ABSTRACT

Connections Between Circuit Polynomials and Certain Areas of Linear Algebra

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This thesis deals with the application of cycle polynomials to functions of matrices such as permanents and determinants.

Let G be a graph. For each cycle, α, of a cycle (or circuit) cover of G, a weight w_α can be associated. The weight of a cycle cover S in G is $w(S) = \prod_{\alpha} w_\alpha$. The cycle polynomial (or circuit) polynomial of G is then, $C(G;\underline{w}) = \sum_{S} w(S)$, where $\underline{w}$ is a vector of weights assigned to the cycles.

With every n×n matrix A = [a_{ij}] over the real numbers, we can associate a digraph D_A, whose node set is $V(D_A) = \{1,2,\ldots,n\}$. Two nodes i and j are joined by a directed arc (i, j) labelled a_{ij} if and only if a_{ij} is non-zero.

In this thesis, we utilise cycle polynomials and associated digraphs, to establish novel proofs of well known results on matrix operations. General classes of matrix functions are found which satisfy results on matrix operations. The permanent and determinant of a graph which are, respectively, the permanent and determinant of an associated matrix, are also studied. We also identify and classify families of graphs whose permanents and determinants are various classes of integers.

Keywords: Akhenaton Daaga; Cycle Polynomial; Associated Digraph; Permanent; Determinant; Immanant; Matrix Operations.