C A R I B B E A N   E X A M I N A T I O N S   C O U N C I L

REPORT ON CANDIDATES' WORK IN THE CARIBBEAN SECONDARY EDUCATION CERTIFICATE® EXAMINATION

MAY/JUNE 2012

ADDITIONAL MATHEMATICS GENERAL PROFICIENCY EXAMINATION

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GENERAL COMMENTS

Additional Mathematics, the newest CXC subject offering at the CSEC level, was tested for the first time in the May/June 2012 examinations. The subject is intended to bridge a perceived gap between the CSEC General Mathematics and the CAPE Unit1 Pure Mathematics. The intent is to offer CSEC Mathematics students a more seamless transition to the advanced thinking and skills needed for CAPE Unit 1 Mathematics courses, although a good CSEC Mathematics student should still be able to meet the skills and thinking demands of the CAPE Unit 1 Mathematics course.

The examinations consist of three papers:

- Paper 01 — a 45 – item multiple choice paper;
- Paper 02 — a structured, 'essay-type' paper consisting of 8 questions;
- Paper 031 or Paper 032 — Paper 03 represents an School-Based Assessment (SBA) project component for candidates in schools, and Paper 032 an alternative to the SBA for out-of-school candidates.

The Additional Mathematics syllabus (CXC 37/G/SYLL 10) tests content in four main topic areas divided as follows: Section 1 — Algebra and Functions, Section 2 — Coordinate Geometry, Vectors and Trigonometry, Section 3 — Introductory Calculus, and Section 4 — Basic Mathematical Applications.

Paper 01 tests content from Sections 1, 2 and 3 of the syllabus. This paper carries 45 items which are weighted up to 60 for the overall examination. Paper 02 tests content from all four sections of the syllabus. This year the paper consisted of four sections, divided as described previously for the outline of the syllabus. Each section contained two problem-solving type questions. The questions in Sections 1, 2 and 3 were all compulsory. The two questions in these sections were worth a total of 28, 24 and 28 marks respectively. Section 4 also contained two questions, one on Data Representation and Probability and the other on Kinematics worth 20 marks each; candidates were to choose one question from this section. Paper 03 represents the SBA component of the examination. Candidates can do a project chosen from two project types, a mathematical modelling project (Project A) and a data handling/statistical analysis project (Project B). The SBA component is worth 20 marks. Alternatively, candidates can sit an alternative to the SBA paper, Paper 032, which consists of a more in-depth extended question from Sections 1, 2 and/or 3 of the syllabus. This paper carries 20 marks.

DETAILED COMMENTS

Paper 01 – Structured Essay Questions

This was a 45 - item paper covering Sections 1, 2 and 3 of the syllabus. A total of 1720 candidates sat this paper of the examination. Candidates scored a mean of 23.22, with standard deviation 8.77.

Paper 02 – Structured Essay Questions

No formula sheet was provided for this sitting of the examination. However, in cases where it was suspected that candidates might have been affected by the absence of a formula sheet the mark scheme was adjusted so that candidates were awarded follow through marks if they used an incorrect reasonable formula, and were not penalized for this wrong formula. For future examinations, a formula sheet will be provided for papers 01 and 02.

Section 1: Algebra and Functions

Question 1

This question tested candidates' ability to:

- determine a composite function, its range and inverse;
- make use of the Factor Theorem;
- use the laws of indices to solve exponential equations in one unknown; and
- apply logarithms to reduce a relationship to linear form.

There were 1746 responses to this question. The mean mark was 5.72 with standard deviation 3.26. Thirty-nine candidates obtained full marks.

