

Casting the Net

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“What is the net of an open-ended cylinder?” This is one of the most productive activities that I use to illustrate concepts related to teaching and learning and for providing meaningful opportunities to learn. Adults and students alike rush to give “the answer”—“*a square*”—quickly amended to the more general category, “*a rectangle*.” **“How do you know?”** I challenge, which is usually greeted by an anguished silence as if the *right* answer was all that would ever be required. A few brave souls eventually attempt an explanation that runs in one of two directions—cutting an open-ended cylinder, such as a paper towel roll, with scissors (which have been strategically provided) in a straight line along the vertical axis and flattening the surface, or folding a rectangular sheet of paper so that the two opposite parallel sides meet. At this point, most are quite satisfied with their mathematical knowledge, and are prepared or can be easily instructed to handle the algorithmic computations that are instrumentally tied to the determination of (surface) areas and volumes of real-world cylinders like pipes, hoses, blood vessels, reeds, rods, and other structures with uniform circular cross section; tasks that frequently appear in SEA and CXC exams. This, however, is where the real lessons about teaching and learning begin.

“Examine the paper towel rolls in front of you. Are there any other nets that can produce a cylinder?” After short work and some discussion, many, having unfurled the roll along the continuous spiral where it is stuck, declare the net to be “*a parallelogram*.” Some begin to make connections between the areas of parallelograms and rectangles, others between the shapes themselves. Continuing the investigation, **“Are there any more?”** Encouraged by having found one more than they had anticipated, and armed with simple concrete materials—scissors and discarded paper towel rolls, investigations and discussions proliferate. Soon all manner of “shapes”—parallelograms of different sizes, V-shaped chevrons, serrated polygons, beautiful obzocky plane figures with no names—populate the learning space. All are nets of the same cylinder, all with the same area, and all with (at least) one other “mathematical” feature in common.

Looking across the seemingly endless variety of forms most beautiful and wonderful, two mathematical truths eventually become apparent to learners. However, it usually takes them a little while longer to be able to articulate their insight in acceptable mathematical language—“*each ‘shape’ has one pair of parallel sides of the same length*”—the reminder that this was and could become a cylinder. Secondly, they complete the generalisation, “*the other two sides fit together like a jigsaw puzzle, they are geometrical ‘opposites’*.” Tracking back now we try to put our growing insights together with what we already know. The challenging question and task becoming the articulation, communication, and inscription (in acceptable mathematical discourse) of how these different nets are generated, that is, to relate the different ways in which “cuts” are made with the final forms obtained, to delineate the forbidden cut(s), and to understand the rectangle and parallelogram as the result of special cases of cutting.

Other questions arise—Why is the parallelogram used so frequently to construct cylinders? What advantages does it have over other forms? These lead in the direction of aesthetics, optimisation, the mathematics of spirals, fault lines, and structural integrity. Unsettling questions emerge—both conceptually and pedagogically. Are these things really nets? Wouldn’t students be confused and disadvantaged in their examination by learning how to think differently about the net of a

cylinder when all they are required to know is what it is and how to find its area and volume? What about the people setting and marking the papers? Would they accept the answer that the net of an open-ended cylinder is something other than a rectangle or parallelogram?

Many teachers declare in our debriefing after the activity that while they have enjoyed the learning experience, “*we were never taught this*” or “*in this way*” or “*I couldn’t do this in my school,*” or ominously invoke the inviolable authority of “*the syllabus.*” “***Neither was I***”, “***yes you can***” and “***So what?***” I respond. Might we find the courage to shift the sole focus away from the requirements of a syllabus or an exam to supporting ***learning***? The curriculum is what we make it in *our* classrooms. I end by rephrasing the opening question and title of the lesson in the plural, more generative, form, “What ***are*** the nets of an open-ended cylinder?” The answer, hopefully, a rich space for experiencing meaningful and memorable learning.

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