ON CIRCUIT POLYNOMIALS OF GRAPHS

Abstract

This thesis presents work done in expanding and developing results on the circuit polynomial of a graph. The circuit polynomial of a graph is defined as follows.

Let F be the family of circuits, each of which is given a specified weight. Let G be any graph, and C a spanning subgraph, or circuit cover of G, the components of which are members of F. The weight of C is defined to be the product of the weights of its components. Then, the circuit polynomial of G is the sum of the weights of all the circuit covers of G. Isolated nodes and single edges are defined to be circuits with one and two nodes respectively. Normally, a circuit on k nodes will be assigned a weight w_k. Thus, the circuit polynomial of a graph G with p nodes will normally be a polynomial in p variables $w_1, w_2, \ldots, w_p$.

Certain analytical properties of the circuit polynomial, namely, differentiation and integration, are examined. Applications of these results to a reconstruction problem are also discussed. It is shown that the circuit polynomial of a graph does not characterize graphs up to isomorphism. To this end, pairs of graphs with the same circuit polynomial are exhibited, along with methods of constructing such sets of graphs. A connection is established between the circuit polynomial of a graph, suitably weighted, and the determinant of a certain class of matrices. This result is then applied to the problem of finding the number of spanning trees in a graph. Finally, the circuit polynomials of certain families of polygonal chains are derived.