Candidates performed best on Parts (a) (i), (iii) and (b) which required them to determine the composite function *g(f(x))* of given functions *f(x)* and *g(x),* and the inverse of the composite function. In Part (a) (iii) some candidates experienced difficulty at the interchange step, that is,

1) (i) $g(f(x)) = x^3 + 6$; Let $y = g(f(x)$ (ii) $\left[*y^3 = x+6*\right]$ commonly seen error

$y = \sqrt[3]{x+6}$

$\therefore \left[gf(x)\right]^{-1} = \sqrt[3]{x+6}$

Part (b) was generally well done by candidates. However, some of them did have difficulty here. The following common errors were seen:

- Substituting $x = 2$ into *f(x)* rather than $x = -2$;
- Some candidates who did correctly substitute $x = -2$ then made errors in their manipulation of the directed numbers;
- A few candidates attempted long division approach to solving this problem; however they very often encountered some difficulty with this.

Parts (a) (ii), (c) and (d) presented the most difficulty to candidates. In Part (a) (ii), which required candidates to state the range of the composite function, generally candidates were unable to do this, or stated discrete values for the range, that is, {6, 7, 14, 33}, not recognizing it as the infinite set of Real Numbers, $6 \leq g(f(x)) \leq 33$.

In Part (c), many candidates attempted to solve the given equation $3^{2x} - 9(3^{-2x}) = 8$ without (appropriate) substitution. Having not recognized a suitable substitution to use in order to transform the equation into polynomial form, candidates incorrectly took logarithms of each term in the equation. Some candidates were able to recognize that $y = 3^{2x}$ or $y = 3^x$ were suitable substitutions and quite often were able to derive an appropriate polynomial equation. Those who derived the quadratic equation were able to easily solve the indices equation. However, those who derived the quartic equation at times found it challenging to solve. A number of candidates who got to the solution stage of the problem did not recognize (or know) that the logarithm of a negative number did not exist.

In Part (d), although many candidates knew how to transform $x^3 = 10^{x-3}$ to logarithmic form, they did not use brackets and so wrote $3 \log_{10} x = x - 3 \log_{10} 10$, instead of the expected $3 \log_{10} x = (x - 3) \log_{10} 10$. Whilst candidates did not lose marks this time given the nature of the specific case here (base is the same as the number so that $\log_{10} 10$ is 1), teachers are asked to encourage their students to use brackets to avoid possible errors in the future. Another common error made was having reached to the stage of 3 $\log_{10} x = x - 3$, a number of candidates then went on to give the gradient as 1, seemingly not recognizing that they had to write the equation in terms of log x, that is. to divide through by 3 so that the coefficient of log x was 1, i.e. $\log_{10} x = \frac{1}{3}x - 1$, so that the gradient of the linear form was $\frac{1}{3}$. Other common errors observed were:

- $\text{Log}_{10} x^3 = \log_{10} 10^{x-3}$
  $\text{Log}_{10} 3x = \log_{10} x - \log_{10} 3$

- $\text{Log}_{10} x^3 = \log_{10} 10x - \log_{10} 30$
  $3\log_{10} x = 10x - 30$

Solutions

(a) (i) $g(f(x)) = x^3 + 6$ (ii) $6 \leq g(f(x)) \leq 33$ (iii) $[gf(x)]^{-1} = \sqrt[3]{x-6}$
(b) $a = 20$
(c) $x = 1$
(d) (i) $\log_{10} x = \frac{1}{3}x - 1$ (ii) gradient $= \frac{1}{3}$

<u>Question 2</u>

This question tested candidates' ability to:

- make use of the relationship between the sums and products of the roots of quadratic equations;
- solve an inequality of the form $\frac{ax+b}{cx+d} \geq 0$;
- identify an AP, obtain an expression for its general term, find its sum to a finite term.

There were 1733 responses to this question. The mean mark was 5.64 with standard deviation 4.27. One hundred and four candidates obtained full marks

Candidates performed best on Part (c) where they had to recognize the problem as an Arithmetic Progression (AP) and find its first term and common difference.

