CARIBBEAN EXAMINATIONS COUNCIL

REPORT ON CANDIDATES' WORK IN THE CARIBBEAN SECONDARY EDUCATION CERTIFICATE® EXAMINATION

JANUARY 2013

MATHEMATICS
GENERAL PROFICIENCY EXAMINATION

GENERAL COMMENTS

The General Proficiency Mathematics examination is offered in January and May/June each year. There was a candidate entry of approximately 12 900 in January 2013; 40 per cent of the candidates earned Grades I–III. The mean score for the examination was 78 out of 180 marks.

DETAILED COMMENTS

Paper 01– Multiple Choice

Paper 01 consisted of 60 multiple-choice items. This year, 29 candidates each earned the maximum available score of 60. Sixty-eight per cent of the candidates scored 30 marks or more.

Paper 02 – Structured Questions

Paper 02 consisted of two sections. Section I comprised eight compulsory questions totalling 90 marks. Section II comprised three optional questions: one each from (i) Algebra, Relations, Functions and Graphs; (ii) Measurement, Trigonometry and Geometry; and (iii) Vectors and Matrices. Candidates were required to answer any two questions from this section. Each question in this section was worth 15 marks.

This year, one candidate earned the maximum available mark of 120 on Paper 02. Approximately 20 per cent of the candidates earned at least 60 marks on this paper.

Compulsory Section

Question 1

This question tested candidates' ability to

- perform the four basic operations with decimals
- use the order of operations to do calculations with decimals
- convert from one currency to another
- find the cost price given the percentage sales tax.

The question was attempted by 99 per cent of the candidates, 4.3 per cent of whom earned the maximum available mark. The mean mark was 7.52 out of 11.

The majority of candidates produced satisfactory responses to this question. In general, they demonstrated competence in the use of the calculator to perform computations in the correct order. However, there were some misconceptions by some candidates as are reflected in the following methods and procedures.

In Part (a), instead of squaring, some candidates multiplied by 2 and 1.3^2 was evaluated as 2×1.3.

In Part (b), the hotel accommodation for 3 nights was misunderstood and the return airfare was also multiplied by 3. Few of the candidates associated the EC$1610.00 charge, which included a 15% sales tax, with 115% of the cost price. Most of them proceeded to calculate 15% of EC$1610.00, and then to subtract the result from this amount to determine the cost price.

Solutions

(a) 9.257
(b) (i) US $647 (ii) US $596.30
(iii) Angie's since $596.30 < $647 (iv) $1400

Recommendations

Teachers are advised to teach proportion using the unitary method. They should also use authentic tasks in the teaching of consumer arithmetic. Attention should be given to squaring and to writing the result of a computation exactly.

Question 2

This question tested candidates' ability to

- use the distributive property of multiplication over addition
- solve a linear equation in one unknown
- factorize quadratic functions involving difference of squares
- solve worded problems involving a pair of simultaneous equations.

The question was attempted by 99 per cent of the candidates, 10.6 per cent of whom earned the maximum available mark. The mean mark was 5.48 out of 12.

The performance of candidates on this question was satisfactory. In Part (a), while most candidates were able to correctly apply the distributive property, they were generally unable to simplify the equation and to correctly proceed to calculating the value of *p*.

In Part (b), candidates encountered difficulties in factorizing the quadratic expressions. Some popular but incorrect methods seen included
$25m^2 - 1 = (25m - 1)(25m + 1)$; $2n^2 - 3n - 20 = n(2n - 3) - 20$;
$2n^2 - 3n - 20 = (n + 4)(2n - 5)$.

In Part (c), candidates correctly translated words into symbols and wrote the pair of equations in x and y. The majority of candidates identified an appropriate strategy for solving the pair of equations such as elimination, substitution, matrices and trial and error. The most popular choice was elimination; but with this, several candidates added the equations and hence could not eliminate either variable.

