However, a common error encountered was the use of u_1 as -3 (disregarding the given data; $u_1 = 1$).

Answer(s): (b) $u_1 = 1$ (given), $u_2 = -3$, $u_3 = -1$, $u_4 = 3$

Question 7

This question tested knowledge about summing a geometric series to n terms as well as finding x and d given the sum and product of three consecutive terms, $x-d$, x and $x+d$ of an arithmetic series.

Most candidates attempted this question with about ninety percent gaining at least six marks.

- (a) Most candidates identified the common ratio and were able to use the sum formula for a GP.
- (b) Many candidates who attempted this part obtained the maximum marks. The majority summed the terms and found the value of x . They then substituted this value in the product equation and solved correctly for d . However, a few candidates ignored the fact that $d > 0$ and left their answer as $d = \pm 2$.

Answer(s): (b) (i) $x = 7$, $d = 2$

Question 8

The question tested the use of the binomial term nC_r , quadratic equations and inequalities.

Approximately 80% of the candidates attempted this question in which 60% were able to earn full marks. However, there were some candidates who experienced difficulties in parts of this question, for example, with the binomial expansion of ${}^{x-2}C_2$.

Such candidates lacked the basic building block on binomial coefficients which seemed not to be known by some of the candidates. Some candidates also failed to answer the questions asked in Part (a) and went straight ahead to solve for x , for example, ${}^{x-2}C_2 = {}^5C_2 \Rightarrow x-2=5 \Rightarrow x=7$.

Answer(s): (b) $x=7$

Question 9

This question focused on the expansion of the expression $(1+ux)(2-x)^3$ in powers of x up to the term in x^2 . Candidates were required to find the value of the constant ' u ', based on a given condition of a specific coefficient in the expansion.

- (a) Many candidates were able to expand the expression properly but a significant number of them did not stop at x^2 ; instead they expanded the expression completely.
- (b) Several candidates had elementary problems with signs and as a result, they were unable to obtain the correct coefficient of x^2 and lost marks. Thus, although they identified the terms properly, they failed to give the correct answer for u .

Answer(s): (a) $8 + (8u - 12)x + (6 - 12u)x^2 + \dots$

$$(b) \quad u = \frac{1}{2}$$

Question 10

The question tested the candidates' knowledge of intersecting graphs and the algebraic equation represented at the point of intersection. It also covered roots in a specific interval of the real number line.

Most candidates were able to write down the equation required. The majority of candidates were able to determine that the function (equation) had values with different signs at the end points of the interval.

However, a large number of candidates did not point out that the function should be continuous. Some were insightful enough to say that the function was both continuous and differentiable.

While candidates established the existence of the root within the interval through a difference in signs of the value of the functions, a large number of them did not use the words 'intermediate value theorem' seemingly unaware that this was the theorem in use.

Generally, this question was well done.

Answer(s): (a) $e^x = -x$ or $e^x + x = 0$

SECTION C

(Module 3: Counting, Matrices and Modelling)

Question 11

This question tested the candidates' skills in using the binomial term nC_r in counting problems.

This question was generally well done by the majority of candidates. A few candidates who were unable to answer the question, applied the concept of 'permutation' rather than 'combination' that was required.

Answer(s): (a) 70 (b) 224 (c) 425

Question 12

The question tested arrangements of objects.

- (a) This question was generally well answered. However, some candidates had difficulty in distinguishing between permutations and combinations.
- (b) This part proved to be more difficult than Part (a) as candidates could not determine the denominator as $(7!)$, in the calculation of the required probability.

Answer(s): (a) (i) 3600 (ii) 2400 (b) $\frac{3600}{7!} = \frac{5}{7} = 0.714$

Question 13

The topics covered related to determinants and methods of their evaluation.

This question was successfully answered by a large majority of the candidates. Two methods were used by the candidates to answer the question. Of these, the more popular method related to the use of 'minors'.

Question 14

The topics covered in this question were systems of equations, cofactors of a matrix, transpose of a matrix, matrix multiplication and determinants.

