

**CARIBBEAN EXAMINATIONS COUNCIL**

**REPORT ON CANDIDATES' WORK IN THE  
CARIBBEAN ADVANCED PROFICIENCY EXAMINATION  
MAY/JUNE 2007**

**APPLIED MATHEMATICS**

**APPLIED MATHEMATICS****MAY/JUNE 2007****INTRODUCTION**

The revised Applied Mathematics syllabus was followed this year for the third time. Of the one hundred and fifty-nine candidates registered for the examination, one hundred and twenty-one wrote Option C, twenty-seven wrote Option B and eleven wrote Option A. One candidate wrote the Alternative to the SBA Paper in Option C.

This is a one-Unit course comprising three papers and three Options. However, a candidate is required to take only ONE Option. Papers 01 and 02 were examined externally, while Paper 03 was examined internally by the teachers and moderated by CXC. Contributions from Papers 01, 02 and 03 to the Unit were 40 per cent, 40 per cent and 20 per cent respectively.

The three options are Option A, Option B and Option C.

Option A consists of Discrete Mathematics, Probability and Distributions and Statistical Inference. Option B consists of Discrete Mathematics, Particle Mechanics and Rigid Bodies, Elasticity, Circular and Harmonic Motion. Option C consists of Discrete Mathematics, Probability and Distributions and Particle Mechanics.

**GENERAL COMMENTS**

Acceptable grades, Grades I to V, were obtained by 79 per cent of the candidates writing Option C, 33 per cent of the candidates writing Option B and 64 per cent of the candidates writing Option A. The standard of work seen from many candidates in this examination was above average. Again this year, candidates appeared to be well prepared in Discrete Mathematics and Probability and Distributions and generally answered the questions well. Many candidates did not do well in the Mechanics module. Despite this fact, there were a number of candidates who appeared to be well prepared in all three of their modules. In general, there were a large number of areas of strength displayed by many candidates. Nevertheless, many candidates still need to pay more attention to their algebraic manipulation.

Areas of strength displayed by most candidates on this paper were as follows:

- conversion from words to logic symbols
- construction of truth tables
- calculation of the earliest start time
- identification of the number of degrees for a vertex
- formulation of the null and alternative hypotheses in symbols
- calculation of test statistics
- naming and justifying the use of given distributions
- calculating probabilities of events combined by unions and intersections using appropriate formulae
- calculating the expected value and variance of a linear combination of two independent random variables
- applying the formula for the binomial distribution
- use of the Poisson, geometric and normal distributions to solve problems.

The areas of the course which the candidates found difficult were as follows:

- inability to convert from words to symbols
- manipulation of the rows in the simplex method
- omission of the non-negativity constraints in the linear programming problem
- calculation of the latest start time and hence the critical path
- inability to construct an activity network algorithm
- inability to calculate the number of degrees of freedom for a goodness-of-fit using a Poisson of unknown mean
- inability to resolve forces correctly

### **Internal Assessment**

Generally, the size of the samples submitted were adequate and marks were entered correctly onto the AMAT1-3 forms.

This year, the overall presentation and quality of samples submitted were satisfactory. Candidates chose topics which were relevant to the objectives of the syllabus and, in most cases, were suitable to their level. In fact, a few candidates excelled in their assignment, employing techniques beyond the level expected.

Projects equally incorporated Discrete Mathematics with Particle Mechanics and Discrete Mathematics with Statistics and Probability, while a small number of candidates did their project on only one of the 3 modules.

Samples were generally well done, with 71 per cent being adequately handled, 24 per cent were successfully accomplished, while the remaining 5 per cent were vague.

Not all candidates gave full descriptions of their plan for carrying out their tasks. 81 per cent gave articulate descriptions of their investigation. The majority showed clear evidence of doing purposeful mathematics in relation to collecting of data. However, the integration and use of diagrams and illustrations could have been more effectively utilized.

The majority of candidates appeared to have a better understanding of the syllabus; they were successful at applying their mathematical knowledge appropriately and within context.

