CARIBBEAN EXAMINATIONS COUNCIL

REPORT ON CANDIDATES' WORK IN THE CARIBBEAN SECONDARY EDUCATION CERTIFICATE

JANUARY 2004

MATHEMATICS

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GENERAL COMMENTS

The General Proficiency Examination is offered in January and June each year. The Basic Proficiency is offered in June only.

There was a candidate entry of approximately 11,672 in January 2004. There is an overall improvement in this year's examination over 2003. The mean mark was 48.21. Fifty-seven per cent of the candidates achieved Grades I -III.

Candidates performed satisfactorily on Number Theory, Algebra and Sets. However, they performed poorly on Measurement, Geometry and Graphs.

DETAILED COMMENTS

The examination consists of two papers.

PAPER 01 - Multiple Choice

Paper 1 consists of 60 multiple choice items. Seven candidates scored full marks, and approximately 85 per cent of the candidates scored more than half the marks.

PAPER 02 - Essay

Paper 2 consists of eight compulsory questions in Section I and six optional questions in Section II, from which candidates are required to answer two. One candidate scored full marks and approximately 26 per cent scored more than half the marks.

<u>Question 1</u>

The question tested the candidates' ability to:

- perform the basic operations with rational numbers
- solve problems involving payments by instalments as in the case of hire purchase

The question was attempted by all of the candidates and the performance was generally good, with almost 28 per cent of the candidates scoring the full 10 marks. The mean mark was 7.6. Most candidates were able to calculate the cash price of the dining room suite and its total hire purchase price. However, candidates experienced difficulty calculating the extra cost of buying on hire purchase as a percentage of the cash price. It appears that many of them did not recognise that $880 was given as the cash price of the suite. Several candidates in attempting to divide by a fraction did not invert the second fraction.

In addition, a number of candidates wrote $1\frac{3 - 10}{15}$ with no further attempt at simplifying.

Solutions:

(a) $\mathbf{\frac{14}{15}}$

(b) **$105**

(c) **(i) $1 056** **(ii) 20%**

Question 2

The question tested the candidates' ability to:

- substitute numbers for algebraic symbols in simple algebraic expres sions
- change the subject of formulae
- solve simultaneous linear equations in two unknowns algebraically
- apply the distributive law to remove brackets in algebraic expressions

The question was attempted by 99.9 per cent of the candidates, about 12 per cent of these scored the full 12 marks. The mean mark was 6.81. Most candidates knew how to substitute numbers into algebraic expressions and how to solve a pair of simultaneous equations in two unknowns. They however experienced difficulties in changing the subject of the formula, multiplying by zero, and in computing $(-3)^3$. Some common occurrences were:

$2(-3)^3 = (2 \times -3) \times (2 \times -3) \times (2 \times -3)$ **OR** $2(-3)^3 = 2 \times 27$

$C = \frac{5}{9}(F - 32) \quad = \quad C = \frac{5}{9}F - 32$ **OR** $C + 32 = \frac{5}{9}F$

Solutions:

(a) (i) 20 (ii) -54

(b) (i) $F = \frac{9}{5}C + 32$ **(ii)** $59^{o}C$

(c) $x = 3; y = 4$

Question 3

The question tested the candidates' ability to:

- construct Venn diagrams to show subsets, intersection and union of two sets
- list the members of a set from a given description
- solve geometric problems using the properties of similar figures

Approximately 99 per cent of candidates attempted this question. The mean mark was 5.19 out of a maximum of 10 marks. Two percent of the candidates scored full marks. Most candidates were able to complete correctly labelled Venn diagrams to illustrate the information. Some candidates experienced difficulty in determining the complement of the union of the sets P and Q. Many candidates did not attempt Part (b) of the question. Most of those who attempted Part (b) could not calculate the scale factor and most of those who did could not use it in calculating the area of the enlarged figure.

Solutions:

(a) (ii) a) = {2, 5} b) = {4, 6, 7}

(b) (i) $\frac{5}{3}$ **(ii)** 50 cm^2

Question 4

The question tested the candidates' ability to:

- convert units of time within the SI system
- solve simple problems involving time, distance and speed
- calculate the area of sectors of circles
- calculate the volume of a right prism

The question was attempted by 98 per cent of the candidates of whom nearly two per cent scored the maximum of 11 marks. The mean mark was 3.71. The candidates demonstrated competence in calculating speed, area and volume. The areas of weak performance included extracting the correct information from the given table, manipulating hours and minutes using consistent units.