Parts (a) and (b) presented the most difficulty to candidates. In Part (a), many candidates attempted to solve the quadratic equation whether by factorizing or using the formula, showing a seeming lack of knowledge about the sum and product of roots of quadratic equations. In Part (b), although some candidates were able to determine the boundary values for the inequality, they had difficulty with the direction of the inequality for these boundary values which would make the overall inequality 'true'. Although more rarely seen, there was generally more success among candidates who attempted a graphical solution to this question.

In Part (c), a common error observed related to candidates finding the 24th term of the AP (T_{24}) rather than the sum of the first 24 terms (S_{24}). Another common error observed was that some candidates could not correctly write the S_n formula, commonly writing it as $S_n = \frac{1}{2}[a + (n-1)d]$, instead of the expected $S_n = \frac{n}{2}[2a + (n-1)d]$.

Solutions

(a) $\alpha^2 + \beta^2 = 4$

(b) $x > \frac{5}{2}$ and $x < -\frac{1}{3}$

(c) Total paid = \$2100

Section 2: Coordinate Geometry, Vectors and Trigonometry

Question 3

This question tested candidates' ability to:

- find the centre of a given circle and showing that a given line passes through this centre;
- find the equation of a tangent to a circle at a given point;
- add and subtract vectors, find displacement vectors and find the magnitude of a vector.

There were 1715 responses to this question. One hundred and sixteen candidates obtained full marks.

Candidates performed best on Part (a) (ii) which required them to calculate the gradient of the radius of the circle (from two points), and hence determine the gradient and equation of the tangent to the circle's radius.

Candidates had difficulty with Part (a) (i) which required them to show that the line $x + y + 1 = 0$ passed through a given circle's centre. Although many could find the coordinates of the circle's centre, some candidates had difficulty in showing (that is, proving) that the given line passed through this centre, indicating a seeming lack of knowledge of what constituted a simple proof in this case. Candidates also had difficulty with Part (b), writing $\overrightarrow{BP}$ in terms of its position vectors, or correctly adding vectors to get $\overrightarrow{BP}$. A common error seen here was that $\overrightarrow{OP} = \overrightarrow{BP}$.

Solutions

(a) (ii) $y = \frac{4}{3}x + 11$

(b) (i) $\vec{BP} = \frac{3}{2}\mathbf{a} - \mathbf{b}$ (ii) $|\vec{BP}| = \frac{1}{2}$

Question 4

This question tested candidates' ability to:

- use the formulae for arc length and sector area of a circle;
- use the compound angle formula to evaluate a value for sin θ in surd form.

There were 1705 responses to this question. The mean mark was 4.20 with standard deviation 2.24. Two hundred and fifteen candidates obtained full marks.

Candidates performed best on Part (a), that is, finding the area and perimeter of a given shape, although some candidates did not seem to understand the instructions to give their answer in terms of π. For example, some candidates having arrived at the area of the figure as $16 + \frac{8\pi}{3}$ then went on to give a decimal answer. Other candidates mistakenly simplified this to $\frac{24\pi}{3}$. In Part (a) (ii), some candidates failed to recognize that one side of the square (or one radius of the sector) should not be included in the calculation for perimeter, and so added the whole perimeter of the square to the whole perimeter of the sector.

Part (b) presented difficulties for many candidates, in particular coming up with an appropriate split for $\frac{7\pi}{12}$; hence many candidates did not recognize the need for use of the compound angle formula. Some candidates who recognized the need to split then wrote it as $\cos\frac{\pi}{3} + \cos\frac{\pi}{4}$ for which no marks were awarded. Of note, a few candidates wrote that $\cos\frac{7\pi}{12} \equiv \cos\left(\frac{7\pi}{3} + \frac{7\pi}{4}\right)$ and used the fact that 2π is one revolution to come to the correct compound angle $\cos\left(\frac{\pi}{3} + \frac{\pi}{4}\right)$. The concept of exactness also appeared to be lost on many of the candidates who proceeded to write the solution, $\frac{\sqrt{2}-\sqrt{6}}{4}$ as a decimal, or some other incorrect simplification, such as $-\frac{\sqrt{4}}{4}$.