Few candidates included in their evaluation the insights into problems encountered in their investigation or what was done to resolve them. The majority of candidates also disregarded or perhaps did not understand what was meant by “suggest (2 or more) ideas for future study of the assignment’s topic and suggestions for improvement.”

Most candidates had little or no difficulty with grammar or structuring of their statements. However, some candidates lost marks for lack of recording their actions at each stage of the investigation, as well as during calculations.

Teachers’ assessments of assignments were generally appropriate since the majority of the marks were in agreement with that of the moderator. Overall, both teachers and candidates displayed a satisfactory understanding of the assessment criteria and the ranking of the candidates was reliable in most of the cases.

**Paper 01 Option C**  
**SECTION A (Module 1: Discrete Mathematics)**  
**same for Options A and B**

Question 1

This question tested the candidates' ability to:

- (a) construct truth tables for given compound propositions that involve conjunctions, disjunctions and negatives
- (b) use the result obtained from the truth table, in Part a, to simplify the given proposition.

This question was well done by most candidates. In Part (a) candidates were able to correctly construct the truth tables for  $p \Rightarrow q$  and  $\sim p \vee q$  and most of these candidates were able to correctly explain why these two propositions are equivalent.

In Part (b) candidates were able to use the truth table to simplify the given proposition. Some candidates used the result obtained in Part (a) along with the distributive law and the complement law to simplify the expression.

**Answers**

1 (a)

p	q	$\sim p$	$p \Rightarrow q$	$\sim p \vee q$
T	T	F	T	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

OR

p	q	$\sim p$	$p \Rightarrow q$	$\sim p \vee q$
1	1	0	1	1
1	0	0	0	0
0	1	1	1	1
0	0	1	1	1

$p \Rightarrow q$  and  $\sim p \vee q$  are logically equivalent since the corresponding terms in their truth tables are the same.

$$\begin{aligned}
 \text{(b) } (p \vee q) \wedge (p \Rightarrow q) &= (p \vee q) \wedge (\sim p \vee q) \\
 &= (p \wedge \sim p) \vee q \\
 &= 0 \vee q \\
 &= q
 \end{aligned}$$

OR

p	q	$p \vee q$	$p \Rightarrow q$	$(p \vee q) \wedge (p \Rightarrow q)$
T	T	T	T	T
T	F	T	F	F
F	T	T	T	T
F	F	F	T	T

OR

p	q	$p \vee q$	$p \Rightarrow q$	$(p \vee q) \wedge (p \Rightarrow q)$
1	1	1	1	1
1	0	1	0	0
0	1	1	1	1
0	0	0	1	0

$(p \vee q) \wedge (p \Rightarrow q) = q$  since the corresponding terms in their truth tables are the same

### Question 2

This question tested the candidates' ability to formulate compound propositions in symbols in terms of the given propositions  $w, x, y$  and  $z$  and the logical connectives  $\wedge, \vee$  and  $\sim$ .

This question was reasonably well done. Candidates were able to answer Parts (a) and (c) correctly. In Part (b) some candidates, incorrectly expressed the proposition as  $\sim y \wedge w$ . In a few instances, candidates used the incorrect symbol for the proposition.

### **Answers**

- (a)  $z \wedge x$
- (b)  $\sim(y \wedge w)$
- (c)  $x \vee y$

### Question 3

This question tested the candidates' ability to

- (a) construct an activity network showing the cost associated with allocating three individuals  $A, B$  and  $C$  to three tasks  $P, Q$  and  $R$
- (b) identify which one of the vertices  $P, Q$  and  $R$  has the highest degree.

This question was well done, with candidates being able to represent individuals and tasks as vertices and costs as edges. It should be noted that very few candidates displayed the correct directions on the edges.

Part (b) was well done by all candidates who were able to identify the vertex with the highest degree.

### Question 4

This question tested the candidates' ability to formulate a linear programming problem in two variables from real-world data.

