Preface

The object of this work is to establish relationships between the numbers of general graphs and the numbers of multigraphs with given partition, with a view to enumerating multigraphs with a given partition.

The method employed involves removing loops from the general graphs on a particular partition thereby creating multigraphs on partitions which are related to the original partition, but are usually different from it. The number of general graphs is equal to the sum of the numbers of multigraphs obtained in this way.

The interrelation of the partitions in these two types of graph is represented in three different ways. The first is by means of a lattice which we call a G-lattice, and over which certain functions are defined which enable us to express the numbers of multigraphs in terms of the numbers of general graphs on related partitions and hence to derive formulae by which the multigraphs can be enumerated. The second representation is a matrix formulation which associates with the G-lattice a particular digraph, for which the adjacency matrix is used along with the appropriate unit matrix to set up the correspondence. The third method is a generating-function formulation in which the elements of the G-lattice are used to form polynomials from which the correspondences between the general graphs and multigraphs are obtained. The formulae arrived at in each instance all display inclusion-exclusion patterns in the numbers of general graphs on the partitions in the appropriate lattices.

The principle of inclusion-exclusion plays a major role in the formulation of our theorems and its importance cannot be overemphasised.

Chapters 1 through 4 deal with establishing general results about G-lattices of which the most important is the " constancy of G-lattices" (chapter 4). Chapter 5 is reserved for the matri: formulation of the problem and several properties of the matrices, of which the correspondence-matrix is the most important, are outlined. Chapter 6 deals with the generating-function aspect of the representation of the problem. Chapter 7 contains applications of the results obtained in chapters 1 to 6 and several enumerative theorems are formulated. The Superposition theorem and S-function are used to obtain numerical results. A few examples are included to illustrate the use of our theorems. Some remarks on possible further developments of the material included in this work constitute the final chapter 8.

Appendix 1 contains several expansions of S-functions which have been used in the calculations. A second appendix 2 contains some correspondence-matrices and their inverses and a third appendi 3 is reserved for generating-functions of a few simple problems.