\textbf{ABSTRACT}

An attempt is made to classify permutation binomials of the type $ax^k + bx^j$, $ab \neq 0$, $i \leq j < k$.

In particular when $a = 1$ and $j = 1$, the binomials $x^k + bx$, $b \neq 0$ are studied. When $\gcd (k-1, q-1) = 1$, $f(x) = x^k + bx$, $b \neq 0$ does not permute $F_q$, and when $\gcd (k-1, q-1) = k-1$, $x^k + bx$ can be written in the form $x^{q+n-1} + bx$ for some integer $n|q-1$. There are necessary and sufficient conditions for these polynomials to permute $F_q$. But the case when $\gcd (k-1, q-1) \neq 1$, $k-1$, remains unresolved. Such polynomials sometimes permute $F_q$ and sometimes not, and a criterion for such polynomials to be permutation polynomials has not been obtained.

Necessary and sufficient conditions for binomials of the form $x^k \left( bx + ax \frac{q+n-1}{n} \right)$, $ab \neq 0$, $n|q-1$ to permute $F_q$ are obtained. Permutation binomials of degrees 7 and 9 are studied. The possible finite fields of prime power order permuted by binomials of degrees 7 and 9 are identified.
It is proved that the union of permutation binomials $H_q$ of $G_q$ and $K_q$ where $G_q = \{ ax + bx \frac{q+1}{2} \mid a, b \in F_q \}

\psi(a^2 - b^2) = 1 \text{ and } K_q = \{ cx^{q-2} + dx \frac{(q-3)/2}{2} \mid c, d \in F_q \}

\psi(c^2 - d^2) = 1 \}$ is a group, and that $G_q$ is normal in $H_q$.

The structure of $H_q$ is studied. Given a permutation binomial $ax + bx \frac{q+1}{2}$, $a, b \in F_q$, the permutation binomial $x^k \left( ax + bx \frac{q+1}{2} \right)$ can be expressed in the form $f(x^k)$ such that $(\ell, q-1) = 1$, $f(x) \in G_q$. Conversely, every permutation polynomial $f(x^k)$ where $f(x) \in G_q$, $\gcd(\ell, q-1) = 1$, can be written in the form $x^k \left( ax + bx \frac{q+1}{2} \right)$, $k$ an integer. The class of permutation binomials of the form $x^k \left( ax + bx \frac{q+1}{2} \right)$ form a group $U_q$.

It is seen that $G_q$ is normal in $H_q$ and $H_q$ is normal in $U_q$. 