- (a) Almost all candidates earned full marks on this part.
- (b) (i) Most candidates confused the matrix of cofactors of the matrix A with the determinant of A.
- (ii) Almost all candidates were able to write the transpose of B but too many were unable to form the matrix product $B^T A$.
- (iii) Few candidates, even among those who calculated $B^T A$ correctly, were able to deduce the $\det A$.

Answer(s): (a)
$$\begin{pmatrix} 1 & 1 & -1 \\ 2 & -1 & 1 \\ 3 & 0 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$$

(b) (i)
$$B = \begin{pmatrix} -2 & -1 & 3 \\ -2 & 5 & 3 \\ 0 & -3 & -3 \end{pmatrix} \quad (ii) \quad B^T A = \begin{pmatrix} -6 & 0 & 0 \\ 0 & -6 & 0 \\ 0 & 0 & -6 \end{pmatrix}$$

(iii) $|A| = -6$

Question 15

The question tested the candidates' knowledge of the differential calculus in determining rates of change. Few candidates attempted this question. The majority of those who attempted the question earned very low marks.

- (a) For Part (a) most candidates earned 1 mark.
- (b) For Part (b) most candidates failed to differentiate the function correctly. Most candidates substituted the values of h and r directly into $A = 2\pi r^2 + 2\pi rh$. Candidates attempted to use various methods to answer this question which had no relationship whatsoever to the question.

Answer(s): (a) $\frac{dr}{dt} = 1.5$

(b) $\frac{dA}{dt} = 54\pi \text{ cm}^2 / \text{sec}$ when $r = 4$, $h = 10$

UNIT 2
PAPER 02
SECTION A
(Module 1: Calculus II)

Question 1

This question tested differentiation of a function of a function by means of the product rule as well as mathematical modelling related to an exponential function.

- (a) (i) Basically, most candidates were able to apply the Product Rule for differentiation. However, many cases were observed where the differential of

$\ln^2 x$ was **incorrectly** given as $\frac{2}{x}$ and $2 \ln x$.

In addition, many candidates lost marks for inability to simplify the differential to show the required result.

- (ii) Those candidates who attempted to apply the Product Rule for the result at Part (a) found it difficult to differentiate the expression $\ln x (3 \ln x + 2)$. Many candidates used the result at Part (a) as $3x^2 \ln^2 x + 2x^2 \ln x$ and proceeded to apply the Product Rule. Failure of these candidates to differentiate $\ln^2 x$ correctly resulted in their inability to show the required result. Many weaknesses in algebra were evident in the candidates' work.
- (b) (i) A number of candidates gave the correct answer to this part of the question. Many of them substituted the values of $N = 50$ when $t = 0$ but failed to determine $e^{-r(0)} = 1$. Instead they carried forward the expression $1 + ke$. This resulted in the value of k given in terms of e . Failure to give the limit of ke^{-rt} for large t complicated the answers.
- (ii) A common error seen in the responses to this question was NOT calculating the EXACT value of r . The majority of students got the equation $e^{-r} = 0.2$ but used the calculator to find r .
- (iii) Having obtained the values for k and r there were no difficulties getting the correct answer to this question.

A few candidates gained full marks for the entire question.

Answer(s): (b) (i) $N = 800$ (ii) $k = 15$, $r = \ln 5$ (iii) $N = 714$

Question 2

The topics tested in this question related to partial fractions, integration of rational functions and reduction formulae.

- (a) (i) A few candidates **erroneously** used the result

$$\frac{1+x}{(x-1)(x^2+1)} = \frac{A}{x-1} + \frac{B}{x^2+1}$$

to find the partial fractions.

Other errors included wrong calculations for the values of x which were substituted.

- (ii) A few candidates incorrectly evaluated $\int \frac{-x}{x^2+1} dx$, particularly with respect to the minus sign. Many candidates failed to include the constant of integration.