Solutions:

(a) (i) 1 hour 36 minutes (ii) 50 km/h

(b) (i) 353 cm^2 (ii) 42 400 cm^3

Question 5

The question tested the candidates' ability to:

- interpret and make use of functional notation and their combinations
- draw, read and interpret graphs of functions
- draw and use graphs of a given quadratic function to determine the roots of the given function

This question was attempted by approximately 98 per cent of the candidates of whom 5 per cent earned the maximum of 13 marks. The mean mark was 7.79. The areas of good performance included the use of correct scales and plotting points on the graph. The areas of weak performance included the inability of several candidates to determine the composite function, drawing a smooth curve to represent the quadratic function and plotting points to draw the line y = x. Many candidates did not recognise that the solutions to the quadratic equation were at the points of intersection of the two graphs.

Solutions:

(a) (i) 5 (ii) 7

(b) (i) 10, -2
(iv) $x = 0$ and $x = 4$

Question 6

The question tested the candidates' ability to:

- use Pythagoras' theorem to solve simple problems
- use the sine, cosine and tangent ratios in the solution of right-angled triangles
- solve problems involving bearings
- find by drawing and/or calculation the gradients and intercepts of graphs of linear functions
- determine the equation of a given line
- state the relationship between an object and its mage in a plane when reflected in a line in that plane

The question was attempted by 92 per cent of the candidates. Of these 3 per cent scored the maximum 12 marks. The mean mark was 3.81. The areas of good performance included the use of Pythagoras Theorem and determining the y intercept of the mirror line drawn. The areas of weak performance included finding the bearing, correctly determining the position of the mirror line and finding the equation of the mirror line.

Solutions:

(a) (i) 32.2 km (ii) 120°
(b) (ii) (0,4) (iii) y = -x + 4

Question 7

The question tested the candidates' ability to:

- use the mid-point of a class interval to estimate the mean of data presented in grouped frequency tables
- construct a cumulative frequency table for a given set of data
- draw and use a cumulative frequency curve
- estimate the median of a set of grouped data
- use theorectical probability to predict the expected value of a given set of outcomes

The question was attempted by 94 per cent of the candidates. Of these 2 per cent scored the maximum 12 marks. The mean mark was 3.57. The areas of good performance were finding the mid-interval values and completing the cumulative frequency column. The areas of weak performance included using the ogive to determine the median score (many candidates used the cumulative frequency value of 25 instead of 20 for this purpose), reading the scales used on the graph and computing the mean of the group distribution.

Solutions:

(a) **15.5**

(b) **18, 32, 37, 40**

(c) (i) **20** **(ii)** **11** **(iii)** **20 or 21** **(iv)** $\frac{9}{40}$

Question 8

The question tested the candidates' ability to:

- use instruments to draw and measure angles and line segments
- use instruments to construct angles and triangles
- calculate the area of the region enclosed by a triangle

The question was attempted by 84 per cent of the candidates. Of these, nearly 3 per cent scored the maximum 10 marks. The mean mark was 4.20. Candidates performed well on the construction of the equilateral triangle and in sub-dividing the larger triangle into nine other triangles. They performed poorly on completing the table and on determining the area of the basic triangle.

Solutions:

(c)

3	**8**
9	**64**

(d) $\sqrt{3}$ **cm**^{2}

Question 9

The question tested the candidates' ability to:

- draw and use distance-time graphs
- use the gradient of a graph of a linear function to determine the rate of change of one variable with respect to another
- determine the maximum and minimum values of quadratic functions by the method of completing the square

The question was attempted by 55 per cent of the candidates. Of these, less than 1per cent scored the maximum 15 marks. The mean mark was 4.06. The areas of good performance included reading and interpreting information from the distance-time graph and determining the speed. However, in part (b) few candidates correctly determined the time that the two persons would meet. In part (c), although most candidates identified a method for completing the square, they were not able to complete the process due to errors in working with fractions or collecting like terms. Some candidates expressed the quadratic in the correct form, but were unable to transfer the results to state the minimum value of the of f(x) or the value of x at which it occurred.

Solutions:

(a) **(i) 04:15** **(ii) 9 km** **(iii) 30 mins** **(iv) 6 km/h**

(b) **(i) 06:42** **(ii) 2 km**

(c) **(i)** $4\left(x-\frac{7}{8}\right)^2-\frac{1}{16}$.

(ii) a) $-\frac{1}{16}$ **b)** $\frac{7}{8}$

Question 10

The question tested the candidates' ability to use linear programming techniques to solve problems involving two variables.

The question was attempted by 30 per cent of the candidates. Of these, 5 per cent scored the maximum 15 marks. The mean mark was 4.53. Good performance were noted in candidates' attempts at interpreting and using the given scales, writing the profit function and substituting values into this function to find the maximum profit. The areas of weak performance were in writing the inequations (many candidates used the equal sign instead of the inequality sign) and in drawing the lines representing the boundaries of the inequations.