The performance on this question was disappointing. Most candidates were unable to correctly obtain the objective and objective expression as well as the correct constraints. Many candidates gave the constraints as  $x + 7y \leq 110$  instead of  $x + 7y \geq 110$   
 $5x + y \leq 80$  instead of  $5x + y \geq 80$

Some candidates also used the equality symbol instead of the inequality symbol. Candidates correctly indicated that  $x$  and  $y$  are non-negative constraints, but did not indicate that they must be integers.

### **Answers**

$$\begin{array}{ll} \text{Minimize} & x + y \\ \text{Subject to} & x + 7y \geq 110 \\ & 5x + y \geq 80 \\ & x, y, \geq 0 \\ & x, y, \text{ integers} \end{array}$$

Question 5

This question tested the candidates' ability to:

- (a) to determine a Boolean expression for a given logic circuit
- (b) identify which one of the given circuits is equivalent to an OR and a NOT gate.

This question was fairly well done. In Part (a) many candidates were able to identify the correct expression for circuits *A*, *B*, *C* and *D*. However, circuit *E* proved challenging to a number of candidates. Only a few of those who were able to write down the expression, simplified it correctly.

In Part (b) most candidates were able to identify the NOT gate, but few were able to identify the OR gate.

**Answers**

- (a) A:  $\sim r$   
 B:  $\sim(r \vee s)$  OR  $\sim r \wedge \sim s$   
 C:  $r \vee s$   
 D:  $\sim(\sim r \vee \sim s)$   
     $= r \wedge s$   
  
 E:  $\sim(\sim r \vee \sim(r \vee s))$   
     $= r \wedge (r \vee s)$   
     $= r$
- (b) (i) Circuit 2 functions as an OR gate  
 (ii) Circuit 1 functions as a NOT gate

**Paper 01 Option C**  
**SECTION B (Module 2: Probability and Distributions)**  
**same for Option A**

Question 6

This question tested the candidates' ability to calculate the number of ordered arrangements of  $n$  objects taken  $r$  at a time, with and without restrictions.

This question was generally fairly well done. In Part (a), many candidates gave the answer as  $7!$  instead of  $\frac{7!}{2!}$ , while in Part (b), most candidates were able to fix the first and last positions, but

encountered difficulty arranging the remaining letters. Errors seen were  $3x \frac{4!}{2!}x2$  and  $3x5!x2$

**Answers**

- (a) 2520
- (b) 360.

Question 7

This question tested the candidates' ability to:

- (a) state TWO properties that must be satisfied by a discrete probability distribution,
- (b) use the given probability distribution table to obtain the value of the constant  $a$ , as well as to determine probability.

In Part (a), a few candidates gave the properties for a binomial distribution, but most candidates indicated that the sum of the probability is one.

In Part (b) (i), all candidates were able to find the value of  $a$  correctly. The error made in

Part (b) (ii) was  $P(2.2 \leq X \leq 4) = P(2 \leq X \leq 4)$

**Answers**

- (a)  $0 \leq P(X = x) \leq 1$  and  $\sum P(X = x) = 1$
- (b) (i)  $a = 0.35$  (ii)  $0.4$

Question 8

This question tested the candidates' ability to apply the Poisson formula in solving problems.

This question was quite well done. Part (a) was well done by all candidates. Part (b) posed problems for a number of candidates who used  $\lambda = 2$  instead of  $\lambda = 4$ .

Another error made in this question by candidates was to find  $P(Y \geq 3) = 1 - P(X \leq 3)$  using  $\lambda = 2$

**Answers**

- (a) 0.271
- (b) 0.762

Question 9

This question tested the candidates' ability to:

- (a) model a practical situation by the geometric distribution
- (b) calculate the expected value and variance of the geometric distribution
- (c) calculate probability using the geometric distribution.

This question was not done as well as expected. Part (a) was well done by a large number of the candidates, however there were a few candidates who identified the distribution as binomial, while one candidate identified it as a Poisson distribution.

In Part (b) almost all candidates were able to calculate  $E(X)$  and  $\text{Var}(X)$  correctly.

