ABSTRACT

Schur Multipliers and the Fourier Interpolation Problem

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The Schur-multiplier problem and the Fourier interpolation problem were found to be bounds for each other and in some specific cases a solution to one problem provides a solution to the other problem.

For any \(m \times n\) matrix \(T\), the Schur multiplier norm, denoted \(\|T\|_s\), is \(\max_{X \neq 0} \|T \circ X\| / \|X\|\) where \(\circ\) denotes the Schur product. In particular, for any \(n \times n\) Hankel matrix \(T_\mu\) with \(\mu = (\mu_{-n+1}, \mu_{-n+2}, \ldots, \mu_{n-1})\), the Schur-multiplier problem is to find the Schur multiplier norm of \(T_\mu\). Let

\[ B_\mu = \{ f \in L_1(\mathbb{R}) : \hat{f}(x_j) = \mu_j \text{ for } -n + 1 \leq j \leq n - 1 \} \]

where \(\hat{f}\) is the Fourier integral transform of \(f\), \(\{x_{-n+1}, x_{-n+2}, \ldots, x_{n-1}\}\) is a set of real numbers in arithmetic progression, and \(L_1(\mathbb{R})\) is the set of complex-valued Lesbesgue-integrable functions on \(\mathbb{R}\). Then the Fourier interpolation problem is to find \(\inf_{f \in B_\mu} \|f\|_1\). Thus, one connection between these two problems is that

\[ \|T_\mu\|_s \leq \inf_{f \in B_\mu} \|f\|_1. \]

In this thesis, necessary and sufficient conditions needed for equality in the 2-dimensional case are stated and proved along with necessary
conditions for the $n$-dimensional case and also an alternative proof to that presented by R. McEachin in [RMn].

In the 2-dimensional case, the analysis couples a hermitian unitary matrix $X$ with a unit vector $y$ that is in the zone of feasibility of $X$ to produce a distinguished Hankel $T_{\mu}$ consequently equating the two problems.

In the 3-dimensional case, the analysis uses a unit vector $y$ as pivot to construct a feasible pair $(X, y)$ and hence a distinguished $T_{\mu}$.

**Keywords:** Nadine McCloud; Schur multiplier; Fourier interpolation problem; hermitian unitary matrix; norming pair; feasible pair; distinguished matrix; zone of feasibility.