Solutions

(a) $p = \frac{3}{2}$

(b) (i) $(5m - 1)(5m + 1)$ (ii) $(2n + 5)(n - 4)$

(c) (i) $5x + 12y = 61;$ $10x + 13y = 89$ $30x$

(ii) a)5 g; b) 3 g

Recommendations

Teachers should emphasize the appropriate use of the distributive, associative and commutative properties and should emphasize the difference between an expression and an equation. Students should practise the factorization of quadratic expressions and solving simultaneous linear equations. The difference between squares is a useful tool in factorization and needs continued attention by teachers and students.

Question 3

This question tested candidates' ability to

- represent information in a Venn diagram
- formulate and solve a linear equation in one unknown using information from a Venn diagram
- solve simple geometrical problems using the properties of parallel lines and angles in isosceles triangles
- use the properties of two triangles, to explain why they are similar but not congruent.

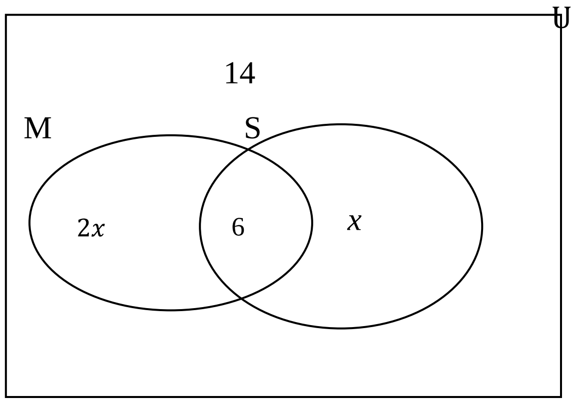
The question was attempted by 99 per cent of the candidates, 1.5 per cent of them earned the maximum available mark. The mean mark was 5.42 out of 12.

The performance of candidates on this question was moderate. In Part (a), most candidates were able to enter 6 in the intersection of the sets M and S, $2x$ in $M \cap S'$ and x in $S \cap M'$, but were less proficient in identifying the number of elements in $(M \cup S)'$. Many of them were unable to form an equation from information represented on the Venn diagram, and in several instances when they arrived at the equation, they lacked the algebraic skills to solve it.

In Part (b), candidates were able to correctly determine the measure of the angle formed at A. However, calculating angle AED proved more challenging, as well as providing a reason why the two triangles in question were similar but not congruent. A common incorrect response was that they were similar because they were two isosceles triangles.

Solutions

(a)(i)



(ii) $x = 10$

(b) (i) a) $B\hat{A}C = 72°$ b) $A\hat{E}D = 54°$

(ii) Similar since corresponding angles are equal

Not congruent because lengths of corresponding sides are unequal.

Recommendations

Teachers are expected to revise on an ongoing basis the content covered in Set Theory at the lower grades of the school. Students should be encouraged to provide reasons why plane figures could be similar but not congruent.

<u>Question 4</u>

This question tested candidates' ability to

- change the subject in a formula
- solve problems involving the inverse and the composite functions
- rearrange a linear equation to determine the gradient of a given line
- determine the equation of the perpendicular bisector of a given line

The question was attempted by 93 per cent of the candidates, 1.6 per cent of them earned the maximum available mark. The mean mark was 2.53 out of 12.

The performance of candidates on this question was generally weak. In Part (a) (i), they seldom attempted to collect terms in r, the variable that they were required to make the subject of the formula. Even when this was done, they did not proceed further since it was not recognized that by factorizing they could isolate r. Their attempt at making r the subject in $v = \pi r^2 h$ was equally poorly done.

In Part (b) (i) a), candidates could not find the inverse of the function and therefore could not evaluate $f^{-1}(19)$.

In Part (c) (i), candidates could not determine the gradient of a line when given its equation. Most candidates were unable to use coordinate geometry to solve the problem.