Part (c) was not marked. However, approximately $\frac{1}{3}$ of the candidates who attempted this part of the question knew that $\sec x = \frac{1}{\cos x}$.

Solutions

(a) (i) $16 + \dfrac{8\pi}{3}$ m^2 (ii) $16 + \dfrac{4\pi}{3}$ m

(b) $\dfrac{\sqrt{2}-\sqrt{6}}{4}$

Section 3: Introductory Calculus

Question 5

This question tested candidates' ability to:

- differentiate a quotient made up of simple polynomials;
- determine the equations of tangents and normal to curves;
- make use of the concept of derivative as a rate of change.
-

There were 1712 responses to this question. The mean mark was 5.89 with standard deviation 4.47. One hundred and thirty-three candidates obtained full marks.

In this question, candidates performed best on the mechanics of differentiation. Many though could not state the correct quotient rule for differentiation, and so performed the mechanics of differentiation in Part (a) as if for a product rule. And, having performed the mechanics of differentiation correctly, a marked number of candidates then could not apply the distributive law correctly with a negative 1 multiplier to expand the brackets, i.e. $-1(3x + 4)$ was often expanded as $-3x + 4$.

Part (b) was generally reasonably done. Some candidates experienced difficulty with the meaning/significance of $\dfrac{dy}{dx}$, in that having found it for the given curve, they could not translate it into finding the value of the gradient at the given point (2, 10). That said, making use of a value for gradient to find the equation of the tangent and the equation of the normal was fairly widely known.

Approximately 50 per cent of candidates did not appear to know the topic 'Rates of Change' and so could not do Part (c). Others, having determined that the area, A, of the square (s) could be written as $A = s^2$ then went on to differentiate this as $\dfrac{dy}{dx} = 2s$ instead of $\dfrac{dA}{ds} = 2s$, showing a lack of understanding of the meaning of differentiation as rate of change. Candidates' difficulty with notation was especially evident in this part of the question; very few candidates wrote their differential equations in terms of dt.

Solutions

(a) $\dfrac{-10}{(x-2)^2}$

(b) (i) $y = 17x - 24$

(ii) $10y = x + 172$

(c) $\dfrac{dA}{dt} = 40$ cm^{2}s^{-1}

Question 6

This question tested candidates' ability to:

- evaluate the definite integral of a function [of the form $(ax \pm b)^n$, $n \neq -1$];
- determine the equation of a curve given its gradient function and a point on the curve;
- find the area of a region bounded by a trigonometric curve and the x-axis;
- find the volume of revolution of the solid formed when a curve of degree 2 is rotated about the x-axis.

There were 1655 responses to this question. The mean mark was 6.23 with standard deviation 4.61. One hundred and two candidates obtained full marks.

Candidates performed best on Part (b), although some lost marks as they did not include or failed to find a value for the constant of integration. Part (c) was also relatively reasonably done, although some candidates experienced difficulties in correctly evaluating cos θ. In Part (a), in their integration of $(16 - 7x)^3$, although many candidates could perform the mechanics of integration to obtain 4 in the denominator, many candidates failed to multiply the coefficient of x in the polynomial by the 4 to obtain -28 for the denominator. A few candidates expanded the polynomial in order to integrate it.

Part (d) presented the most difficulty for candidates. Many candidates could set up the problem as $\pi \int_0^3 (x^2 + 2)^2 \, dx$, although some did forget the 'π' in their formula, but a number of candidates were unable to expand $(x^2 + 2)^2$ correctly, very often obtaining $x^4 + 4$.