Part (c) posed difficult to a number of candidates who wrote  $P(X < 26) = 1 - P(X > 26)$  rather than

$$P(X < 26) = 1 - P(X > 25) = 1 - q^{25} \text{ where } q = \frac{29}{30}$$

**Answers**

- (a) Geometric distribution with  $p = \frac{1}{30}$
- (b) (i)  $E(X) = 30$  and  $\text{Var}(X) = 870$ .
- (ii) 0.572

**Question 10**

This question tested the candidates' ability to use the cumulative distribution function to

- (a) find the value of a constant
- (b) calculate probability
- (c) determine the probability density function,  $f(x)$
- (d) calculate the expected value for a linear combination of variable  $X$ .

This question was well done by a number of candidates. However, the common error seen in Part (a) was the use of integration to find the constant  $a$ . For some candidates this error continued in Part (b).

In Part (c), all candidates recognised that they needed to differentiate in order to find the probability function  $f(x)$ , however it was noted that some of these candidates did not indicate the value of  $f(x)$  when  $x < 1$  and when  $x > 2$ .

Part (d) was well done by almost all candidates.

**Answers**

- (a)  $f(x) = \frac{3}{8}x^2 \quad 1 < x \leq 2$   
 $0 \quad \text{otherwise}$
- (b) 0.19225

**Paper 01 Option C**  
**SECTION C (Module 3: Particle Mechanics)**  
**same for Module 2 Option B**

**Question 11**

This question tested the candidates' ability to find the magnitude and direction of a third force  $Q$ , if  $R$  is the resultant of  $P$  and  $Q$ , given the magnitude and direction of two forces  $P$  and  $R$ .

This question was attempted by the large majority of candidates but was not well answered. They however interpreted the question as "a body in equilibrium under the action of forces  $P$ ,  $Q$ , and  $R$ ." Few candidates produced a triangle of forces showing  $R$  as the resultant of  $P$  and  $Q$ .

**Answer**

$Q = 373 \text{ N}$ ; Direction - An angle of  $19.57^\circ$  to the vertical.

Question 12

This question tested the candidates' ability to construct a velocity-time graph from data, and use it to determine the distance travelled, and the acceleration of a body.

Few candidates used the graph to find the time, given the acceleration. They did not relate the gradient to the acceleration. They however were more familiar with associating the area under the graph with distance traveled. The question was generally well answered.

**Answer**

$$\text{Total time} = 3\frac{1}{3} \text{ seconds.}$$

$$\text{Total distance} = 6\frac{2}{3} \text{ metres.}$$

Question 13

This question tested candidates' ability to:

- (a) apply the principle of conservation of momentum to the direct impact of two inelastic bodies moving in a straight line.
- (b) calculate the kinetic energy of a body.

This was a popular and well answered question. Candidates demonstrated their knowledge of the principle and it was evident that they had adequate practice in solving this type of problem.

**Answer**

- (i) velocity before impact =  $337.4 \text{ ms}^{-1}$
- (ii) 11.8 J

Question 14

This question tested the candidates' ability to:

- (a) find the tension and acceleration, given two particles connected by a light inextensible string passing over a smooth, weightless pulley
- (b) find the distance travelled by one of the particles before coming to rest.

In attempting to solve this problem, many candidates failed to indicate the forces acting on the particles. This led, in many cases, to errors when they attempted to apply Newton's equation, Force = mass  $\times$  acceleration. The net force was too often incorrect.

In part (b), candidates experienced difficulty as they tried to apply the principle of conservation of energy to determine the required height reached by the 8 kg mass.

**Answer**

$$\begin{aligned} \text{(a) Acceleration} &= 1.96 \text{ ms}^{-2} . \\ \text{Tension} &= 94.08 \text{ N} \end{aligned}$$

$$\text{(b) } 0.24 \text{ m}$$

Question 15

This question tested the candidates' ability to:

- (a) express resistance in terms of speed,  
 (b) calculate the maximum speed, given the power developed by an engine, its speed and resistance.