Solutions:

(a) **(i)** $x \geq 15$; $y \geq 20$ **(ii)** $40x + 30y \leq 2400$
(c) **(i)** $P = 25x + 6y$ **(ii) 45 dresses and 20 shirts**
(iii) $1 245

Question 11

The question tested the candidates' ability to:

- calculate the distance between two points on the earth, treated as a sphere, measured along the parallels of latitude or meridians
- calculate the area of a triangle given two sides and the included angle by means of the formula:
  Area of $\Delta ABC = \frac{1}{2} ab \sin C$
- calculate the area of a sector of a circle
- calculate the area of a segment of a circle.

The question was attempted by 17 per cent of the candidates. Of these, 3 per cent scored the maximum 15 marks. The mean mark was 3.22. The areas of good performance included knowing how to calculate the distance between two points on the Earth's surface and determining the area of the segment of the circle. The areas of weak performance were candidates' inability to determine the angle subtended at the centre by the arc joining the two points on the Earth's surface and in calculating the radius of the given circle in part (b).

Solutions:

(a) 4 331 km

(b) (i) 9.33 cm (ii) 60.74 cm^2 (iii) 17.88 cm^2

Question 12

The question tested the candidates' ability to:

- solve problems using the theorems related to properties of a circle
- use simple trigonometric ratios to solve the problems based on measures in the physical world: heights and distances

The question was attempted by 18 per cent of the candidates. Of these less than 4 per cent scored the maximum 15 marks. The mean mark was 4.59. The areas of good performance included candidates' knowledge of the circle theorems and their ability to use them in calculating the size of the unknown angles. There was also satisfactory performance in part (b) in recognising that angle MTN was 23^o. An area of weak performance included not writing the reasons to support the calculations in part (a).

Solutions:

(a) **(i)** **52^o** **(ii)** **38^o** **(iii)** **142^o** **(iv)** **76^o**

(b) **(ii)** **23.6 m**

Question 13

The question tested the candidates' ability to:

- associate a position vector with a given point P(a, b) where O is the origin (0,0)
- determine the magnitude of a vector
- use vectors to represent and solve problems in Geometry.

The question was attempted by 36 per cent of the candidates. Of these less than 2 per cent scored the maximum 15 marks. The mean mark was 3.74. The areas of good performance include writing the column vectors representing the given position vectors and in determining the position vector of the point D by a graphical method. The area of weak performance included the inability to add vectors and in recognising that they were required to find the lengths of at least two sides of the triangle to prove that it was isosceles.

Solutions:

(a) **Position Vector of:**

A: $\begin{pmatrix} 2 \\ 2 \end{pmatrix}$ **B:** $\begin{pmatrix} 6 \\ 2 \end{pmatrix}$ **C:** $\begin{pmatrix} 4 \\ 6 \end{pmatrix}$

(b) **(i)** $\begin{pmatrix} 4 \\ 0 \end{pmatrix}$ **(ii)** $\begin{pmatrix} 2 \\ 4 \end{pmatrix}$ **(iii)** $\begin{pmatrix} -2 \\ 4 \end{pmatrix}$

(c) **The magnitude of vectors AB, AC and BC are $\sqrt{16}$, $\sqrt{20}$ and $\sqrt{20}$ respectively.**

(d) **Position vector of D is**

Question 14

The question tested the candidates' ability to:

- identify a 2 x 2 singular matrix
- obtain the inverse of a non-singular 2 x 2 matrix
- use matrices to solve simple problems in Algebra
- determine the 2 x 2 matrices associated with the following transformations: enlargements, rotations, reflections
- use matrices to solve simple problems in Geometry.

The question was attempted by 27 per cent of the candidates. Of these two per cent scored the maximum 15 marks. The mean mark was 5.88. The areas of good performance included understanding and using the definition of a singular matrix, expressing the simultaneous equations in matrix form, finding the inverse of the matrix and multiplying two matrices.

The areas of weak performance included not being able to describe the transformation represented by the matrix W and in correctly ordering the matrices to determine the single matrix for the combined transformation.

Solutions:

(a) $\frac{5}{3}$

(b) **(i)** $\begin{pmatrix} 3 & 5 \\ 2 & 4 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 19 \\ 14 \end{pmatrix}$

(ii) $\frac{1}{2} \begin{pmatrix} 4 & -5 \\ -2 & 3 \end{pmatrix}$

(iii) $\begin{pmatrix} 3 \\ 2 \end{pmatrix}$

(c) (i) **a reflection in the *x* axis or y = 0**

(ii) **a rotation of 180^o about (0,0) or an enlargement by scale factor –1, centre (0,0)**

(iii) $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$

(iv) $\begin{pmatrix} -6 \\ -4 \end{pmatrix}$