Solutions

(a) $\frac{935}{4}$ (b) $y = \frac{3x^2}{2} - 5x + 4$ (c) $\frac{2\sqrt{3} + 3}{2}$ units^2

(d) $96.6\,\pi$ units^3

Section 4: Basic Mathematical Applications

Question 7

This question tested candidates' ability to:

- make use of the laws of probability, specifically the sum of all probabilities in a sample and the addition rule;
- calculate the probability of an event occurring;
- construct and use a tree diagram and calculate conditional probability.

Candidates performed best on Part (a). Many used their knowledge of Sets and Venn Diagrams from CSEC Mathematics to represent the given problem and successfully solve it. However, for candidates who used a formula method to solve, a common error observed was forgetting to subtract the intersection $P(L \cap D)$ (that is, owned both a laptop and a desktop computer) from the $P(L) + P(D)$.

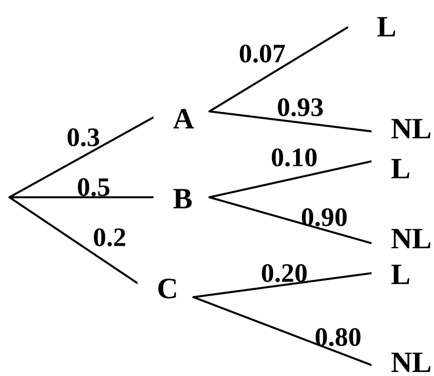
Parts (b) and (c) presented the most difficulty for candidates. In Part (b), a common error observed related to candidates presenting a solution representing sampling with replacement, although the question stated that the items were withdrawn without replacement.

In Part (c) (i), many candidates had difficulty drawing the tree diagram and it was clear that some did not know what this was as they drew diagrams of actual trees, or drew stem and leaf diagrams. Of those who did know what to do, many could only correctly draw the first branch, and had problems completing the second branches for 'late/not late'. Two common errors observed were candidates who correctly wrote the probability of a taxi being late from garage A as P(L) = 0.07 but then calculated P($\overline{L}$) = 0.23, and candidates who incorrectly wrote the probability of a taxi being late from garage A as P(L) = 0.7 instead of P(L) = 0.07 as given. In Part (c) (ii) b, candidates recognized that conditional probability was involved but did not know the formula for conditional probability; they knew that some sort of division was necessary but did not know what to divide by, with many not using the probability they had computed in Part (c) (ii) a) as the denominator here.

Solutions

(a) 17% (b) (i) $\frac{1}{20}$ (ii) $\frac{1}{2}$

(c) (i)



(c) (ii) (a) 0.111 (b) $\frac{40}{111}$

<u>Question 8</u>

This question tested candidates' ability to:

- draw and interpret and make use of velocity-time graphs;
- apply rates of change to the motion of a particle.

There were 879 responses to this question. The mean mark was 6.96 with standard deviation 5.02. Seven candidates obtained full marks.

Candidates performed best on Parts (a) (i) and (ii) which required them to draw a velocity-time graph from given information, and also to determine the distance.

Part (a) (iii) however presented the most difficulty for candidates. This part dealt with a second car starting 3 seconds after the first car from the same starting point, moving at a constant (faster) velocity and meeting the first car sometime later. Most candidates did not know that the distances covered would have been the same at the meeting point, or that the second car would have taken 3 seconds less than the first car to reach that point. Candidates' experience of difficulty with Part (b) related mainly to notations and their failure to incorporate the constant of integration into their solutions.

Solutions

(a) (ii) 264 m (iii) 6 seconds

(b) (i) velocity = 32 ms^{-1} (ii) displacement = $\frac{52}{3}$ m

Paper 031 - School -Based Assessment (SBA)

Generally, many of the projects submitted were of a high quality and related to real-world situations. Many showed a high level of ingenuity, and were obviously scenarios in which students were interested (for example, modelling the trajectory of a basketball to maximize the number of points that could be gained from free throws). A number of the submissions showed evidence that students had applied a high degree of effort to the task, understood the focus of their topic selection and had conducted the requisite research thoroughly. Projects were generally well presented. Marks were generally in the range of 14 to 20, with many of the sample submissions receiving the full score of 20 marks.