- (b) (i) Some candidates found this part of the question difficult. It was clear that integration by parts was not fully understood by these candidates. Other candidates had difficulty evaluating.

$$(xe^x - e^x)_0^1$$

- (ii) Generally most candidates demonstrated that they knew how to proceed with this question. However, writing the integral as

$$I_n = x^n e^x - \int_0^1 e^x nx^{n-1} dx$$

without including the limits of integration in the first integral, resulted in

$$I_n = x^n e^x - nI_{n-1}.$$

Candidates simply quoted the given result for I_n thus obtaining partial credit.

- (iii) Many candidates attempted to find $\int_0^1 x^3 e^x dx$. The attendant difficulties were expected. Some candidates found it very difficult to use the formula for I_n and the link to I_1 .

A few candidates gained full marks for the question.

Answer(s): (a) (i)
$$\frac{1+x}{(x-1)(x^2+1)} = \frac{1}{x-1} - \frac{x}{x^2+1}$$

(ii)
$$\int \frac{1+x}{(x-1)(x^2+1)} dx = \ln|x-1| - \frac{1}{2} \ln(x^2+1) + k \text{ (const)}$$

(b) (i) $I_1 = 1$ (iii) $I_3 = 6 - 2e$

SECTION B

(Module 2: Sequence, Series and Approximations)

Question 3

This question tested arithmetic progressions, mathematical induction and sequences.

- (a) (i) This part required that candidates show that $\sum_{r=1}^m \ln 3^r$ is an arithmetic progression. Many candidates listed the terms in the progression with a comma between them, for example, $\ln 3, \ln 3^2, \ln 3^3$, and not $\ln 3 + \ln 3^2 + \ln 3^3 + \dots \ln 3^m$ as was expected. Many answers did not indicate the m^{th} term of the progression. Approximately 80 per cent of the candidates showed working to indicate the first term 'a' and the common difference, 'd'. Approximately 60 per cent of these candidates calculated the numerical value of $\ln 3$, rounding it off to the 3 s.f. for the most part, but some also rounded off to 1 s.f.

It must be noted that it is **unnecessary** to use 10 significant figures in the value for $\ln 3$. Almost fifty per cent of the candidates made a final statement to indicate that they understood that the progression was arithmetic in nature.

- (ii) Few candidates added all 20 terms. Approximately eighty per cent of candidates earned full marks for this question.
- (iii) Many candidates gained full marks for this part.
- (b) (i) Approximately five per cent of the candidates performed exceptionally well on this item. The remaining ninety per cent had a vague idea as to the steps involved in proof by mathematical induction. Steps such as:
Prove true for $n = 1$, assume true for $n = k$, were missing for the most part.

The final statement was also missing.
- (ii) Approximately ninety-five per cent of the candidates did not do this question correctly. Most of them did not understand that they should apply the technique of completing the square.

Answer(s): (a) (i) A.P. with first term $\ln 3$ and common difference $\ln 3$.
(ii) Sum to 20 terms is $210 \ln 3$.

(b) (ii)
$$x_{n+1} - x_n = \left(x_n - \frac{1}{2}\right)^2 > 0$$

$$\Rightarrow x_n < x_{n+1}$$

Question 4

This question tested the ability of the candidate to sketch curves and use the Newton-Raphson method to find the non-zero root of $\sin x - x^2$.

Part (a) was poorly done as all but a few students had problems sketching $y = \sin x$ and $y = x^2$. Most drew $y = x^2$ as either the straight line $y = x$ or as a V-shaped curve. Candidates also had some problems sketching $y = \sin x$. Candidates need to be reminded of the need to use an appropriate scale on each axis as many did not show the point of $(\frac{\pi}{2}, 1)$ for $\sin x$ nor the $(1, 1)$ for x^2 . In too many cases, the points of intersection were way off. Candidates should also have stated the domain for the sketches.

Part (b) was attempted by several candidates although they just mentioned that the 2 curves 'intersected at 2 points, hence there were 2 real roots'. Hardly anyone wrote that $x = 0$ and $x = \alpha$ (the non-zero root).

