C A R I B B E A N   E X A M I N A T I O N S   C O U N C I L

REPORT ON CANDIDATES' WORK IN THE CARIBBEAN ADVANCED PROFICIENCY EXAMINATION MAY/JUNE 2008

APPLIED MATHEMATICS (TRINIDAD AND TOBAGO)

APPLIED MATHEMATICS

MAY/JUNE 2008

INTRODUCTION

The revised Applied Mathematics syllabus was followed this year for the first time. The revised Applied Mathematics syllabus is a two-Unit course comprising three papers. Papers 01, multiple choice items, and Paper 02, essay questions, were examined externally; while Paper 03 was examined internally by the teachers and moderated by CXC. Contributions from Papers 01, 02 and 03 to each Unit were 30 per cent, 50 per cent and 20 per cent respectively.

Unit 1: Statistical Analysis consists of Collecting and Describing Data, Managing Uncertainty and Analysing and Interpreting Data.

Unit 2: Mathematical Application consists of Discrete Mathematics, Probability and Distributions and Particle Mechanics.

GENERAL COMMENTS

In Unit 1 93 per cent of the candidates obtained Grade I-V. In Unit 2 approximately 95 per cent of the candidates obtained acceptable grades, Grades I – V, while nine per cent obtained Grade VI. The standard of work from most of the candidates in this examination was above average. Again this year, candidates appeared to be well prepared in Discrete Mathematics and Probability and Distributions and generally answered the questions well.

However, the questions on Particle Mechanics were not well answered. As in previous years, candidates need to pay more attention to their algebraic manipulation.

APPLIED MATHEMATICS

MAY/JUNE 2008

PAPER 02 - UNIT 1

Question 1

This question tested candidates' ability to:

- distinguish between a cluster sample and a stratified random sample
- use stratified random sampling techniques
- calculate mean and Standard Deviations

This question was attempted by the majority of the candidates. For the most part high scores were achieved by candidates.

Part (a) (i) of the question was well attempted with few errors. Most candidates were awarded three or two marks.

Part (a) (ii) of the question, required candidates', knowledge of the differences between cluster sampling and stratified random sampling. Most candidates had a fairly good understanding of stratified random sampling, but far too many were not adequately familiar with cluster sampling.

Part (a) (iii) was well done.

In part (b), most candidates scored full marks in compositing the angles for the pie chart and were able to successfully construct the pie charts. Many, however, did not follow the instructions to use a radius of 4 cm.

The majority of the candidates scored high marks in part (c)(ii), where they were required to compute the standard deviation, however, a few candidates made mistakes in the arithmetic. Teachers are encouraged to provide the students with more practice in this area.

Question 2

This question tested candidates' ability to:

- construct a cumulative distribution table and the cumulative frequency curve
- draw box and whisker plots
- estimate median and inter-quartile range

This question was generally well done.

In part (a), many candidates omitted the first row with the "0" cumulative frequency. A few candidates did not attempt the cumulative frequency table.

In part (b), quite a few candidates plotted mid points rather than the upper class boundaries, so their graphs were translated to the left, hence they obtained incorrect values from the graph in (c). In addition, many candidates did not close their cumulative frequency curves by plotting the "0" cumulative frequency. A few candidates used rulers to draw a polygon instead of a smooth curve.

In part (c) (i), since "35" fell between the small blocks, a few candidates experienced problems reading off the values – they instead read the nearest integer values.

Part (c) (ii) was well done by most candidates. A few used "120", the upper class limit of the last call interval instead of using the sum of the frequencies to find the position of the quantities.

In part (c) (iii), some candidates incorrectly read the values from the graph and did not compute the position of the upper and lower quartiles correctly.

For Part (d), most candidates' box and whisker plots had the correct shape. A few of them did not show the whiskers. Some started and ended their whiskers at the lower and upper quartiles respectively.

Part (e) was well done by most of the candidates. Some common mistakes were:

- Candidates divided by "5" instead of "Σf"
- Candidates used mid points instead of boundaries.

In part (f), the majority of candidates had some idea of what the answer should be. However, a few confused the concepts positively and negatively skewed.

Question 3

This question tested candidates' knowledge and application in the following topics:

- probability
- mutually exclusive events
- independent events
- mean and standard deviation.

This question was done well by the majority of the candidates.

All candidates who attempted Question 3, did Part (a) (i) well.

In part (a) (ii), most candidates scored at least 1 mark. Many candidates only stated the possible combinations of colours rather than all the possible arrangements.

In part (b), most candidates understood when two events are mutually exclusive and independent and had few problems with this section.

In part (c), the majority of candidates had a generally good understanding of the probability concepts and were able to use a venn diagram to solve the problems on this part of the question.