Solutions

(a) (i) $r = \frac{h}{(1-h)}$ (ii) $r = \sqrt{\left(\frac{V}{\pi h}\right)}$

(b) (i) $f^{-1}(19) = 7$ (ii) $gf(3) = 4$

(c) (i) gradient of GH $= -\frac{3}{2}$ (ii) Equation of JK is: $3y = 2x - 5$

Recommendations

Teachers should give special attention to the balancing nature of equations and should expose the students to a variety of questions requiring the change of the subject of a formula. Students need to be taught how to determine the gradient of a line when the equation of the line is given. Teachers should provide more practice for students in finding the inverse of a function.

<u>Question 5</u>

This question tested candidates' ability to

- measure and state the length of a line and the measure of an angle
- determine the bearing of one point from another
- solve problems related to scales and distances
- construct an angle of 120° using ruler and compasses only.

The question was attempted by 93 per cent of the candidates, 1.5 per cent of whom earned the maximum available mark. The mean mark was 5.25 out of 12.

In Part (a), candidates were able to correctly measure and state the length of RT and the measure of an angle although a number of them could not correctly identify the bearing. In Part (b), candidates demonstrated some proficiency in correctly applying the scale to determine the value of RM. However, the limited knowledge of bearings prohibited successful completion of the problem.

Solutions

(a) (i) 5.8 cm (ii) 65° (iii) 174 m

(b) (i) 10 cm (iii) $N\hat{R}M$

Recommendations

Teachers need to use mathematical language when teaching concepts in geometry. Efforts should also be made to teach related concepts at the same time instead of in isolation. Questions should be set using authentic situations to assist in the transfer of learning to everyday experiences.

Question 6

This question tested candidates' ability to

- determine the radius when the diameter is given
- determine the circumference of the cross section of a cylinder
- use information from the net of a cylinder to determine its curved surface area
- determine the depth of water in a cylinder when given the volume of water, in litres.

The question was attempted by 91 per cent of the candidates, 2.5 per cent of whom earned the maximum available mark. The mean mark was 3.35 out of 11.

In Part (a), candidates easily obtained the radius of the circle and the circumference of the cross section of the cylinder. In Part (b), when the net of the same cylinder was given, many candidates did not see the relationship between the two. As a consequence, they invariably arrived at the incorrect answer for the curved surface area of the cylinder.

In Part (c), many candidates did not convert litres to cubic centimeters as was required. They also had difficulty formulating an equation for the volume of water in the cylinder; and in situations where they formed the equation, many of them could not correctly make h the subject of the formula.

Solutions

(a) (i) 6 cm (ii) 37.68 cm
(b) $a = 37.68$ cm; $b = 8$ cm
(c) $h = 4.4$ cm

Recommendations

Teachers should ensure that students are familiar with the nets of solids. Attention should also be given to the consistency of units within equations.

<u>Question 7</u>

This question tested candidates' ability to

- complete a grouped frequency table
- identify the modal class of a grouped frequency distribution
- identify the class interval in which a given score would lie
- calculate an estimate of the mean of grouped data
- estimate the proportion of the data set above a given value.

The question was attempted by 89 per cent of the candidates, 3.1 per cent of whom earned the maximum available mark. The mean mark was 3.66 out of 10.

Although candidates were generally familiar with the term *modal class*, they were unable to identify the correct interval in which a given score would lie.

When calculating the mean, many candidates divided by the sum of the mid-point values by 6 instead of dividing the sum of the values $f \times x$ by 100. Generally, candidates did not know why the value calculated for the mean was just an estimate rather than the exact value.

To determine the probability that a student chosen at random would score 40 or more, several candidates divided 40 by 100, while others attempted to divide the interval 40−49 by 100.

Solutions

(a) (i) modal class is $20 - 29$ (ii) 19.4 lies in the class $10 - 19$

(b) (i)

Score	**Mid-point (x)**	**Frequency (f)**	**f × x**
20−29	24.5	25	612.5
30−39	34.5	22	759
40−49	44.5	20	890
50−59	54.5	12	654

(b) (ii) Sample mean $= 31.4$

(c) The mean calculated is an estimate because the assumption was made that the scores in any interval are all equal to the mid-value of the class.