Part (c) was also attempted by many candidates. Most of them knew that they were supposed to use the Intermediate Value Theorem, but many used values of $x < 0$ and values other than $\frac{\pi}{4}, \frac{\pi}{2}$. Some used 0 and then said that 0 was positive. Many did not mention that the function was continuous.

Part (d) was attempted by almost all candidates. This was handled well by many, but the common mistake was the use of 0.7 as degrees instead of radians, hence, an incorrect answer was obtained. A few quoted the Newton-Raphson formula incorrectly. Many candidates did not notice that only one iteration was required and went on to find $x_2, x_3 \dots$ and up to x_8 (in a few cases).

Answer(s): (c) Interval: $[0, \frac{\pi}{2})$, (d) $x_2 = 0.943$

SECTION C

(Module 3: Counting, Matrices and Modelling)

Question 5

This question tested the candidates' knowledge of simple permutations and probability in Part (a) while Part (b) examined their knowledge based on a probability model.

- (a) (i) The majority of candidates who attempted this question realised that they were dealing with a 4-digit number, although a few considered a 6-digit number. Many candidates realised that the number must start with the digits 3, 4 or 5, but a few included the digit 6.
- (ii) In general this question was poorly answered, however, responses of candidates indicated that they were familiar with the concept of probability.
- (b) (i) It was clear that many candidates were not prepared to apply an unfamiliar formula to calculate the required probability, for example, binominal model. Some candidates exhibited serious misconceptions of probability as values were given outside of the interval $[0, 1]$

- (ii) Some candidates gave no consideration to possibilities that would have exhausted the sample space and chose to perform longer, rigorous calculations in order to arrive at their solution.

Answer(s): (a) (i) 180 (ii) $\text{Prob} = \frac{96}{180} = 0.533$

(b) (i) 0.543 (ii) 0.457

Question 6

The question tested basic knowledge about the product of conformable matrices and of finding the inverses of invertible matrices. Modelling is also included.

Almost all candidates attempted this question with many good responses. Part (a) focused on standard routine matrix operations while Part (b) focused on mathematical modelling incorporating matrices.

In Part (a), candidates frequently made arithmetic errors in calculating AB and this made it difficult to deduce A^{-1} from (a)(i). Others resorted to alternative methods of finding A^{-1} .

In Part (b), the weaker candidates seemed to have been challenged by the wording of the problem. The majority of them interchanged c, b, z , with p, q, r , and generated meaningless equations.

Another common error made by candidates was attempting to find M^{-1} although it was given in Part (b)(v).

Answer(s): (a) (i) $AB = 4I$ (ii) $A^{-1} = \frac{1}{4}, B = \frac{1}{4} \begin{pmatrix} 2 & -1 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}$

(b) (i) $Z - \text{grass} = 2p + 2q + 6r$

(ii) $2p + 4q = c$

$2p + 2q + 6r = z$

$6p + 4q + 4r = b$

(iii) $\begin{pmatrix} 2 & 4 & 0 \\ 2 & 2 & 6 \\ 6 & 4 & 4 \end{pmatrix} \begin{pmatrix} p \\ q \\ r \end{pmatrix} = \begin{pmatrix} c \\ z \\ b \end{pmatrix}$

(iv) $x = M^{-1}D$

(v) $p = 3, q = 6, r = 2.$

UNIT 2
PAPER 03/B (ALTERNATIVE TO INTERNAL ASSESSMENT)

SECTION A
(Module 1: Calculus II, Algebra and Fractions)

Question 1

The question tested the candidates' knowledge of functions, natural logarithms and differential equations based on a simple biomathematical model.

There were some good responses to this question from among the few candidates who wrote the examination. Some experienced difficulty in writing down the differential equation in Part (a), but for those candidates who got past Part (a) the results were encouraging. Part (b)(i) also posed some minor challenges.