This question presented little difficulty to candidates. They knew that at the maximum speed, the pull of the engine was equal to the resistance, and were familiar with the definition of power. They were able to derive the correct equations and solve the problem.

**Answer :**

- (a)  $R = 30v$   
 (b) Maximum speed =  $51.9\text{ms}^{-1}$ .

**Paper 02 Option C**  
**SECTION A (Module 1: Discrete Mathematics)**  
**same for Options A and B**

Question 1

This question tested the candidates' ability to use the Simplex method to solve a linear programming model in two variables.

This question was well done by some of the candidates, while the responses obtained from the remainder of the candidates were disappointing. Some of the latter candidates were only able to complete one tableau correctly, while others completed only two of them with some of the rows incorrectly calculated. In setting up the first tableau some candidates omitted to rewrite the objective function as  $P - 5x - 2y = 0$ . Those candidates entered 1 5 2... in the tableau rather than 1 -5 -2...

All candidates put in the two slack variables. Many arithmetic errors were seen in this question.

**Answer**

Operations					Operations	P	x	y	$s_1$	$s_2$	RHS	
x	y	$s_1$	$s_2$	RHS		1	0	$-\frac{13}{4}$	$\frac{5}{4}$	0	5	
-5	-2	0	0	0	$\frac{1}{4}R_2$	0	1	$-\frac{1}{4}$	$\frac{1}{4}$	0	1	
4	-1	1	0	4		0	0	4	-1	1	32	$\frac{1}{4}R_3$
4	3	0	1	36								
x	y	$s_1$	$s_2$	RHS	Operations	P	x	y	$s_1$	$s_2$	RHS	Operations
-5	-2	0	0	0		1	0	$-\frac{13}{4}$	$\frac{5}{4}$	0	5	
1	$-\frac{1}{4}$	$\frac{1}{4}$	0	1	$R_1 + 5R_2$	0	1	$-\frac{1}{4}$	$\frac{1}{4}$	0	1	$R_1 + \frac{13}{4}R_3$
4	3	0	1	36	$R_3 - 4R_2$	0	0	1	$-\frac{1}{4}$	$\frac{1}{4}$	8	$R_1 + \frac{13}{4}R_3$
P	x	y	$s_1$	$s_2$	RHS	Operations						
1	0	0	$\frac{7}{16}$	$\frac{13}{16}$	31							
0	1	0	$\frac{13}{16}$	$\frac{1}{16}$	3							
0	0	1	$-\frac{1}{4}$	$\frac{1}{4}$	8							

Valid termination of Simplex Method P= 31

## Question 2

This question tested candidates' ability to:

- (a) use the activity network algorithm to construct an activity network diagram in a real-world situation
- (b) from a given activity network diagram
  - (i) calculate the earliest start time, latest start time and float time
  - (ii) identify the critical path
  - (iii) use the critical path in decision making

In Part (a), although the question clearly indicated that the network algorithm should be used, very few candidates did so. It was found that less than 3 per cent of the candidates used this method to answer the question. The result was that the activity network was poorly constructed by the majority of candidates. The major errors seen were

1. Activity C was positioned after activities F and G
2. Activities F and G were not obliquely opposite each other. The same occurred for activities B and E.
3. Activity D, though clearly the last activity, was found within the activity diagram.
4. The times between activities were incorrectly recorded.

In Part (b), the responses to this part of the question showed that candidates were more comfortable interpreting an activity network than constructing it.

Candidates were able to determine the earliest start time for the activities. A few had some difficulty in correctly stating the latest start time.

Candidates used a variety of diagrams to display the earliest, latest and float times, but the tabular form was the most popular. Though the float time in quite a few instances was incorrect, most knew that the difference of the two times was required.

Over 80 per cent of the candidates were able to quote the critical path correctly. Furthermore, over 60 per cent were able to determine the minimum completion time of the project. Very few candidates related it to the time associated with critical path and added one month to the time associated with the critical path.