There were nonetheless a number of observed weak areas related to sample submissions. These included:

- Some SBA project titles and aims/purpose were not clear, not concise and not well defined. Some titles were too vague, others too wordy. In some cases, students simply restated the project's title as its aim/purpose;
- In a number of cases, the project's aim did not relate to the given title and sometimes not linked to any syllabus objective;
- In some cases for Project B, students did not identify their variables which would be a key aspect of this project type related to the analysis and discussion of the results;
- In some cases for both Projects A and B assumptions were not always stated;
- Some students' data collection method(s) were flawed, with the resulting negative impact on analysis and findings;
- It appeared that all (or most) of the students at some centres conducted their research using the same topic or a limited number of topics. Although there is nothing untoward with this approach, teachers must be aware that the probability of students presenting identical results, analyses and conclusions is very remote. In essence, students must demonstrate individuality in their work;
- In presenting their data or mathematical formulations, many students were short on explanations of what was happening so it was sometimes difficult to follow their presentations;
- Many students had difficulty presenting an appropriate, coherent and full analysis of their data, hence finding(s) and conclusion(s) were deficient;
- The majority of students did not incorporate or suggest any future analysis in their submissions;
- Some students failed to adequately connect their finding(s), discussion and/or conclusion(s) to their original project aim(s)/purpose, their data collected and their analysis of the data. In some cases, the conclusions stated did not arrive out of the presented findings;
- A few submissions appeared to be downloaded from the internet, and there were cases where submitted samples were blatantly plagiarized from CXC/CSEC Additional Mathematics material. Teachers must be alert to any suspected plagiarism. Plagiarism must not be condoned and marked accordingly;

- Some students presented projects that were more applicable to the sciences with an inadequate application of Additional Mathematics concepts;
- Whilst projects were generally well presented, a marked number of them showed deficiencies in grammar, spelling and punctuation.

Other general issues encountered in moderating the SBA samples included:

- There was no CSEC Additional Mathematics rubric/mark scheme accompanying the samples submitted by some centres;
- Some teachers created their own rubric or made adjustments to the CSEC Additional Mathematics rubric;
- Some projects were incorrectly categorized and assessed, that is, a Project A being labelled and assessed as a Project B, and vice versa. This did create some problems as for example, Project B requires the collection and analysis of data from an experimental-type activity.

The following recommendations hold for continued improvement in this aspect of the Additional Mathematics examinations:

- All projects should have a clear and concise title, and well defined aim(s) or purpose;
- Where possible the SBA should identify with authentic situations;
- The variables that are being used or measured (Project B) must be clearly stated and described. Variables can be controlled, manipulated and responding;
- The type of sample and sample size if relevant must be clearly stated;
- Teachers must ensure that projects which integrate other subject areas utilize concepts as contained in the CSEC Additional Mathematics syllabus;
- If students collect their data in a group setting, they must demonstrate their individual effort in relation to analysis (interpretation) and finding(s)/conclusion(s);
- Projects involving dice or playing cards must be more expansive so that students can present a more in-depth analysis of the topic under consideration;
- As good practice, students should be encouraged to cite all sources and insert a reference/bibliography page;
- Teachers should guide students using the assessment criteria found in forms 'Add Math 1 – 5A' and 'Add Math 1 – 5B' which are both available on the CXC website. Teachers can give their students the rubric as a means of guidance in developing their projects.

Paper 032 - Alternative to School-Based Assessment (SBA)

This paper tested candidates' ability to:

- formulate a mathematical problem and derive values to maximize its solution;
- make use of the laws of indices and logarithms to evaluate a specific n^{th} value of an AP.

There were 54 responses to this paper. The mean mark was 5.24 with standard deviation 4.22. The highest mark awarded was 18/20.