(d) Probability $= \frac{32}{100}$

Recommendation

Students need to practise all aspects of statistics to maximize their chances of solving questions of this nature.

Question 8

This question tested candidates' ability to

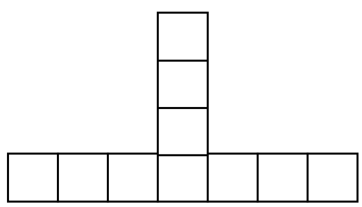
- draw the fourth diagram in a sequence of diagrams in which the first three diagrams in the sequence are given
- recognize and generate the terms of a sequence
- determine the term of the sequence for which the value of the function is given
- derive an equation connecting the value of the function with the term of the sequence.

The question was attempted by 97 per cent of the candidates, 3.9 per cent of whom earned the maximum available mark. The mean mark was 4.83 out of 10.

A large number of candidates recognized the pattern of squares in the sequence and were able to correctly draw the fourth diagram. They were able to identify the pattern of numbers within the table and used this to arrive at the numerical values that were missing from the table. However, the majority of candidates were unable to describe the pattern using algebraic symbols.

Solutions

(a)



(b) (i) $a = 10$ (ii) $b = 28$ (iii) $c = 14$

(c) $3n - 2$

Recommendation

Teachers should assist students with writing expressions and formulae derived from number sequences.

Optional Section

Question 9

This question tested candidates' ability to

- calculate values of *y* for given values of *x* in a reciprocal function
- plot points associated with a reciprocal function and draw a smooth curve through the points
- write a quadratic function in the form $a(x-h)^2+k$
- determine, for the quadratic function, its minimum value and the equation of its line of symmetry
- solve a quadratic equation.

The question was attempted by 71 per cent of the candidates, 2.9 per cent of whom earned the maximum available mark. The mean mark was 5.75 out of 15.

In Part (a) (i), candidates were able to correctly calculate the missing *y*-coordinates for the function $y = \frac{3}{x}$. However, many candidates found it difficult to use the scales given to plot points where the *x*-coordinates were in decimal form. They also had difficulty using uniform scales on both axes.

In Part (a) (ii), they were able to correctly represent the scales given on the graph.

In Part b (i), candidates were able to use an appropriate strategy to solve a quadratic equation. However, they experienced some difficulty obtaining correct values for *a*, *h* and *k* when they attempted to complete the square. Candidates also had difficulty identifying the minimum value from the expression for the completed square. However, the coordinates for the minimum point were often quoted correctly. Applying the correct coefficients to the quadratic formula and simplifying the expression was also problematic for candidates. As a result $\frac{-5\pm\sqrt{5^2-4\times3\times1}}{2\times3}$ was often seen, and $-5\pm\sqrt{\frac{13}{6}}$ = -5 ± $\sqrt{2.16}$ or $5\pm\sqrt{\frac{13}{6}}$ = 5 ± $\sqrt{2.16}$ was the penultimate stage of the simplification leading to incorrect solutions.

Solutions

(a) (i)

$x\ (sec)$	0.5	3	5
$y\ (^{m}/_{s})$	6	1	0.6

(b) (i) $3\left(x-\frac{5}{6}\right)^2-\frac{13}{12}$ (ii) $f_{min}=-\frac{13}{12}\ when\ x=\frac{5}{6}$ (iii) $x=1.43, 0.23$

Recommendations

Teachers should provide students with more practice questions involving fractional coordinates and solutions of quadratic equations with $b < 0$. They should also help students to recognize their weaknesses and encourage them not to choose questions for which they were not adequately prepared.

Question 10

This question tested candidates' ability to

- use circle theorems to evaluate the measure of three different angles
- use the cosine formula to solve for an unknown angle in a triangle
- determine the area of a triangle in which the measures of three sides are given
- use information derived from three-dimensional geometry to draw a given triangle showing the angle of elevation
- use trigonometric ratios to calculate an unknown height.