In (d) (ii), most candidates had a basic understanding of expected value, but too many of them experienced difficulty in the determination of Var (x) and the standard deviation.

Question 4

This question tested candidates' ability to solve problems involving probabilities of the normal distribution.

In part (a), most candidates were able to state the three conditions that described a binomial distribution.

In part (b), most candidates were able to state which experiment may be modeled by a binomial distribution, although some did not clearly state a valid reason.

Parts (c) (i) and (ii) were very well done.

In part (d), most candidates answered well, with only a few candidates not being able to show the appropriate region. E.g. $P(-0.75 < Z < 1) = \Phi(0.75) - \Phi(1)$.

Question 5

This question tested candidates' ability to:

- calculate unbiased estimates of the mean and standard deviation
- state the critical region for a given test
- state conclusions drawn from hypothesis testing
- calculate the appropriate test statistics

Question (a) (i)a. was answered correctly by most candidates.

Part (a) (i)b. was generally not well answered due to use of incomplete formula – omitting of $\frac{n}{n-1}$ or $\frac{1}{n-1}$ and at times $\sqrt{\ }$.

In part (a) (ii)a., most candidates used χ^2 value rather than t-test. Most could identify the appropriate tail but a few forgot to apply the negative sign although they used the less than (<)sign.

Part (a) (ii)b. was done fairly well with the major error being the numerator. Candidates used (45 – 43) instead of (43 – 45).

Part (a) (ii)c. was well done based on the critical region and computed value.

Part (b) (i) was fairly well done with the main error being 12/180 instead of 1/9.

In part (b) (ii), there was weakness in determining the correct parameter. Some candidates wrote an English statement instead of using symbols as required by the question.

Part (b) (iii) was well done by most candidates.

In part (b) (iv), most candidates did not apply the continuity correction factor and used incomplete formula.

In part (b)(v), the conclusion was well stated in most cases based on the critical region and computed value.

Question 6

This question tested candidates' knowledge to:

- plot a scatter diagram from a given set of data
- calculate and interpret product movement correlation coefficient
- find the equation of a regression line y on x in the form $y = abx$.

Part (a) was well done by the majority of candidates.

In part (b), calculation of the product moment correlation coefficient was well done. The interpretation of the correlation coefficients was not clearly stated. A possible interpretation was "There is a very strong degree of positive (linear) correlation between x only".

In part (c) (i), finding the regression line of y on x in the form $y = a + bx$, was well done, with the exception of a few rounding off errors.

In part (c)(ii), the drawing of the line was correctly done by many, but some candidates produced a "line of best fit".

In part (d) (i) and (ii) candidates did not interpret the regression coefficient, b, and the constant a, in relation to the context of the question.

Part (e) (i) was well done by the majority of candidates.

In part (e) (ii), the reliability of the value obtained was not clearly stated, since some candidates could not conclude that the value fell outside of the range.

UNIT 2

PAPER 02

General Comments

Question 1

This question tested candidates' ability to:

- formulate a linear programming to determine programming model to determine the number of each product that would maximize the profit
- determine the feasible region for a given linear programming problem.

This question was not well done by most candidates.

In part (a), many candidates did not use the term "maximize". Some omitted the non-negativity constraints and did not state that "x and y are integers".

Part (b) was well done. Most of the candidates were able to identify the feasible region. A few interchanged the axes.

In part (c), many candidates did not write down the coordinates of the vertex and did not test all the vertices. Some candidates only tested the point they thought would yield the maximum value. Some candidates used the simplex method instead of using the graph. Quite a few candidates did not treat (0, 0) as a valid vertex in the feasible region.

Question 2

This question test candidates' ability to:

- employ four liable techniques to establish the validity of statements
- represent Boolean expression as a switching circuit

This question was reasonably well done.

Most candidates got part (a) correct. However, a few incorrectly stated "he not is tall and happy". The solution was "He is not tall and he is not happy" or "He is neither tall nor happy".

Part (b) was not well done. Quite a few candidates used truth tables, but did not identify the truth values of the statements.

The truth tables in part (c) were well done by most candidates. However, a few did not have a correct concept of a contradiction – they showed that they believed that if it is not a contradiction, then it must be a tautology.

In part (d)(i), some candidates confused switching and logic circuits. Some tried to use truth tables instead of drawing switching circuits. A few candidates confused parallel and series circuits.