**Paper 02 Option C**  
**SECTION B (Module 2: Probability and Distributions)**  
**same for Option A**

Question 3

This question tested candidates' ability to:

- (a) (i) use the Normal Distribution  $X \sim N(\mu, \sigma^2)$   
(ii) use  $z = \frac{x - \mu}{\sigma}$   
(iii) use the Probability Distribution tables
- (b) (i) use the Binomial Distribution  $Y \sim B(n, p)$   
(ii) state the values of the parameters  $n$  and  $p$
- (c) (i) use the inverse of the formula  $z = \frac{x - \mu}{\sigma}$   
(ii) use algebra to simplify.

In Part (a) the majority of candidates were able to identify the distribution as Normal and correctly stated its parameters. These candidates were able to standardize correctly, but a few of them used the continuity correction factor in error.

In Part (b) the majority of the candidates were able to recall and use the binomial distribution formula. However, problems were encountered by the candidates in the substitution process. The minority did not identify the correct values for the parameters  $n$  and  $p$ . Overall the question was well done and the majority of the candidates obtained full marks.

In Part (c) some candidates were able to interpret the question correctly. The most common error was in the interpretation of  $P\left(Z > \frac{167 - a}{5}\right) = 0.825$ . These candidates expressed the previous equation as follows:

$$1 - P\left(Z < \frac{167 - a}{5}\right) = 0.825 \Rightarrow P\left(Z < \frac{167 - a}{5}\right) = 1 - 0.825 = 0.175$$

They then encountered the problem of finding the z-value for 0.1750 in their standard normal tables.

Candidates who expressed it this way failed to recognize that  $P\left(Z > \frac{167 - a}{5}\right) = 0.825$  means that

the z-value for  $\frac{167 - a}{5}$  must be negative (because 0.825 is greater than 0.5). Those who expressed

it as  $P\left(Z < \frac{a - 167}{5}\right) = 0.825$  were able to obtain the correct answer. The candidates who avoided

the error stated above obtained full marks.

**Answers**

(a) 0.497 (b) 0.167

(c) 172 cm to 3 sig. fig

**Question 4**

This question tested candidates ability to:

- (a) use the probability function  $f(x) = P(X=x)$  where  $f$  is a simple polynomial or rational function
- (b) calculate the expected values  $E(X)$
- (c) calculate the variances  $\text{Var}(X)$
- (d) calculate  $E(aX+bY)$ , where  $X$  and  $Y$  are independent random variables.
- (e) calculate  $\text{Var}(aX+bY)$  where  $X$  and  $Y$  are independent random variables.
- (f) apply the properties  $f(x) \geq 0$  and  $\int_{-\infty}^{+\infty} f(x)dx = 1$ , where  $f$  is a probability density function ( $f$  will be restricted to simple polynomials)

In Part (a) the majority of candidates were able to calculate the exact value of  $a$ . They found the solution by integrating and had no difficulties in obtaining a final solution.

In Part (b) candidates did not recall the formula correctly and used other methods to arrive at a solution. These methods were not accurate and it was reflected in the overall performance. Approximately 60 per cent of the candidates found  $E(X)$  correctly, but a number of these same candidates experienced difficulty calculating  $\text{Var}(X)$ .

In Part (c) most candidates correctly calculated  $E(Y)$  and  $\text{Var}(Y)$ . However, a few candidates incorrectly used the formula when calculating  $\text{Var}(Y)$ . These candidates subtracted  $E(Y)$  instead of  $E(Y)^2$

In Part (d) some candidates failed to recall  $E(5) = 5$ . These candidates gave  $E(5) = 0$  and so were not able to arrive at the correct answer.

In part (e) the formula was incorrectly used as candidates omitted to square the coefficients of the variables when calculating the variance.