Answer(s):

(a) $f'(t) = k f(t)$

(b)(i) $f(0) = 10^6 = 1\,000\,000$; $f(2) = 2 \times 10^6 = 2\,000\,000$.

(iii) $f(7) = 10^6 (2^{7/2}) \approx 11\,313\,709$

SECTION B
(Module 2: Sequences, Series and Approximations)

Question 2

This question examined sequences by means of mathematical modelling.

There were some excellent solutions to this problem from the small number of candidates. The only aspects of real difficulty for one or two candidates were Parts (a)(ii) and (iii).

Answer(s): (a)(i) row 1 - entries: Year 4 $- P(1 - \frac{1}{q})^3$; Year 5 $- P(1 - \frac{1}{q})^4$

row 2 - entries: Year 3 $- P(1 - \frac{1}{q})^3$; Year 4 $- P(1 - \frac{1}{q})^4$

Year 5 $- P(1 - \frac{1}{q})^5$

(ii) G.P. with common ratio $1 - \frac{1}{q}$

(iii) $P(1 - \frac{1}{q})^n$.

(b)(i) $q = 5$

(ii) \$6553.60

(iii) $n = 17$

SECTION C
(Module 3: Counting, Matrices and Modelling)

Question 3

The question covered the topic of random selections and row reduction of the augmented matrix of a given system of equations.

The candidates found this question manageable. Only one or two had difficulty concluding the inconsistency of the system in Part (b).

Answer(s): (a) (i) $\frac{56}{2002} = 0.028$ (ii) $\frac{560}{2002} = 0.280$ (iii) 0.972

PAPER 03

INTERNAL ASSESSMENT

Module Tests

Approximately 138 centres (161 Teachers) in Unit 1 and 90 centres (98 Teachers) in Unit 2 were moderated.

In general, there was a marked improvement in the quality, consistency of marks awarded and the presentation of the Internal Assessment (Module Tests) by the teachers throughout the participating territories. Unit 2 was of a very good standard.

Although most of the questions used were taken from past CAPE Pure Mathematics Examination Papers, it was evident that few teachers made a conscientious effort to be original and creative in the tests designed.

This year, the majority of the samples of tests were submitted with question papers, solutions and detailed marking schemes with the marks allocated to the cognitive levels as specified in the syllabus. There were eight of 161 teachers in Unit 1 and two out of 98 teachers of Unit 2 who submitted samples without the required documents, therefore, making the moderation process more difficult.

It should be noted that the majority of teachers satisfied the objectives outlined by CXC CAPE Pure Mathematics Syllabus, Unit 1 and Unit 2. However, some common mistakes observed throughout the moderation process included:

- (1) Test items examined were not consistent with the allotted time for the examination as some were either too long or too short. In one instance, there were 15 items with several parts to be completed by the candidates in 1 hour.

Teachers are reminded that the module test should be of 1 1/2 hours' duration.

- (2) A few teachers continued to award fractional marks.

- (3) On the question papers, teachers should indicate the time allotted, the total score for the examination, as well as instructions for the test.
- (4) In a few cases, teachers tested topics in Unit 1 that were not in the Unit 1 Syllabus, for example, Logarithms, Partial Fractions and Implicit Differentiation. It must be noted that '3-dimensional vectors' is not on the CAPE syllabus.
- (5) A maximum of 5 samples is required for moderation. Please note that additional samples are not needed, unless there is a specific request from the Council. (Refer to FORM PMATH - 2)
- (6) A few teachers are using the incorrect PMATH form to record marks. The scores for the three Modules (1, 2 and 3) scores must be recorded on the same form. (Form PMATH 2-3 Unit 2).
- (7) Marks for the candidates should be clearly identified for each question at the side of the student's solution; and the total at the top.
- (8) The maximum mark that is allocated to each question on the question paper should be reflected in the allocation of marks on the solution and the mark scheme.

Overall, for the efficiency of the moderation process, teachers should make every effort to adhere to the guidelines provided in the CAPE Pure Mathematics Syllabus.