In part (d)(ii), a few candidates used the "NOT' gate without the circle ("o"). A few candidates confused the "AND" and the "OR" gates.
Part (e) was very poorly done. A large number of candidates tried to use truth tables rather than use the laws of Boolean algebra as instructed. A few candidates confused complement and idempotent laws. Some did not use the distributive and deMorgan's laws appropriately. A possible solution is

$$(A \wedge B) \vee (A \wedge \sim B) \vee (\sim A \wedge \sim B)$$
$$= [A \wedge (B \vee \sim B)] \vee (\sim A \wedge \sim B)$$

$$= (A \wedge 1) \vee (\sim A \wedge \sim B)$$
$$= (A \vee (\sim A \wedge \sim B)$$
$$= (A \vee \sim A) \wedge (A \vee \sim B)$$
$$= 1 \wedge (A \vee \sim B)$$
$$= A \vee \sim B$$

Question 3

This question tested candidates' ability to calculate probability of events combined by intersections using appropriate counting techniques.

In part (a), candidates were generally able to identify odd numbers and numbers greater than 500 000. Most of the errors came in determining repetition for the other 4 digits.

In part (b), most candidates correctly recognized this as a combination.

In part (b)(i), most candidates were able to write ${}^{13}C_4$.

Part (b)(ii) was well done by most candidates.

In part (b)(iii), although many candidates were able to identify 3 girls and 1 boy or 4 girls and 1 boy, many did not state both groups or they added the combinations rather than multiply.

For example the 3 girls and 1 boy was calculated as $6C_3 + 7C_1$ instead of $6C_3 \text{ x } 7C_1$. The correct answer was $6C_3 \text{ x } 7C_1 + 6C_4 \text{ x } 7C_{0=}155$.

In part (c), many candidates did not recognize that if A and B are independent

$P(A/B) = P(A)$

These candidates calculated P(B) and then used $P(A/B) = \dfrac{P(A \cap B)}{P(B)}$

Candidates recognized that since A and B are independent then
$P(A' \cap B') = P(A') - P(B')$

Few candidates used $P(A' \cap B') = 1 - P(A \cup B)$

In part (d), most candidates were able to identify the correct solution and then gave
$P(A \cap B) \neq 0$.

Some candidates however, confused the condition for independence with the condition for mutually exclusive.

Question 4

This question tested candidates' ability to:

- use the probable function *f(x)=P(x=x)* when f is a simple polynominal
- calculate the probability of a random variable for a poisson distribution with a given mean value.

In part (a)(i), there were some candidates who attempted to integrate they were able to write

$$\int_1^4 k\, x\, dx = 1$$

However, none of these candidates integrated correctly. All of the other candidates obtained the correct answer.

In part (a)(ii), again those candidates who attempted to integrate reached as far as stating

$$\int_1^4 x\, f(x) = E(x).$$

Most other candidates correctly calculated $E(x) = \Sigma x\, P(X = x) = 3$.

Some candidates missed the final mark by incorrectly calculating $\frac{30}{10} = 3.5$ or 3.6.

In part (a)(iii), many candidates gave only 1 or 2 combinations. Those who got the 3 combinations correct proceeded to correctly finish the equation.

In part (a)(iv), few candidates used the table as a calculation for the values that Y could take.

Few candidates obtained all the values of Y giving values of 3, 4, 5, 6, 7 only. Several candidates appeared not to understand the concept of adding two independent variables. Calculating the probabilities for the values of Y is therefore a problem.

The solution is shown in the table below:

Y	2	3	4	5	6	7	8
P(Y=y)	1/100	1/25	1/10	1/5	1/4	6/25	4/25

In part (v), most candidates were able to write

$$E(Y) = E(X_1) = E(X_2)$$

but still had problems substituting values for $E(X_1)$ and $E(X_2)$.

Candidates did not understand what is meant by X_1 and X_2. In calculating Var (Y) some candidates calculated the $E(Y^2)$ from the table.

Few candidates used the property Var (Y) = 2 Var (X).

Part (a)(vi) E(3X + 2Y)

Though candidates were able to state E(3X + 2Y)
= 3 E(x) + 2 E(Y)

many did not complete the questions because they could not make the substitution for E(Y).

One candidate recognized

$$E(Y) = 2\,E(X)$$
$$\therefore\ 3E(X) + 2\,E(Y)$$
$$= 3\,E(X) + 4\,E(X)$$
$$= 7\,E(X).$$

For Part (b) $P(X \geq 2)$, most candidates interpreted this correctly as $1 - P(X < 2)$
Few candidates wrote $P(X < 2) = P(X = 0) + P(X = 1) + P(X = 2)$

Most candidates were able to correctly apply the Poisson formula.

<u>Question 5</u>

This question tested candidates' ability to apply the principle of conservation of

(i) energy and
(ii) linear momentum

involving the collision of two bodies moving in a straight line.

The candidates correctly equated the momentum before and after impact, to obtain the velocity of the combined masses after the collision.