**Answer**

- (a) 1            (b)  $E(X)=3/4$ ,  $\text{Var}(X)=3/80$  (c)  $E(Y)=2$ ,  $\text{Var}(X)=1/2$  (d)  $193/20$  (e)  $93/20$

**Paper 02 Option C**  
**SECTION C (Module 3: Particle Mechanics)**  
**same for Module 2 Option B**

**Question 5**

This question tested candidates' ability to:

- (a) model the motion of a projectile as a particle moving under constant gravitational force neglecting air resistance
- (b) use the equations of motion of a projectile to determine the time of flight, the horizontal range, the greatest height reached and the Cartesian equation of the trajectory of a projectile.

Part (a) was attempted by 90 per cent of the candidates. Most of them got the correct solution, but a few students used a variety of methods to find the greatest height rather than doing a simple subtraction ( $10 \text{ m} - 2 \text{ m} = 8 \text{ m}$ ).

Part (b) was poorly done by most of the same candidates. Some candidates who misinterpreted Part (a) and used  $45^\circ$  as the angle to attain maximum height, went on and get the same  $45^\circ$  for Part (b).

In Part (c), candidates were required to use the following equations:  $x = ut \cos \theta$  and  $x = ut + \frac{1}{2} at^2$  to find the height above the floor. The performance on this part of the question was poor. Common errors were: using the formula;  $\frac{u \sin 2\theta}{g}$  instead of  $\frac{u^2 \sin^2 \theta}{g}$  for greatest height.

**Answers**

- (a) 8 m
- (b)  $44.1^\circ$
- (c) 6.76 m

Question 6

This question tested candidates' ability to:

- (a) resolve forces
- (b) use the principle that when a particle is in equilibrium, the vector sum of its forces is zero or the sum of the components in any direction is zero
- (c) solve problems involving concurrent forces in equilibrium.

In part (a), most of the candidates produced diagrams, but some of them left out important details such as the directions of the tensions in the strings and the angles. A few candidates used Lami's theorem correctly at point Q. Most candidates used resolution of forces.

Part (b) (i) was well done by most of the candidates.

Part (c) was well done. M could be found by resolving at R or using applying Lami's theorem at point R

**Answer**

- (b) (ii) 12.1 kg

**Paper 02 Option A**  
**SECTION B (Module 3: Statistical Inference)**

Question 5

This question tested the candidates' ability to:

- (a) justify the use of the z- test.
- (b) formulate null and alternative hypotheses
- (c) (i) define a Type I error  
(ii) state the value of P(Type I error)
- (d) state the decision rule
- (e) Calculate the test statistic.

This question was well done by most candidates, who were able to justify the use of the z- test, formulate null and alternative hypotheses, define a Type I error, state the value of P(Type I error), state the decision rule but a few of the difficulty calculating the test statistic.

**Answers**

- (a) In this test of the difference between two populations means, data are derived from random, independent samples taken from two normal distributions of known/equal variance.
- (b)  $X_1$  = scores of females candidates  
 $X_2$  = scores of male candidates  
 $H_0 = \mu_1 = \mu_2$   
 $H_1 = \mu_1 > \mu_2$
- (c) (i) A Type I error is made when the null hypothesis is rejected when it is actually true.  
(ii) P(Type I error) = 0.05 OR 5%
- (d) Reject  $H_0$  if  $z$  test  $> 1.645$
- (e) 
$$z \text{ test} = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sigma \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

$$= \frac{190.3 - 185.2 - 0}{12.4 \sqrt{\frac{1}{60} + \frac{1}{55}}}$$

$$\approx 2.203 \text{ to 3dp}$$
- (f) Reject  $H_0$  since 2.203  $>$  1.645 and conclude that there is sufficient evidence at the 5 % level of significance that the score of females candidates are greater than the scores of male candidates in the CAPE exam.

**Question 6**

This question tested the candidates' ability to test the difference between two population means taken from normal distributions of known variances using a t- test.

This question was well done by almost all candidates, who were able to carry out the test correctly and obtain full marks. A few candidates gave the incorrect  $t$  value.

