ABSTRACT

On a Statistical Manifold Defined by A Class of Extreme Value (Or Gumbel) Distributions

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We study the geometrical structure that is induced by the parameter space of a density function. The extreme value density function

$$f(x,\beta,\theta) = \exp\left\{-e^{-\left(\frac{x-\beta}{\theta}\right)}\right\} e^{-\left(\frac{x-\beta}{\theta}\right)} \cdot \frac{1}{\theta};$$

$$-\infty < \beta < \infty, \quad \theta > 0, \quad -\infty < x < \infty,$$

satisfies the regularity conditions [10], and the family $\{f(x,\beta,\theta)\} \subset \mathbb{R}$ defines a 2 - dimensional differentiable manifold (i.e. a regular surface) with coordinate system (β,θ).

We define, on this manifold, the Riemannian metric, g, whose components are the Fisher Information:

$$g_{ij} = E[\partial_i l(x,\phi)\, \partial_j l(x,\phi)],$$

where $\partial_1 = \frac{\partial}{\partial\beta}$, $\partial_2 = \frac{\partial}{\partial\theta}$ and $l(x,\phi) = \log[f(x,\beta,\theta)]$.

The manifold, equipped with g, along with a symmetric trivalent tensor - the skewness tensor, defines the statistical manifold.

On this manifold, the Riemannian connection as well as the α - connections, are defined and computed. It is from these that we calculate the curvature tensor and the Gaussian curvature.

Our calculations show that the manifold has constant negative Gaussian curvature.