In Part (a), candidates were able to formulate the mathematical problem as directed. However, a number of candidates lost marks for not finding the second derivative $\frac{d^2A}{dx^2}$ (for both sports clubs) and confirming that it was a maximum. Candidates also made various arithmetic errors in their solutions so few were able to come up with the expected dimensions for both sports clubs.

All candidates who attempted Part (b) approached a solution using the given information that the series was an AP. A number of candidates were able to get one expression for the common difference d by subtracting two consecutive terms given. However, many failed to obtain a second expression for d and so could not equate these two to get an expression for a in terms of b. Many did not realize that they needed to substitute for a and so ended up with a final expression in ab, which, in some cases even though correct, would not have given them the requested value for n.

Solutions

(a) (i) Maximize $A = 6xy$ Subject to $9x + 8y = 600$

(ii) Maximum area Q = 7500 m^2 (iii) Maximum area P = 22 500 m^2

(b) $n = 112$

Appendix: Suggested Solutions to Selected Question Parts in Paper 02

1(c) Solve $3^{2x} - 9(3^{-2x}) = 8$ [1]

Let $3^{2x} = m$; Substitute into [1] gives

$m - \dfrac{9}{m} = 8$

$m^2 - 9 - 8m = 0$

$m^2 - 8m - 9 = 0$

$(m - 9)(m + 1) = 0$

$m = 9$ OR $m = -1$

that is, $3^{2x} = 9$ OR $3^{2x} = -1$ (INVALID)

From $3^{2x} = 9$

$2x \log 3 = \log 9$

$2x = \dfrac{\log 9}{\log 3} = 2$

$x = 1$

2(b) $\dfrac{2x-5}{3x+1} > 0$

For LHS > 0, numerator AND denominator must BOTH have the same sign

For $2x - 5 > 0$ AND $3x + 1 > 0$

$x > \dfrac{5}{2}$ AND $x > \dfrac{-1}{2}$

Both true when $x > \dfrac{5}{2}$ [1]

AND for $2x - 5 < 0$ AND $3x + 1 < 0$

$x < \dfrac{5}{2}$ AND $x < \dfrac{-1}{2}$

Both true when $x < \dfrac{-1}{2}$ [2]

From [1] AND [2], range of values when $x > \dfrac{5}{2}$ AND $x < \dfrac{-1}{2}$

5(c) $\dfrac{ds}{dt} = 4 \text{ cm s}^{-1}$

$A = s^2$; $\dfrac{dA}{ds} = 2s$

$\dfrac{dA}{dt} = \dfrac{dA}{ds} \times \dfrac{ds}{dt}$

$= 2s \times 4 = 8s$

When $s = 5$, $\dfrac{dA}{ds} = 8 \times 5 = 40 \text{ cm}^2 \text{ s}^{-1}$

7(b) Let R = red marble, B = blue marble, K = black marble

(i) P(all of same colour) = P(RRR) + P(BBB) + P(KKK)

$$= \frac{4}{10} \times \frac{3}{9} \times \frac{2}{8} + \frac{3}{10} \times \frac{2}{9} \times \frac{1}{8} + \frac{3}{10} \times \frac{2}{9} \times \frac{1}{8}$$

$$= \frac{1}{20}$$

(ii) P(exactly one Red) = P($R\bar{R}\ \bar{R}$) × 3

$$= \frac{4}{10} \times \frac{6}{9} \times \frac{5}{8} \times 3$$

$$= \frac{1}{2}$$

7(c) (ii) (a) P(L) = P($A \cap L$) + P($B \cap L$) + P($C \cap L$)

= 0.3 × 0.07 + 0.5 × 0.1 + 0.2 × 0.2

= 0.021 + 0.05 + 0.04

= 0.111

(b) $$P(C/L) = \frac{P(C \cap L)}{P(L)}$$

$$= \frac{0.2 \times 0.2}{0.111}$$

$$= \frac{0.04}{0.111}$$

$$= \frac{40}{111}$$