**Answers**

- $X$ : masses of packets of margarine (in grams)  
 $H_0$ :  $\mu = 250$   
 $H_1$ :  $\mu \neq 250$

$$\Sigma x = 2004.4$$

$$\Sigma x^2 = 502\,245.72$$

$$v = 8 - 1 = 7$$

$$\text{Reject } H_0 \text{ if } |t_{\text{test}}| > 3.499$$

$$\begin{aligned} \bar{x} &= \frac{\Sigma x}{n} \\ &= \frac{2004.4}{8} \\ &= 250.55 \end{aligned}$$

$$\begin{aligned}
&= \frac{1}{n-1} \left[ \sum x^2 - \frac{(\sum x)^2}{n} \right] \\
&= \frac{1}{7} \left[ 502245.72 - \frac{(2004.4)^2}{8} \right] \\
&= \frac{43.3}{7} \approx 6.185714286 \\
t_{\text{test}} &= \frac{\bar{x} - \mu}{\frac{\hat{\sigma}}{\sqrt{n}}} \\
&= \frac{250.55 - 250}{\sqrt{\frac{6.185714286}{8}}} \\
&\approx 0.625 \text{ to 3sf}
\end{aligned}$$

since  $|t_{\text{test}}| \not\geq 3.499$ , accept  $H_0$  and conclude that there is sufficient evidence at the 1 % level of significance that the machine produces 250g packets of margarine.

**Paper 01 Option B**  
**SECTION C (Module 3: Rigid Bodies)**

**Question 11**

This question tested the candidates' ability to:

- (a) draw a diagram showing clearly the forces acting on the rod
- (b) find the inclination of the rod to the horizontal
- (c) calculate the force exerted by the peg on the rod
- (d) calculate the vertical component of the force exerted by the plane on the rod at its point of contact with the ground.

Given a uniform rod resting in equilibrium, with a point of the rod on a smooth peg, and one of its ends on a rough horizontal plane.

The candidates experienced difficulty inserting the forces correctly. For example, the direction of the force exerted on the rod by the smooth peg was drawn in the vertical direction, instead of perpendicular to the rod, by the majority of candidates.

Another weak area was that of “taking the moment of a force about a point”. They were not taking the product of the force and the **perpendicular distance** of the force from the point. This question was not well answered.

**Answer**

- (a) (ii) 30 degrees.
- (b) (i) 75.1 N (ii) 65 N

Question 12

This question tested the candidates' ability to calculate the period of oscillation of a particle moving with SHM, given its velocity and distance from the centre of oscillation in two specific cases.

This did not turn out to be a popular question. Those who attempted it seemed to be unfamiliar with the equation  $v^2 = \omega^2 (a^2 - x^2)$ , and the period of oscillation in terms of  $\omega$ , i.e.

$$T = \frac{2\pi}{\omega}. \text{ This resulted in low scores for the candidates.}$$

**Answer**                       $T = 1.6 \text{ sec.}$

**Question 13**

This question tested the candidates' ability to calculate the distance of the centre of mass of the composite body from the base given a solid hemisphere surmounted by a solid cone of equal radius..

Candidates did not display any knowledge of a satisfactory method of solving this problem. The problem was easily solved by using "moment of composite body = sum of moments of hemisphere and cone". Instead of this approach, candidates calculated the mean of the distances of C.G. of hemisphere and cone from the base. It was not a popular question.

**Answer:** 13 cm

Question 14

This question tested candidates' ability to use Hooke's law and apply Newton's law.

This proved to be an easy question for candidates, as they experienced little difficulty quoting Hooke's law and finding the correct resultant force to be used in applying Newton's law of motion. .

**Answer :**      Acceleration =  $7.13\text{ms}^{-2}$  .

Question 15

This question tested candidates' ability to solve a problem in which a body was performing circular motion on a rough horizontal surface.

This proved to be one of the easier questions for the candidates. They showed an appreciation of the acceleration  $r\omega^2$ , towards the centre of the circle, and correctly stated that on the point of slipping,  $F = \mu R$ .

**Answer :**

(a) 0.9 8 (b) 1 N (c) 1.02