

Healthy Mathematical Dispositions

Steven Khan

A quarter century and more of re-conceptualising globally what it means to know, learn, and do mathematics has witnessed a change from the view of mathematics as a “collection of abstract concepts and procedural skills to be mastered” to a set of “human sense-making and problem-solving activities based on mathematical modeling of reality.” Consequently, the goal of a mathematics education at present is seen to be the development of a healthy “mathematical disposition.” The components of such a disposition include mastery of specific content knowledge, mastery of heuristics (problem-solving strategies), meta-knowledge proficiency (knowledge about one’s thinking while thinking), self-regulatory skills (the ability to alter one’s thinking/actions/strategies based on meta-awareness and to effectively manage negative affect), and positive mathematics-related beliefs. This last category includes seeing mathematics as a sensible, useful, and worthwhile subject of inquiry, beliefs about one’s own efficacy and the role of effort in achievement in the discipline, and beliefs about the functioning of mathematics in wider society. The general public, anti-intellectual cultures, and societal institutions, though, harbour and transmit many beliefs, negative attitudes, and misconceptions about mathematics and its teaching and learning, which make the developing of healthy dispositions a difficult task.

Among the most common beliefs that students (and adults) have about mathematics are that: 1) mathematics problems have one and only one right answer; 2) there is only one correct way to solve any mathematics problem; 3) ordinary students cannot expect to understand mathematics—they simply memorise it and apply what they have learnt mechanically and without understanding; 4) mathematics is a solitary activity done by individuals in isolation; 5) students who understand mathematics will be able to solve any problem in five minutes or less; 6) the mathematics learned in school has little or nothing to do with the real world; and 7) methods of formal proof are irrelevant to processes of discovery or invention of new mathematics. Such wide-ranging and varied beliefs are inimical to the development of a healthy mathematical disposition and may reflect forms of sociocultural conditioning. Teacher practices and instructional materials do much to support this conditioning.

Educators must work at helping students to develop more productive dispositions towards mathematics. This entails providing students with more authentic mathematical experiences than have traditionally been found in mathematics textbooks. For example, few students, before university, will ever encounter a mathematical problem that will require days of thinking, hours of frustration, intense collaboration and discussion, and numerous dead ends before a solution is arrived at. For most teachers, there simply isn’t enough time for this type of inquiry activity. But this is exactly what real mathematicians do. In March 2007, the American Institute of Mathematics announced that after four years of collaboration, among 18 mathematicians and computer scientists, the structure of the 57 dimensional Lie Group E_8 , a 120-year-old problem, was complete. The solution was represented as a 453,060 by 453,060 Matrix. The accomplishment was described as monumental in that mathematics could “now be viewed as a team sport.”

In attempting to develop healthy mathematical dispositions, mathematics educators must provide learners with sufficient opportunities to engage in meaningful mathematical activities that

require more than five minutes or even one session of engagement, and which may require a subdivision of mathematical tasks among individuals. This is not an easy task, but it is an essential one.

Students' beliefs about themselves as mathematics learners and about mathematics exert a strong influence on their engagement with mathematics and their subsequent achievement. Such beliefs include confidence in one's ability (self-efficacy) and attribution of success or failure, among others. High-achieving students tend to have a realistic view of their ability. Some low-achieving students overestimate their ability, while others' lack of confidence undermines their performance. High-achieving students also tend to correctly attribute their success or failure to their own efforts as opposed to luck with questions, chance, or some other external factor.

The research literature suggests that the early years of secondary school may be the most important period for determining students' beliefs about mathematics as it relates to their later achievement in mathematics. Students' mathematics-related beliefs tend to develop very early and remain fairly stable. Significant evidence suggests that traditional mathematics instruction is the main source of debilitating beliefs. However, weak instructional materials, such as inappropriate textbooks, are increasingly being implicated in the reinforcement of such beliefs. Thus, in attempting to help *all* students develop healthy or enabling mathematical dispositions, both institutional practices and resources must be appropriately addressed.

School of Education, UWI, St. Augustine