However, some incorrectly equated the kinetic energies before and after impact, and therefore obtained incorrect values for the velocity.

Part (b)(ii) presented difficulty to many candidates, who appeared not to know where to start. The required distance could have been obtained by equating the work done and the loss of kinetic energy.

The time required to bring the particle to rest may then be obtained by

(i) Applying the formula $F = ma$ to obtain acceleration a and
(ii) using $v = u - at$, when $v = 0$ to calculate t.

Question 6

This question tested candidates' knowledge of the motion of a projectile.

Candidates showed that they were familiar with this type of question and performance was good.

In part (a), candidates quoted the expression for the greatest height and used it correctly. A small number of the candidates attempted to use the equation of the projectile, and then substitute $y = 0$.

This clearly showed a lack of understanding of "greatest height".

In part (b), expressions for the horizontal and vertical distances were correctly stated and the correct values obtained.

In part (b) (iii), there was a misprint of $u \cos t\ \alpha$ for $u \cos\pi 5\ \alpha$.

The majority of the candidates did not notice the difference and proceeded to find $u \cos \alpha$. Full marks were however awarded to all candidates attempting this part of the question.

Parts (iv) – (vi) were generally well done.

In part (v), candidates recognized that

$$u^2 = u^2 \sin^2\alpha + u^2 \cos^2\alpha, \text{ and}$$

used this to calculate u.

However, incorrect values of $u \sin \alpha$ and $u \cos \alpha$, resulted in an incorrect value for u in a few cases.

In part (vi), the method, $\tan \alpha = \dfrac{u \sin \alpha}{u \cos \alpha}$, was recognized, and all candidates applied this to the selection.

Approximately 24 per cent of the candidates were awarded a score of less than 10 out of 25.

INTERNAL ASSESSMENT

UNIT 1

Again this year, the overall presentation and quality of the samples submitted were satisfactory. Generally, candidates chose topics that were suitable to their level, and were relevant to the objectives of the syllabus. There were a few candidates who employed techniques that went beyond the level expected.

Project Title:

Some project titles did not clearly indicate what the project was about. Candidates had problems with making the title relate to the project, for example, "internal versus external exams". There is need for school teachers to clarify this area with candidates.

Purpose of Project:

Variables were not clearly defined. In some cases no variables were stated.

In cases where there were investigations to be done, the purpose of the project was not stated. There was a clear lack of responses to guide the project to a suitable conclusion.

Method of data Collection:

In many cases there was no sampling. Candidates used a fair amount of secondary data. Candidates were to state how they collected at the data in addition to stating the data.

Presentation of Data:

In general the presentation of data was well done. Use of more sophisticated 'A' level standard methods, like box and whisker plots, stem and leaf, plots is required.

Statistical Knowledge/Analysis of Data:

There was inappropriate use of concepts, for example, linear regression was attempted without calculation of the final equation of the line and the correlation coefficient was used with simplistic methods. In some cases, calculations were done that were unrelated to the statement of task. For example, correlation coefficients were calculated without any justification.

Discussion of Findings/Conclusion:

This section was often ignored or candidates were not prepared enough to relate the results to the purpose of the project.

Communication of Information:

In only a minority of cases did candidates lose marks here. However, it is recommended that candidates familiarize themselves with the technical terms of statistics to produce a higher level of analysis and to state the conclusions more efficiently.

List of References:

In some cases candidates failed to state title, author, reference numbers or dates of publication.

UNIT 2

PAPER 02

MATHEMATICAL APPLICATIONS

INTERNAL ASSESSMENT

Generally the topics chosen were suitable for candidates at this level. Again this year, most of the projects submitted by candidates were from the Discrete Mathematics and Probability and Distribution sections. These projects were appropriate and were given adequate treatment by candidates. There was an understanding of what was required in the assessment but a number of candidates who tried to incorporate Mathematics from two or more modules including Module 1 experienced difficulty in doing so. It was noted that the majority of the topics chosen were based on the distribution of a product within small communities and "get rich quick" schemes.

It appeared that the 'hands-on' approach through which candidates were afforded the opportunity to apply their Mathematics skills in real life situations served them well. Candidates appeared to have developed an overall appreciation of the mathematical concepts used in their projects and many used them with confidence. A few candidates were even able to do more than what was expected of them.

Teachers' marking of the projects showed that they understood the criteria used and are well aware of the type of assessment used. There was generally close agreement between the marks awarded by the teachers and those by the CXC Moderator.

RECOMMENDATIONS

It is recommended that candidates practise more problems involving the use of the normal distribution and questions from the Mechanics module. Exercises involving the use of algebraic manipulation are strongly recommended. It is necessary for teachers to complete the entire syllabus so that candidates can answer all of the questions adequately.