The question was attempted by 48 per cent of the candidates, less than one per cent of whom earned the maximum available mark. The mean mark was 2.49 out of 15.

In Part (a), most candidates correctly identified angle RMQ as a right angle and, as a result, were able to derive the measure of angle MRQ. Some attempted to solve the problem by using angles in a triangle with no attention given to the circle theorems. A large number of candidates identified triangle PMN as isosceles and proceeded to correctly calculate the measure of angle PMN.

In Part (b) (i), most candidates opted to use the cosine rule to calculate the measure of angle ABC. However, there were those who attempted to use the sine rule, and in some cases the sine ratio, ignoring the fact that triangle ABC was not right angled. Many of the candidates who attempted to use the cosine rule, could not follow through correctly. In most cases, they substituted into the formula incorrectly, and ended up calculating angle BAC instead. Also, in an attempt to determine the area of triangle ABC, the majority of them used the formula $Area = \frac{1}{2}bh$ rather than $Area = \frac{1}{2}ab\ sinC$.

In Part (b) (ii), most candidates could not illustrate triangle TAB with the right angle at A. In most cases they proceeded to incorrectly use AB as the hypotenuse of the triangle rather than as the adjacent side.

Solutions

(a) (i) $M\hat{R}Q = 70°$ (ii) $P\hat{M}R = 20°$ $(iii)\ P\hat{M}N = 63°$

(b) (i) a) $A\hat{B}C = 137.1°$ b) $3849.5\ m^2$ $(ii)\ a)$ T b) $TA = 73.9\ m$

Recommendations

Teachers should provide students with sufficient practice in solving problems associated with circle theorems and in calculating areas of triangles using the formula $A = \frac{1}{2}ab\, SinC$. In addition, students need greater exposure to problems involving three-dimensional geometry.

Question 11

This question tested candidates' ability to

- determine the resultant of two or more vectors from a vector diagram
- solve for the object point given the coordinates of the image and the transforming matrix
- write the matrix which represents an enlargement
- determine the coordinates of the image which results when an object undergoes a combination of two transformations
- write a matrix to represent an authentic situation.

The question was attempted by 41 per cent of the candidates, less than one per cent of whom earned the maximum available mark. The mean mark was 1.77 out of 15.

In Part (a), while candidates were generally able to write a correct route for each vector sum, they were unable to correctly substitute into their expressions or to simplify by collecting like terms.

In Part (b), candidates did not recognize that they could use the inverse of J to obtain the coordinates of the object when given those of the image. In many instances, they incorrectly multiplied J by (5, 4).

In Part (c), candidates were not able to write the matrix to represent the enlargement and very few of them were able to obtain and write the combined transformation HJ in matrix form.

In Part (d), candidates appeared to be confused by the difference between a 3×2 *and a* 2×3 matrix and between a 1×3 *and a* 3×1 matrix. As a consequence, it was common to find incompatible matrices set up for multiplication.

Solutions

(a) (i) $\overrightarrow{MK} = -u + v$ (ii) $\overrightarrow{SL} = \frac{2}{3}u + \frac{1}{3}v$ (iii) $\overrightarrow{OS} = \frac{1}{3}u + \frac{2}{3}v$

(b) $P(4, -4)$

(c) (i) $H = \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$ (ii) $(21, 15)$

(d) (i) $\begin{pmatrix} 2 & 0 \\ 5 & 6 \\ 3 & 10 \end{pmatrix}$ (ii) $(40 \quad 55 \quad 120)$ (iii) $(40 \quad 55 \quad 120)\begin{pmatrix} 2 & 0 \\ 5 & 6 \\ 3 & 10 \end{pmatrix}$

Recommendations

Teachers should provide students with adequate practice in the multiplication of matrices with emphasis on the compatibility of matrices for multiplication. Attention should also be given to the use of the inverse matrix in determining the object coordinates when given the coordinates of the image under a given transformation.